Profit-sharing rules and the taxation of multinational internet platforms

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Abstract

This paper analyzes taxation of an internet platform attracting users from different jurisdictions. When corporate income tax rates are different in the two jurisdictions, the platform distorts prices and outputs in order to shift profit to the low tax country. We analyze the comparative statics effects of an increase in the tax rate of one country. When cross effects are present in both countries, the platform has an incentive to increase the number of user in the high tax country and decrease the number of users in the low tax country. When externalities only flow from one market to another, an increase in the corporate tax rate results either in a decrease or an increase in the number of users in both countries depending on the direction of externalities. We compare the baseline regime of separate accounting (SA) with a regime of formula apportionment (FA), where the tax bill is apportioned in proportion to the number of users in the two countries. Under FA, an increase in the corporate tax rate increases the number of users in the low-tax country and decrease the number of users in the high-tax country. We use a numerical simulation to show that the high-tax country prefers SA to FA whereas the low-tax country prefers FA to SA.

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1 Introduction

Internet platforms often connect agents living under different fiscal jurisdictions. Facebook and Google users receive targeted advertising from companies headquartered outside their country of residence. Sellers and buyers on E-bay transact with agents living in different countries. Booking or Expedia users book flights and hotels all over the world. As users benefit from positive exernalities due to the presence of other users on the platform, the surplus created by the platform cannot easily be ascribed to a specific fiscal jurisdiction, raising difficult issues when the jurisdictions involved fix different corporate income tax rates. In this paper, our objective is precisely to analyze the effect of differences in corporate tax rates on the behavior of internet platforms operating in different jurisdictions.

The problem of profit-sharing across multiple jurisdictions is not specific to multinational internet platforms. Any multinational firm, operating and creating value in different countries (or different states in a federation), will face a similar problem. In order to analyze the global value chain, one needs to clearly describe the activities and assets of the firm, analyze the exact sequence of operations leading to value creation, and ascribe each operation to a specific jurisdiction. In practice, multinational firms have the capacity to select transfer prices across divisions located in different jurisdictions in order to minimize their tax bill. In order to limit these incentives to evade corporate income taxation, the OECD has launched a global program on Base Erosion and Profit Shifting (BEPS) with two actions specifically designed to address the tax challenges in the digital economy (Action 1) and align transfer pricing with value creation (Action 8) (OECD, 2016).

Different methods for allocating the profit of the platform to different jurisdictions have been proposed. If the multinational declares profits separately in the different jurisdictions (Separate Accounting), profit is apportioned according to the profit declared in the different jurisdictions. Alternatively, profit can be apportioned using a formula based on other indicators of the activity of the firm (Formula Apportionment). Formula apportionment is used to allocate profit for corporate income taxation across members of federal states (states in the United States, provinces in Canada, cantons in Switzerland). In the United States, the formula uses three indicators: sales, assets and wages. Formula appointment has been advocated in the European Union for corporate income taxation of large corporations operating in different members of the union.[1] Separate Accounting (SA) and Formula Apportionment (FA) create different incentives for a multinational facing different corporate income tax rates. Under SA, the multinational has an incentive to use transfer prices to shift profit away from the high tax jurisdiction to the low tax jurisdiction. Under FA, profit shifting is irrelevant, but the multinational has an incentive to distort its activities in order to reduce the value of the indicators in high tax jurisdiction and increase them in low tax jurisdictions. In effect, FA imposes a specific tax on each of the activities of the multinational which are used to determine the apportionment formula.

In this paper, we analyze the behavior of a multinational platform under the two different profit-sharing rules. In the Separate Accounting régime, profit is allocated according to the

[1] Devereux and Lorenz (2008) estimate that this would multiply the amount of taxes collected on multinational firms inside the European Union by 2.

declared profit in each jurisdiction. In the Formula Apportionment régime, profit is allocated according to the number of users in each jurisdiction.

We start by analyzing the situation where profits are declared in each jurisdiction (Separate Accounting). We show that, even in the absence of transfer prices, demand externalities across jurisdictions can be exploited to shift profit from the high-tax to the low-tax country. In response to an increase in the corporate tax rate in one of the countries, the platform has a direct incentive to shift profit by increasing the number of users in the high-tax country and decreasing the number of users in the low-tax country. This direct effect must however be balanced with an indirect effect: by increasing the number of users in the high tax country, the platform generates an increase in demand in the low tax country which can dominate the direct effect and result in the end in an increase (rather than a decrease) in the number of users in both countries. Similarly, by decreasing the number of users in the low tax country, the platform generates a decrease in demand in the high tax country which can dominate the direct effect and result in the end in a decrease in the number of users in both countries. We show that when markets are symmetric and tax rates are similar, the direct effect dominates the indirect effect so that an increase in the tax rate increases the number of users and decrease prices in the high-tax country and reduces the number of users and increases prices in the low-tax jurisdiction. When externalities only flow from one country to another, the direct effect of an increase in the tax rate on the number of users in the receiving country vanishes. Hence, an increase in the corporate tax rate results in an increase in the number of users in both jurisdictions (if externalities flow from the high tax to the low tax country) or a reduction in the number of users in both jurisdictions (if externalities flow from the low-tax to the high-tax country). We use a linear model to analyze further the effect of the role of asymmetries in externalities and in the sizes of jurisdictions on the behavior of the platform.

We then analyze a profit-sharing rule for peer-to-peer platform where profit is apportioned according to the number of users in jurisdiction (Formula Apportionment). We show that the direction of the direct effect of an increase in the corporate tax rate on the number of users in the two countries are reversed with respect to Separate Accounting. In order to lower its tax bill, the platform has an incentive to increase the number of users in the low-tax jurisdiction and decrease the number of users in the high-tax jurisdiction. Externalities across jurisdictions and the apportionment key give rise to an indirect effect: if the number of users in the low-tax country increases, demand in the high-tax country increases, pushing up the number of users in the high-tax country. We characterize two situations where the direct effect dominates the indirect effect around the uniform tax rate: when countries are symmetric and when externalities across jurisdictions disappear. In both these situations, an increase in the corporate tax rate results in a decrease in the number of users and an increase in price in the high-tax country and in an increase in the number of users and a decrease in price in the low-tax country.

We finally use a numerical simulation to compare profits and tax revenues of a peer-to-peer platform under Separate Accounting and Formula Apportionment. We first show that the comparative statics results obtained locally around identical tax rates are robust, and that the direct effect dominates the indirect effect for the entire range of tax rate values. Distortions in the number of users are stronger under FA than under SA, resulting in lower pre-tax profit.

However, the tax bill of the platform is higher under SA than under FA, so that post-tax profits are comparable in the two régimes. Tax revenues in the high-tax country are higher under SA than under FA, but tax revenues in the low-tax country are higher under FA than under SA, suggesting that the two countries will disagree on the profit-sharing régime.

The rest of the paper is organized as follows. We discuss the related literature in the next subsection. We present the model in Section 2. In Section 3, we analyze the benchmark model under Separate Accounting. The model with Formula Apportionment is discussed in Section 4. Section 5 presents the results of a numerical simulation comparing the two régimes. Section 6 concludes.

1.1 Relation to the literature

This paper is related to two different strands of the literature: the literature on taxation of two-sided platforms and the literature on formula apportionment. Optimal taxation of two-sided monopolistic platforms have been studied by Kind et al. (2008, 2009, 2010, 2013) and Bourreau, Caillaud and de Nijs (2018). The main focus of these papers is not on corporate income taxes but on unit and ad valorem taxes. The studies of Kind et al. (2008, 2009, 2010, 2013) have generated two main results. First, it shows that ad valorem taxes (like VAT) do not necessarily dominate unit taxes. The classical result in public finance on the domination of ad valorem taxes no longer holds for two-sided markets. Second, the price of a good may decrease with the ad valorem tax. The introduction of a tax on the value added for one side of the market can lead to a change in the entire business model of the platform. For example, the increase in VAT on the price of access for users could induce the platform to set a zero price for Internet access and switch all its revenues to the advertisers side. Bourreau, Caillaud and de Nijs (2018) supplement the model of the two-sided platform by considering data collection and letting consumers select the flow of data uploaded to the platform. They compare taxes levied on the flow of data uploaded by users with taxes paid by advertisers, and analyze the interaction between VAT and taxes based on the flow of data.

In a paper more closely related to our analysis, Kotsogiannis and Serfes (2010) address the issue of taxation with multi-sided market from the point of view of tax competition between countries. They consider competition between two countries that choose two tax instruments and the provision of local public goods, taking into account that each instrument is designed to attract both sides of a two-sided market, namely consumers and businesses. Consumers are located along a Hotelling segment, and two platforms are formed at both ends of the segment. Each firm chooses in which platform to go, and consumers choose to go on either platform based on the number of companies on each platform and the distance to the consumer platform. The time sequence of the model is as follows: the two jurisdictions first choose their levels of public good, and their level of taxation, and consumers and businesses simultaneously choose their platforms. Suppose that jurisdiction A provides more public goods that platform B. If the difference is large, vertical differentiation between platforms is important, and each platform specializes in a segment of the population. If the difference is small, competition between platforms is intense, and it is possible that all consumers and all businesses meet on a single platform. Comparative statics results show that an increase in externalities between the two sides of the market may lead to a decrease in the tax rate in both jurisdictions, an increase in the number of firms on platform A and a decrease in the number of firms on platform B. The model of Kotsogiannis and Serfes (2010) differs from ours in several respect. First they consider perfectly mobile users on the two sides of the market, second they assume that two platforms compete, one in each of the jurisdictions. Finally, they consider taxation on firms and consumers whereas we analyze corporate income taxes paid by the monopolistic platform.

The literature on formula apportionment started with a paper by Gordon and Wilson (1986). They show that the formula used in the United States, which puts positive weight on sales, wages and assets induces distortions in the optimal choice of inputs by the firms. In addition, it results in discriminatory treatment of companies in the same jurisdiction as they will face different effective tax rates. They advocate using an accounting system which replicates the separate accounting in each state. Anand and Sansing (2000) provide a clear account of the history of formula apportionment in the United States They analyze a model where two states bargain over the weights to place on different indicators and show that the weights placed on sales and inputs are typically inefficient in a decentralized equilibrium. Nielsen, Raimondos Moller and Schjelderup (2003) compare SA and FA in a model where transfer prices are used as a way to manipulate the behavior of a subsidiary in an oligopolistic market. Because transfer prices have an additional role in SA, where they allow for profit shifting, the incentives to manipulate are higher under SA than under FA. Kind, Midelfart and Schjelederup (2005) extend the model by considering a first stage of tax competition where two countries simultaneously select their corporate income tax rate to maximize fiscal revenues. The main result of the analysis shows that when transportation costs are low (countries are more integrated), equilibrium tax rates are higher under FA than under SA whereas the opposite holds when transportation costs are high. Nielsen, Raimondos-Moller et Schjelderup (2010) analyze capital investment decisions of a multinational under the two régimes of SA and FA. They analyze the effect of an increase in the tax rate on the capital accumulation of the multinational around symmetric tax rates, and show that the effect is higher for FA than for SA. The rationale is that capital levels not only affect the profit of the multinational but also the apportionment formula. This result echoes our result on output choices by the multinational two-sided platform, where FA leads to larger manipulations than SA. Finally, Gresik (2010) compares SA and FA when the production cost of the intermediate output is privately known by the multinational. Under FA, as transfer prices are irrelevant, the equilibrium tax rate only depends on the average production cost. Under SA, equilibrium tax rates are more complex as the multinational manipulates transfer prices. Gresik (2010) shows that equilibrium tax rates are higher under SA and that all firms prefer FA.

2 The model

We analyze the strategies of a monopolistic internet platform with activities in two jurisdictions with possibly different tax rates. The services provided by internet platforms are very diverse, ranging from online retailers intermediaries (such as Amazon, which connects customers and sellers, Booking, which connects customers and hotels), social media (such as Facebook, which allows users to be connected and connect advertisers to users), search engines (such as Google, which connect advertisers to users), collaborative and peer-to-peer platforms (such as E-bay, Meetic, Spotify, Airbnb, which connect users.) The simple model we introduce in the next section describes some of these situations but not all.

To clarify, let us distinguish platforms according to their users' types. Some platforms connect users of the same type residing in different jurisdictions, as in the case of peer-to-peer or collaborative platforms. Other platforms connect two types of users and the market is intrinsically two-sided, as in the case of advertisers and consumers or hotels and consumers. For two-sided platforms, we assume that users on the same side of the market are located in the same jurisdiction. This situation arises for example if customers only search for hotels located in a foreign country for if (mobile) advertisers all locate in a small low-tax country, whereas (immobile) consumers reside in a large high-tax country.

2.1 Utilities of users and pre-tax profit of the platform

We consider a monopolistic internet platform with users living in two separate jurisdictions, denoted A and B. Users is a generic term, which represents different types of agents according to the specific platform. We do not distinguish between types of users and only track down the total number of users in the two jurisdictions, x_A and x_B. We suppose that the platform follows a business model whereby all users pay a fixed fee (independent of usage) to access the platform. The platform can discriminate according to the residence of users, and charges a fee p_A for users in country A and a fee p_B for users in country B. The volume of use of the platform is supposed to be fixed and identical across users. In each jurisdiction, the utility of users is the sum of two components: an idiosyncratic utility for the platform, which is heterogeneous across users, and a positive externality term which depends on the number of other users in the platform, distinguishing between users in jurisdictions A and B. Formally, the utilities of users in the two jurisdictions are given by $u_B(x_A, x_B)$, nondecreasing in A and nonincreasing in x_B.

$$\begin{aligned} U_A &= \theta_A + u_A(x_A, x_B) - p_A, \\ U_B &= \theta_B + u_B(x_A, x_B) - p_B \end{aligned}$$

where θ_A is distributed according to a continuous distribution with full support F_A on $[\underline{\theta}, \overline{\theta}]$, and θ_B is distributed according to a continuous distribution with full support F_B on $[\underline{\eta}, \overline{\eta}]$.

Externalities across jurisdictions are always nonnegative: u_A is weakly increasing in x_B and u_B is weakly increasing in x_A. Externalities arising from the participation of users in the same jurisdiction can either be positive or negative. To illustrate let us map out the model with some examples.

1- Peer-to-peer platforms: The users only care about a weighted total number of users: The externality for users in A and in B are described by weakly increasing functions $u_A(x_A + bx_B)$ and $u_B(ax_A + x_B)$, where a and b represents the weight placed on the users from abroad. There are positive externalities both across and within jurisdictions.

2- Social media and search engines. Users are in A and advertisers in B. Users of a social media are positively affected by the number of users of the media: there are positive externalities within A. If they do not care about advertising, there are no externalities from B to A: The externality for users in A is described by an increasing function $u_A(x_A)$. As for the advertisers, they benefit positively from a large number of users, but negatively from other advertisers, due to a competition effect which is not modeled explicitly: The externality $u_B(x_A, x_B)$ is weakly increasing in x_A and weakly decreasing in x_B. Externalities only flow from jurisdiction A to jurisdiction B. For a search engine, the model is similar, but with no externalities among users within A.

3- Online retailers and online intermediaries. Customers reside in A and suppliers in B. There are positive externalities across jurisdictions but negligible externalities within each country: they are described by weakly increasing functions $u_A(x_B)$ and $u_B(x_A)$.

We now derive the demand associated to (p_A, p_B). Assume that users in country A have an expectation x_B over the number of users in country B. There is an indifferent consumer given by

$$\widehat{\theta_A} = p_A - u_A(x_A, x_B).$$

provided $p_A - u_A(x_A, x_B)$ is in the support of F_A; otherwise take $\widehat{\theta_A}$ at one of the extreme values. and similarly in B. We normalize the measure of users in jurisdiction B to 1, and let s denote the measure of users in jurisdiction A. Assuming rational expectations on the participation decisions of users, the demand thus satisfies

$$x_A = s(1 - F_A(p_A - u_A(x_A, x_B))),$$

Similarly,

$$x_B = 1 - F_B(p_B - u_B(x_A, x_B)).$$

There is a one-to-one relationship between the prices chosen by the monopolistic platform p_A and p_B and the number of users x_A and x_B. As argued by Weyl (2010), it will prove easier to write the profit in terms of numbers of users instead of prices.[2] The interpretation is that the platform chooses x_A and x_B, knowing the prices for which the numbers of users will be x_A and x_B. From above, the prices are[3]

$$P_A(x_A, x_B) = u_A(x_A, x_B) + F_A^{-1}(1 - \frac{x_A}{s}), \tag{1}$$

$$P_B(x_A, x_B) = u_B(x_A, x_B) + F_B^{-1}(1 - x_B). \tag{2}$$

$P_A(x_A, x_B)$ is always increasing in x_B. We suppose that externalities are not too strong, so that the price function $P_A(x_A, x_B)$ is decreasing in x_A. This assumption is satisfied if externalities are non-positive within A (u_A is decreasing in x_A) or when they are positive but with a slope which is sufficiently low relative to the distribution F ($\partial u_A/\partial x_A.F'(1-x_A) \leq 1$) Similarly,

[2]Such a construction does not work for competitive platforms, as in the case of a single market where competition in prices (Bertrand) leads to different results than competition in quantities (Cournot). See Belleflamme and Toulemonde (2016) for a study of two platforms competing in prices.

[3]The price functions are akin to inverse demand functions, with some subtleties: due to coordination issues, a given couple of prices (p_A, p_B) could lead to different demands Here we always select the largest demands.

in jurisdiction B the price function $P_B(x_A, x_B)$ increasing in x_A and assumed to be decreasing in x_B.

The user surplus in jurisdiction A is computed as

$$\begin{aligned} CS_A &= \int_{p_A - u(x_A, x_B)}^{\underline{\theta}} [\theta + u_A(x_A, x_B) - p_A] f(\theta) d\theta, \\ &= \int_{F_A^{-1}(1-x_A)}^{\underline{\theta}} [\theta - F_A^{-1}(1 - x_A)] f(\theta) d\theta \end{aligned}$$

Hence, taking into account the participation decision of the users, the user surplus in jurisdiction A can be written as a function only of the number of participants in jurisdiction A, x_A. Furthermore, it is easy to check that

$$\begin{aligned} \frac{\partial CS_A}{\partial x_A} &= \int_{F_A^{-1}(1-x_A)}^{\underline{\theta}} [\frac{1}{f[F_A^{-1}(1 - x_A)]} d\theta, \\ &= \frac{1 - F_A^{-1}(1 - x_A)}{f[F_A^{-1}(1 - x_A)]}, \\ &> 0. \end{aligned}$$

Hence, as intuition suggests, an increase in the number of participants x_A results in an increase in user surplus. Similarly, we obtain

$$CS_B = \int_{F_B^{-1}(1-x_B)}^{\underline{\theta}} [\theta - F_B^{-1}(1 - x_B)] f(\theta) d\theta,$$

and

$$\frac{\partial CS_B}{\partial x_B} = \frac{1 - F_B^{-1}(1 - x_B)}{f[F_B^{-1}(1 - x_B)]} > 0.$$

We suppose, following empirical evidence, that the operating costs of the platform are negligible so that the pre-tax profit in each jurisdiction is given by

$$\begin{aligned} V_A &= x_A P_A(x_A, x_B), \\ V_B &= x_B P_B(x_A, x_B) \end{aligned}$$

and the total pre-tax profit as

$$V = V_A + V_B.$$

The choices of the platform will depend on how marginal revenues in a jurisdiction behave with respect to the number of users in the other country, i.e. on the signs of $\frac{\partial^2 V_A}{\partial x_A \partial x_B}$. We have

$$\frac{\partial^2 V_A}{\partial x_A \partial x_B} = \frac{\partial P_A}{\partial x_B} + x_A \frac{\partial^2 P_A}{\partial x_A \partial x_B}.$$

The first term $\frac{\partial P_A}{\partial x_B}$ represents the positive marginal effect of the number of users in B on the price in A. The cross-derivative is positive when this effect does not decrease too rapidly with x_A, more precisely when the elasticity of $\frac{\partial P_A}{\partial x_B}$ with respect to x_A is larger than -1.

We relate this condition to the fundamentals in country A. Using (1) when the market is not covered, we obtain

$$\frac{\partial^2 V_A}{\partial x_A \partial x_B} = \frac{\partial u_A}{\partial x_B} + x_A \frac{\partial^2 u_A}{\partial x_A \partial x_B}.$$

The first term $\frac{\partial u_A}{\partial x_B}$ is positive due to positive cross-externalities across jurisdictions . We will say that the two markets are *complements* (respectively *substitutes*) when both $\frac{\partial^2 V_A}{\partial x_A \partial x_B}$ and $\frac{\partial^2 V_B}{\partial x_A \partial x_B}$ are non-negative (respectively non-positive).

The case of market complements is the most natural case. Markets are complements when the cross derivatives $\frac{\partial^2 u_A}{\partial x_A \partial x_B}$ and $\frac{\partial^2 u_B}{\partial x_A \partial x_B}$ are non-positive. Markets are complements when the functions u_A and u_B are linear. Markets are complements when there are no externalities inside jurisdictions, so that $\frac{\partial^2 u_A}{\partial x_A} = 0$. In the peer-to-peer model, where $u_A(x_A, x_B) = u(x_A + bx_B)$, markets are complements when the elasticity of the function u is smaller than 1.[4]

More generally, markets are complements when the marginal cross-externality in market A, $\frac{\partial u_A}{\partial x_B}$, increases or decreases with the number of users in A with an elasticity less than 1.

For markets to be substitutes, the marginal cross-externality $\frac{\partial u_A}{\partial x_B}$ must decrease with the number of users in A at a very fast rate, with an elasticity greater than 1.

2.2 Separate Accounting and Formula Apportionment

We suppose that the two jurisdictions charge corporate income tax rates t_A and t_B. We consider two regimes of profit sharing. Under Separate Accounting (SA), the platform pays taxes according to the profit declared in each jurisdiction. The post-tax profit of the platform is then given by

$$\Pi = (1 - t_A)V_A + (1 - t_B)V_B.$$

and the fiscal revenues of the two countries are computed as

[4] To see this, note that

$$\frac{\partial^2 V_A}{\partial x_A \partial x_B} = u'(x_A + bx_B) + x_A u''(x_A + bx_B)$$

Since u is concave,

$$\frac{\partial^2 V_A}{\partial x_A \partial x_B} \geq u'(x_A + bx_B) + (x_A + bx_B) u''(x_A + bx_B)$$

Thus the cross-derivative is positive if the elasticity of u', $-x\frac{u''}{u'}$, is less than 1.

$$R_A = t_A V_A,$$
$$R_B = t_B V_B.$$

Under Formula Apportionment (FA), the total profit of the platform, V is attributed to each jurisdiction using the ratio of users, so that the post-tax profit is given by

$$\Pi = V[1 - t_A \frac{x_A}{x_A + x_B} - t_B \frac{x_B}{x_A + x_B}],$$

and the fiscal revenues of the two countries are computed as

$$R_A = t_A V \frac{x_A}{x_A + x_B},$$
$$R_B = t_B V \frac{x_B}{x_A + x_B}.$$

Formula apportionment only makes sense when the apportionment key uses the same type of users in the two countries. Hence FA is an appropriate profit-sharing regime for peer-to-peer platforms, but not for two-sided platforms, where different types of users reside in different jurisdictions.

3 Separate accounting

Out objective in this Section is to compute the optimal choice of the platform under SA for a fixed choice of tax rates (t_A, t_B) and analyze the comparative statics effects of an increase in one of the corporate tax rates, t_A. We first derive the first-order conditions for an (interior) choice of the number of users by the platform. Because of externalities across jurisdictions, the optimal choices of the number of users in countries A and B are interdependent. Assuming that the post-tax profit function is concave in x_A and x_B, the solution to the problem is given by the two equations:

$$(1 - t_A)\frac{\partial V_A}{\partial x_A} + (1 - t_B)\frac{\partial V_B}{\partial x_A} = 0, \quad (3)$$
$$(1 - t_A)\frac{\partial V_A}{\partial x_B} + (1 - t_B)\frac{\partial V_B}{\partial x_B} = 0. \quad (4)$$

Let $\phi_A(x_B; t_A, t_B)$ and $\phi_B(x_A; t_A, t_B)$ be the implicit functions defined by equations (3) and (4). As in a game between two players, the optimal choice of the platform is obtained at the intersection of these two "reaction functions". A simple computation using equations (3) and (4) shows that

$$\frac{\partial\phi_A}{\partial x_B} = -\frac{(1-t_A)\frac{\partial^2 V_A}{\partial x_A \partial x_B} + (1-t_B)\frac{\partial^2 V_B}{\partial x_A \partial x_B}}{\frac{\partial^2 \Pi}{\partial x_A^2}},$$

$$\frac{\partial\phi_B}{\partial x_A} = -\frac{(1-t_A)\frac{\partial^2 V_A}{\partial x_A \partial x_B} + (1-t_B)\frac{\partial^2 V_B}{\partial x_A \partial x_B}}{\frac{\partial^2 \Pi}{\partial x_B^2}}$$

Hence the signs of the slopes of ϕ_A and ϕ_B depend on the cross-derivative $\frac{\partial^2 V_A}{\partial x_A \partial x_B}$. If the markets are complements, the reaction functions are increasing. When the number of users in one country increases, the optimal number of users in the other country increases. If, on the other hand, the markets are substitutes, the two reaction functions are decreasing. An increase in the number of users in one country leads the platform to decrease the number of users in the other country.

We next compute the comparative statics effects of a change in the corporate tax rate t_A on the optimal choices of the platform denoted X_A and X_B. A straightforward computation shows that

$$\frac{dX_A}{dt_A} = \frac{\frac{\partial\phi_A}{\partial t_A} + \frac{\partial\phi_A}{\partial x_B}\frac{\partial\phi_B}{\partial t_A}}{1 - \frac{\partial\phi_A}{\partial x_B}\frac{\partial\phi_B}{\partial x_A}},$$

$$\frac{dX_B}{dt_A} = \frac{\frac{\partial\phi_B}{\partial t_A} + \frac{\partial\phi_B}{\partial x_A}\frac{\partial\phi_A}{\partial t_A}}{1 - \frac{\partial\phi_A}{\partial x_B}\frac{\partial\phi_B}{\partial x_A}}.$$

Because the post-tax profit is assumed to be concave, the Hessian is negative semi-definite and the determinant $1 - \frac{\partial\phi_A}{\partial x_B}\frac{\partial\phi_B}{\partial x_A}$ is always positive. Hence the sign of the effect of a change in t_A on the number of users in country $i = A, B$, X_i, is the same as the sign of

$$\frac{\partial\phi_i}{\partial t_A} + \frac{\partial\phi_i}{\partial x_j}\frac{\partial\phi_j}{\partial t_A}.$$

We can thus decompose the effect of an increase in the tax rate t_A on the number of users in country $i = A, B$ into (i) a direct effect $\frac{\partial\phi_i}{\partial t_A}$ and (ii) an indirect effect linked to the change in the number of users in the other country, $\frac{\partial\phi_i}{\partial x_j}\frac{\partial\phi_j}{\partial t_A}$.

We use the first order conditions (3) and (4) to compute the direct effects

$$\frac{\partial\phi_A}{\partial t_A} = \frac{\frac{\partial V_A}{\partial x_A}}{\frac{\partial^2 \Pi}{\partial x_A^2}},$$

$$\frac{\partial\phi_B}{\partial t_A} = \frac{\frac{\partial V_A}{\partial x_B}}{\frac{\partial^2 \Pi}{\partial x_B^2}}.$$

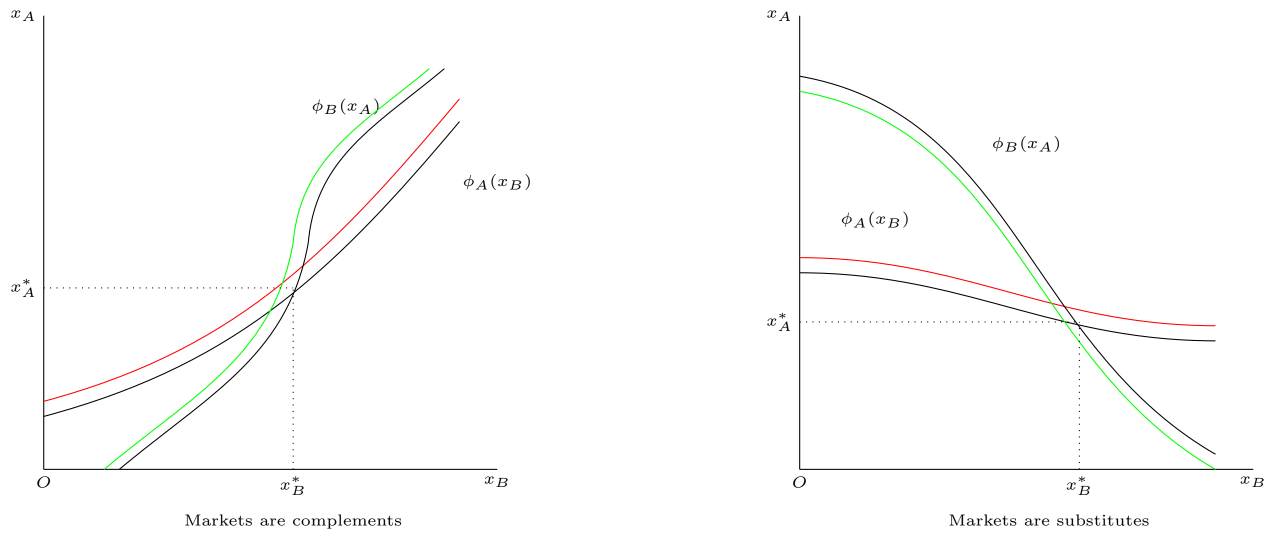

Figure 1: Effect of an increase in t_A under SA

We note that *direct effect of an increase in the corporate tax rate t_A is positive on the number of users in jurisdiction A and negative on the number of users in jurisdiction B.* To see this, observe first that, by concavity, $\frac{\partial^2 \Pi}{\partial x_A^2}$ and $\frac{\partial^2 \Pi}{\partial x_B^2}$ are both negative. Because externalities across jurisdictions are positive, $\frac{\partial V_B}{\partial x_A} = x_B \frac{\partial P_B}{\partial x_A} > 0$, we have at the optimum,

$$\frac{\partial V_A}{\partial x_A} = -\frac{1-t_B}{1-t_A}\frac{\partial V_B}{\partial x_A} < 0.$$

Hence, $\frac{\partial \phi_A}{\partial t_A} > 0$. and $\frac{\partial \phi_B}{\partial t_A} < 0$, leading to the conclusion that the direct effect of an increase in t_A is positive on x_A and negative on x_B.

To understand the signs of the direct effects, recall that, when t_A increases, the platform has an incentive to shift profit from jurisdiction A to jurisdiction B. To do so, the platform should increase the number of users in jurisdiction A, thereby increasing profits V_B in the low-tax jurisdiction through the positive externality of users of jurisdiction A on the price P_B. Similarly, because of positive externalities, the platform should reduce the number of users in jurisdiction B in order to reduce the price P_A and hence the profits V_A in the high-tax jurisdiction.

Figure 1 illustrates the effect of a change in the corporate tax rate t_A on the two reaction functions ϕ_A and ϕ_B when markets are complements (left panel) and substitutes (right panel). The curves in black depict the functions ϕ_A and ϕ_B when the tax rates are identical. The intersection of the two curves gives the optimal numbers of users. The curves in red and green describe the same functions following an increase in t_A. The new choices of the platform are given by the intersection of the red and green curves. In both cases, an increase in t_A leads to an increase in ϕ_A and a decrease in ϕ_B. When markets are complements, the direct and indirect effects work in opposite directions: the direct effect of an increase in t_A on x_A (respectively x_B) is positive (respectively negative), whereas the indirect effect is negative (respectively positive).

Which of the two dominates depends on the exact specifications of the model. When, on the other hand, markets are substitutes, direct and indirect effects work in the same direction. An increase in t_A unambiguously results in an increase in the number of users in jurisdiction A and a decrease in the number of users in jurisdiction B.

When the reaction functions are decreasing or when the direct effect dominates the indirect effect, an increase in t_A results in an increase in x_A and a decrease in x_B. Hence the price in market A, P_A goes down while the price in market B, P_B, goes up. When the tax rate t_A is sufficiently high, market A will end up being covered, and the fee p_A will converge to zero. The platform eventually chooses to extract revenues from users in the low-tax jurisdiction, while not charging any fee to participants in the high-tax jurisdiction. This corresponds to a business model where the platform only charges participants on one side of the market, as in the case of search engines and digital social networks.

We also immediately observe that an increase in the corporate tax rate t_A always results in a reduction in the profit of the platform. By the envelope theorem, only the direct effect of t_A on the post-tax profit matters, and this direct effect is given by $\frac{\partial \Pi}{\partial t_A} = -V_A$ and is always negative.

Consider next the effect of an increase in t_A on the tax revenues of country A,

$$\begin{aligned} \frac{\partial R_A}{\partial t_A} &= V_A + \frac{\partial V_A}{\partial t_A}, \\ &= V_A + \frac{\partial V_A}{\partial x_A}\frac{\partial x_A}{\partial t_A} + \frac{\partial V_A}{\partial x_B}\frac{\partial x_B}{\partial t_A} \end{aligned} \qquad (5)$$

An increase in the tax rate t_A has two effects on tax revenues: a positive price effect (measured by the first term V_A) and an effect on the tax base (measured by the second term $\frac{\partial V_A}{\partial x_A}\frac{\partial x_A}{\partial t_A} + \frac{\partial V_A}{\partial x_B}\frac{\partial x_B}{\partial t_A}$). When the markets are substitutes, or when markets are complements and the direct effect dominates the indirect effect, x_A is increasing in t_A and x_B decreasing in t_A, so that the effect on the tax base is negative. An increase in the corporate tax rate then has an ambiguous effect on the tax revenues of country A.

Similarly, we compute the effect of an increase in t_A on the tax revenues of country B as

$$\frac{\partial R_B}{\partial t_A} = \frac{\partial V_B}{\partial t_A} = \frac{\partial V_B}{\partial x_A}\frac{\partial x_A}{\partial t_A} + \frac{\partial V_B}{\partial x_B}\frac{\partial x_B}{\partial t_A}.$$

The price effect disappears and the only effect is the effect on the tax base. When the direct effect dominates, x_A is increasing in t_A and x_B decreasing in t_A, so that the effect on the tax base is positive. An increase in the corporate tax rate in country A increases the tax base in country B, resulting in an increase in the tax revenues of country B.

We now investigate in detail the magnitude of the direct and indirect effects in two simple situations: when externalities are one-sided and when the two countries are symmetric, and one country contemplates increasing the corporate tax rate above a common tax rate t.

3.1 One-sided externalities

Suppose that externalities are one-sided. Consider first the case where users in jurisdiction A do not benefit from the presence of users in jurisdiction B on the platform, $\frac{\partial P_A}{\partial x_B} = 0$. The platform's profits in country A do not depend on the number of users in country B $\frac{\partial V_A}{\partial x_B} = 0$ so that a change in the corporate tax rate t_A has no direct effect on the choice of the number of users in country B, $\frac{\partial \phi_B}{\partial t_A} = 0$. In that case, a change in the tax rate t_A only has a direct effect on the number of users in country A, and only an indirect effect on the number of users in country B. Hence an increase in t_A always leads to an increase in the number of users in country A, an increase in the number of users in country B when markets are complements and a decrease in the number of users in country B when markets are substitutes.

Conversely, suppose that users in jurisdiction B fo not benefit from the presence of users in jurisdiction A on the platform, $\frac{\partial P_B}{\partial x_A} = 0$. Then the number of users in country B does not affect the profits in country A, and $\frac{\partial V_A}{\partial x_B} = 0$. But then, the optimal choice of the platform in country A does not depend on the number of users in country B, and by condition (3), $\frac{\partial V_A}{\partial x_A} = 0$. This implies that a change in the corporate tax rate has no effect on the number of users in country A, $\frac{\partial \phi_A}{\partial t_A} = 0$. Hence an increase in the corporate tax rate t_A only has a direct effect on the number of users in country B, and an indirect effect on the number of users in country A. An increase in the corporate tax rate t_A thus always results in a decrease in the number of users in country B, a decrease in the number of users in country A when markets are complements, and an increase in the number of users in country A when markets are substitutes. We summarize in the following Proposition

Proposition 1 *(one-sided) Suppose that externalities only flow from market A to market B. An increase in the corporate tax rate t_A always results in an increase in the number of users in country A. It results in an increase in the number of users in country B if markets are complements and a decrease in the number of users in country B if markets are substitutes. Conversely, if externalities only flow from market B to market A, an increase in the corporate tax rate t_A always results in a decrease in the number of users in country B. It results in a decrease in the number of users in country A if markets are complements and an increase in the number of users in country A when markets are substitutes.*

3.2 Symmetric countries

Consider a situation where demands are symmetric, $P_A(x_A, x_B) = P_B(x_B, x_A)$ for all x_A, x_B, and tax rates are identical, $t_A = t_B = t$. At that point, by symmetry, the direct effects of an increase in the corporate tax rate t_A on the number of users in the two countries have opposite signs, but the same magnitude. Furthermore, concavity implies that the slope of the reaction functions ϕ_A and ϕ_B with respect to x_B and x_A are smaller than 1. Hence when markets are complements, the direct effect of a change in the corporate tax rate t_A always dominates the indirect effect. We summarize this finding below.

Proposition 2 *Suppose that countries are symmetric and $t_A = t_B = t$. An increase in the corporate tax rate t_A always results in an increase in the number of users in country A and a decrease in the number of users in country B.*

3.3 A linear model

When externalities are two-sided and the markets are not symmetric, the comparison of the direct and indirect effects becomes intractable. In order to make progress, and study in particular the optimal choices of the platform for any possible values of t_A and t_B, (and not only when the two tax rates are close to each other), we assume the externalities functions, u_A and u_B, to be linear in the number of users and the distributions of the parameters to be uniform over $[0, 1]$. Under these assumptions, the price functions are linear. Specifically, externalities utilities are given by

$$u_A(x_A, x_B) = ax_A + \beta x_B \text{ and } u_B(x_A, x_B) = bx_B + \alpha x_A.$$

The parameters α and β are non-negative, reflecting the cross -externalities from A to B and B to A. The parameters a and b reflect the externalities within jurisdictions A and B respectively, hence can be either sign, as shown in the illustrations. Recall that the size of market B is normalized to 1 and s measures the size of market A relative to market B. The numbers of users x_A and x_B satisfy the constraints: $0 \leq x_A \leq s$ and $0 \leq x_B \leq 1$. A market is said to be 'covered' when the upper-bound is reached, meaning that all users in the jurisdiction participate in the platform: A is covered if $x_A = s$ and B is covered if $x_B = 1$.

Following the computations presented in Section 2.1, the inverse demand functions for $x_A \leq s$ and $x_B \leq 1$ are given by:

$$P_A(x_A, x_B) = 1 - \sigma_A x_A + \beta x_B \text{ with } \sigma_A = \frac{1}{s} - a \quad (6)$$

$$P_B(x_A, x_B) = 1 - \sigma_B x_B + \alpha x_A \text{ with } \sigma_B = 1 - b. \quad (7)$$

The parameter σ_A measures the sensitivity of the price in jurisdiction A to the number of users in jurisdiction A, and similarly for σ_B. These parameters are positive, as we assume the price in a jurisdiction to be decreasing in the number of users in that jurisdiction. According to the above expressions, the sensitivity is decreasing both in the size of the market and in the externalities within the jurisdiction.

Effect of tax distortions on output The optimal choices of the platform depend on the tax levels only through the ratio $\rho = \frac{1-t_B}{1-t_A}$. The ratio ρ is increasing in t_A. We let A denote the high-tax country so that $\rho \geq 1$. In line with the evidence on corporate tax rates, country A is likely to be larger than country B, so that we assume $s \geq 1$, although the next proposition does not require this assumption.

Following the approach introduced in Section 3, we compute the optimal number of users in each jurisdiction for a fixed number of users in the other, i.e. the functions ϕ_A and ϕ_B. For a fixed x_B, the profit is concave in x_A. Thus, given ρ and x_B, the optimal number of users in A, $\phi_A(x_B)$, is given by

$$\begin{aligned}\phi_A(x_B) &= \frac{1}{2\sigma_A}[1+(\rho\alpha+\beta)x_B] \text{ if it is less than } s, \\ &= s \text{ otherwise.}\end{aligned} \tag{8}$$

Similarly, given ρ and x_A, the optimal number of users in B, $\phi_B(x_A)$, is given by

$$\begin{aligned}\phi_B(x_A) &= \frac{1}{2\sigma_B}[1+\frac{1}{\rho}(\rho\alpha+\beta)x_A] \text{ if it is less than } 1, \\ &= 1 \text{ otherwise.}\end{aligned} \tag{9}$$

The functions $\phi_A$and ϕ_B are non-decreasing, as expected from the general analysis since markets are complements.

The numbers of users at the intersection of the two curves, abstracting from the bounds, are given by $(X_A(\rho), X_B(\rho))$ where

$$X_A(\rho) = \frac{1}{2\sigma_A}[\frac{1+\frac{\rho\alpha+\beta}{2\sigma_B}}{1-\frac{(\rho\alpha+\beta)^2}{4\rho\sigma_A\sigma_B}}], \tag{10}$$

$$X_B(\rho) = \frac{1}{2\sigma_B}[\frac{1+\frac{1}{\rho}\frac{\rho\alpha+\beta}{2\sigma_A}}{1-\frac{(\rho\alpha+\beta)^2}{4\rho\sigma_A\sigma_B}}]. \tag{11}$$

These numbers are admissible when they are respectively within $[0, s]$ and $[0, 1]$.

Consider first the lower bounds. The numbers are negative when $1-\frac{(\rho\alpha+\beta)^2}{4\rho\sigma_A\sigma_B}$ is negative. This case arises when the profit is not concave: Since $\frac{\partial^2\pi}{\partial^2 x_A} = 2\sigma_A$, $\frac{\partial^2\pi}{\partial^2 x_B} = 2\rho\sigma_B$, and $\frac{\partial^2\pi_A}{\partial x_A \partial x_B} = \rho\alpha+\beta$, the condition on the cross derivatives for the concavity of profit $\frac{\partial^2\pi}{\partial^2 x_A}\frac{\partial^2\pi}{\partial^2 x_B} \geq \frac{\partial^2\pi_A}{\partial x_A \partial x_B}$ is equivalent to $(\rho\alpha+\beta)^2 < 4\rho\sigma_A\sigma_B$. In that case, since there are always incentives to have positive numbers in each country, the optimal choice is to cover at least one market, as described below. Such a situation surely arises for ρ large enough if there are externalities, i.e. if α or β is positive.

Consider now the upper bounds. Assume that the profit function is concave. The numbers $X_A(\rho)$ and $X_B(\rho)$ are positive. When both are within they are below the upper bound, they are the optimal choices (due to the concavity of Π). When one or both numbers is greater than the maximum market shares, surely at least one market is covered. (This is also true if the profit function is not concave.)

When market A is the only covered market, the optimal number of users in B is given by $\phi_B(s)$ which is less than 1. Hence

$$X_A^*(\rho) = s, \text{ and } X_B^*(\rho) = \frac{1}{2\sigma_B}[1+\frac{1}{\rho}(\rho\alpha+\beta)s] \text{ if it is less than 1.} \tag{12}$$

Similarly, when market B is the only covered market, the optimal number of users in A satisfy

$$X_A^*(\rho) = \frac{1}{2\sigma_A}[1 + (\rho\alpha + \beta)] \text{ if it is less than } s, \text{ and } X_B^*(\rho) = 1. \tag{13}$$

Both markets are fully covered

$$X_A^*(\rho) = s \text{ and } X_B^*(\rho) = 1. \tag{14}$$

For large enough ρ one of the markets needs to be covered. We show that market A is necessarily covered: If B were fully covered, the optimal number x_A in A given by (13), would increase with ρ and eventually reach s.

The following proposition describes the optimal platform's choice as a function of the externalities' parameters, α and β, the sensitivity parameters σ_A and σ_B, and the tax distortion ρ. The proposition considers the situation where the platform's profit is concave and the platform does not cover any market when it faces no distortion, $\rho = 1$.

Proposition 3 *Assume* $0 < X_A(1) < s$ *and* $0 < X_B(1) < 1$. *Let* ρ_A *be the minimum value of* ρ, $\rho > 1$ *for which* $X_A(\rho) \geq s$ *and* ρ_B *be the minimum one for which* $X_B(\rho) \leq 1$. *The optimal number of users in the two jurisdictions* $(X_A^*(\rho), X_B^*(\rho))$ *is characterized as follows.*

For $\rho < \min\{\rho_A, \rho_B\}$, *none of the markets is fully covered and the optimal quantities are given by:* $X_A^*(\rho) = X_A(\rho), X_B^*(\rho) = X_B(\rho)$.

For $\rho \geq \min\{\rho_A, \rho_B\}$, *two configurations arise when* ρ *increases:*

1. $\rho_A \leq \rho_B$. *Then for any* $\rho \geq \rho_A$, *market* A *is fully covered, but not market* B, *with numbers of users given by (12): market* B *is never covered.*

2. $\rho_A > \rho_B$. *Then, for values larger than* ρ_B, *market* B *is first fully covered, but not market* A, *with numbers of users given by (13) until both markets are covered, and, finally, only market* A *is covered if* $\frac{1}{2\sigma_B}[1 + \frac{1}{\rho}(\rho\alpha + \beta)s] < 1$, *which happens when* $1 + \alpha s < 2\sigma_B$.

Proposition 3 characterizes the optimal choices of the platform under different parameter configurations. When the difference in tax rates is sufficiently small, none of the markets are covered and the optimal number of users is computed as $X_A(\rho)$ and $X_B(\rho)$. When the difference in tax rates becomes large, one of the markets ends up being fully covered. Which market becomes fully covered first depends on the parameters (however, for large enough ρ market A is necessarily covered).

If ρ_A is smaller than ρ_B, market A is the first market to be covered. In that case, an increase in ρ unambiguously decreases X_B. The reasoning is the following: in order to shift profit from market A to market B, the platform can no longer increase the number of users in market A and will only reduce the number of users in market B, which is too large with respect to the efficient level in B, i.e. the level maximizing the sole profit in B, given that $x_A = s$.

If ρ_A is larger than ρ_B, market B is the first market to be covered. When ρ increases, the platform increases the number of users in platform A to shift profit towards B. This eventually leads to both markets being fully covered for ρ sufficiently large. Then two situations may arise, as explained below. For ρ large enough, the platform seeks to maximize the profit in jurisdiction B, given full coverage of market A. The platform has an incentive to reduce coverage in market B, if covering market B given $x_A = s$ is not efficient, i.e. it does not maximize the profit in B. In that case, when ρ becomes large enough, the platform reduces coverage in B, whereas market A is fully covered. If on the other hand, given that market A is fully covered, the platform would choose to cover market B in order to maximize the profit in B, then both markets end up being fully covered in equilibrium.

An inspection of the optimal choices shows that they may be increasing or decreasing in ρ depending on the parameters, and the type of equilibrium. Their behavior is easy to analyze when at least one market is covered, as discussed above. This occurs when the differences in taxes are large enough so that ρ is large enough. When instead the difference in tax levels is small, no market is covered, and the effect of an increase in ρ is more complex due to the presence of direct and indirect effects. To illustrate this point, consider the impact of an increase in ρ when ρ is close to one. Easy computations show that:

$$\begin{aligned}\frac{X_A'(1)}{X_A(1)} &= \frac{\alpha}{2+\alpha+\beta} + \frac{\alpha^2-\beta^2}{4\sigma_A\sigma_B-(\alpha+\beta)^2},\\ \frac{X_B'(1)}{X_B(1)} &= -\frac{\beta}{2+\alpha+\beta} + \frac{\alpha^2-\beta^2}{4\sigma_A\sigma_B-(\alpha+\beta)^2}.\end{aligned}$$

As expected, it is never optimal for the platform to increase the number of users in B and decrease the number of users in A simultaneously.

When externalities are symmetric, $(\alpha = \beta)$, an increase in ρ results in an increase in X_A and a decrease in X_B, as expected from the previous analysis.

Consider now non-symmetric externalities. Recall that the denominator $4\sigma_A\sigma_B - (\alpha+\beta)^2$ is assumed to be positive. When externalities only flow from A to B, $\beta = 0$ and $\alpha > 0$, an increase in ρ results in an increase in both X_A and X_B, as seen in the model with one-sided externalities. The result extends to the case where externalities are sufficiently strong from A to B relative to B to A, in particular when $\alpha > \beta$.

When externalities only flow from market B to A, $\alpha = 0$ and $\beta > 0$, an increase in ρ results in a decrease in both X_A and X_B, i.e. in a reduction in the number of users in both countries. The result extends to the case where externalities are sufficiently strong from B to A relative to A to B, $\alpha < \beta$, and the prices' sensitivities $\sigma_A\sigma_B$ are small enough. An increase in ρ results in a reduction in the number of users in both countries. Let us relate the sensitivity parameters to the externalities inside each country by (6) and (7), $\sigma_A = \frac{1}{s} - a$ and $\sigma_B = 1 - b$. Without externalities within jurisdictions, $a = b = 0$, the larger the size of the high tax rate country A, the more likely an increase in its tax level leads to a reduction of the number of users in both markets. This could correspond to the example 3 of section 2.1 with an online intermediary connecting tourists in the large and high tax rate country A and hotels in the small and low tax rate country B. Externalities are positive across markets, and negligible within markets.

4 Formula apportionment

We now turn to formula apportionment, and characterize the optimal choice of the platform. Assuming that the post-tax profit function is concave in x_A and x_B, the optimal number of users is given by the solution to the two equations:

$$(1 - t_A\frac{x_A}{x_A + x_B} - t_B\frac{x_B}{x_A + x_B})\frac{\partial V}{\partial x_A} - \frac{x_B(t_A - t_B)}{(x_A + x_B)^2}V = 0, \quad (15)$$

$$(1 - t_A\frac{x_A}{x_A + x_B} - t_B\frac{x_B}{x_A + x_B})\frac{\partial V}{\partial x_B} + \frac{x_A(t_A - t_B)}{(x_A + x_B)^2}V = 0. \quad (16)$$

Under formula apportionment, a change in the number of users in any of the two countries affects the platform's post-tax profits through two channels. First, it changes the apportionment key, modifying the tax bases and the tax burdens in the two jurisdictions. Second, it changes the pre-tax profit V. Even in the absence of externalities between users in the two countries, the first effect creates an interdependence between the optimal choices of the platform in the two jurisdictions. The coexistence of these two channels also greatly complicates the analysis of the optimal choice of the platform when the jurisdictions set different corporate tax rates.

As in the case of Separate Accounting, let $\psi_A(x_B; t_A, t_B)$ and $\psi_B(x_A; t_A, t_B)$ be the implicit functions defined by equations (15) and (16). The intersection of these two reaction functions define the platform's choices. The signs of the slopes of the reaction functions are given by

$$\frac{\partial \psi_A}{\partial x_B} = -\frac{\frac{(1-t_A)\frac{\partial V}{\partial x_A}+(1-t_B)\frac{\partial V}{\partial x_B}}{(x_A+x_B)} + (1 - \frac{t_A x_A}{x_A+x_B} - \frac{t_B x_B}{x_A+x_B})\frac{\partial^2 V}{\partial x_A \partial x_B}}{\frac{\partial^2 \Pi}{\partial x_A \partial x_A}},$$

$$\frac{\partial \psi_B}{\partial x_A} = -\frac{\frac{(1-t_A)\frac{\partial V}{\partial x_A}+(1-t_B)\frac{\partial V}{\partial x_B}}{(x_A+x_B)} + (1 - \frac{t_A x_A}{x_A+x_B} - \frac{t_B x_B}{x_A+x_B})\frac{\partial^2 V}{\partial x_A \partial x_B}}{\frac{\partial^2 \Pi}{\partial x_B \partial x_B}}.$$

The functions $\psi_A(x_B)$ and $\psi_B(x_A)$ are not necessarily monotonic. The sign of the derivative $\frac{\partial \psi_A}{\partial x_B}$ depends on the sign of two terms. The first term, $(1 - t_A)\frac{\partial V}{\partial x_A} + (1 - t_B)\frac{\partial V}{\partial x_B}$, can either be positive or negative, depending on the tax rates t_A and t_B and the optimal choices of the numbers of users x_A and x_B. This term vanishes when the tax rates are equal, since then there is no distortion in the optimal number of users and $\frac{\partial V}{\partial x_A} = \frac{\partial V}{\partial x_B} = 0$. It is positive whenever $t_A > t_B$ and the number of users in country B is greater than the number of users in country A. The sign of the second term $(1 - \frac{t_A x_A}{x_A+x_B} - \frac{t_B x_B}{x_A+x_B})\frac{\partial^2 V}{\partial x_A \partial x_B}$ depends on the sign of the cross-derivative $\frac{\partial^2 V}{\partial x_A \partial x_B}$. It is positive when markets are complements and negative when markets are substitutes.

We thus observe that the reaction functions are increasing when (i) markets are complements and (ii) the number of users in country B is at least as large as the number of users in country A. In all other cases, it is not possible to ascertain whether the reaction functions are increasing or decreasing.

As in the case of Separate Accounting, we can decompose the effect of a change in the corporate tax rate t_A on the number of users in jurisdiction i, X_i, into a direct and indirect effect, as $\frac{dX_i}{dt_A}$ has the same sign as

$$\frac{\partial \psi_i}{\partial t_A} + \frac{\partial \psi_i}{\partial x_j}\frac{\partial \psi_j}{\partial t_A}.$$

We compute the direct effect using equations (15) and (16):

$$\begin{aligned}
\frac{\partial \psi_A}{\partial t_A} &= \frac{\frac{x_A}{x_A+x_B}\frac{\partial V}{\partial x_A} + \frac{V x_B}{(x_A+x_B)^2}}{\frac{\partial^2 \Pi}{\partial x_A \partial x_A}}, \\
\frac{\partial \psi_B}{\partial t_A} &= \frac{\frac{x_A}{x_A+x_B}\frac{\partial V}{\partial x_B} - \frac{V x_A}{(x_A+x_B)^2}}{\frac{\partial^2 \Pi}{\partial x_B \partial x_B}}.
\end{aligned}$$

The *direct effect of an increase in t_A is negative on the number of users in jurisdiction A, and positive on the number of users in jurisdiction B.* To see this, recall that, by equation (15),

$$(1 - t_A\frac{x_A}{x_A+x_B} - t_B\frac{x_B}{x_A+x_B})\frac{\partial V}{\partial x_A} = \frac{x_B(t_A - t_B)}{(x_A+x_B)^2},$$

so that when $t_A \geq t_B$, $\frac{\partial V}{\partial x_A} > 0$. An increase in the tax rate t_A results in a downward shift of the optimal choice on market A. By a similar computation, $\frac{\partial V}{\partial x_B} < 0$, so that an increase in the tax rate t_A results in an upward shift of the optimal choice on market B.

Under FA, an increase in the corporate tax rate in country A induces the platform to reduce its coverage in the high tax country and increase its coverage in the low tax country. This is easily explained: when the number of users in the low tax country is fixed, the platform has an incentive to lower the number of users in the high tax country in order to reduce the share of profit allocated to the high tax jurisdiction. This first order effect dominates the second-order effect of a reduction in total profit due to the distortion in output. By a similar reasoning, when the number of users in the high tax country is fixed, the platform has an incentive to increase the number of users in the low tax country.

We thus observe that a change in the corporate tax rate t_A has *opposite direct effects under SA and under FA*. Under SA, it leads to an increase in x_A and a reduction in x_B whereas under FA, it results in a decrease in x_A and an increase in x_B.

Figure 2 illustrates the effect of an increase in the corporate tax rate on the choice of the platform when the reaction functions are increasing. As opposed to the case of Separate Accounting, an increase in the corporate tax rate in country A shifts the reaction function in country A downwards (the curve in red is below the curve in black) and the reaction function in country B upwards (the curve in green is to the right of the curve in black). The total effect on equilibrium depends on the balance between the direct and indirect effects which have opposite signs. In Figure 2, the direct effect dominates the indirect effect so that the platform reduces

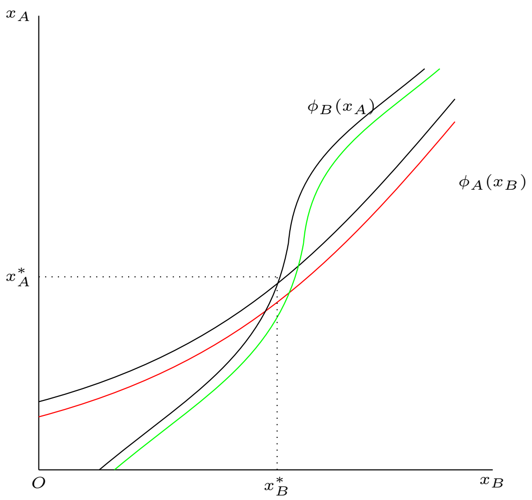

Figure 2: Effect of an increase in t_A under FA

the number of users in jurisdiction A and increases the number of users in jurisdiction B in response to the increase in the corporate tax rate.

As in the case of Separate Accounting, an increase in the corporate tax rate t_A always reduces the profit of the platform. By the envelope theorem, an increase in t_A affects the profit only through the direct effect $\frac{\partial \Pi}{\partial t_A} = -\frac{x_A}{x_A+x_B}V < 0$.

When the corporate tax rate t_A increases, the variation of tax revenues in country A is given by

$$\frac{\partial R_A}{\partial t_A} = \frac{x_A}{x_A + x_B}V + t_A \frac{x_A}{x_A + x_B}\frac{\partial V}{\partial t_A} + \frac{t_A}{(x_A + x_B)^2}(x_B \frac{\partial x_A}{\partial t_A} - x_A \frac{\partial x_B}{\partial t_A})V. \quad (17)$$

The first term captures the tax level effect, which is positive. The second effect captures the effect on the tax base. When $t_A \geq t_B$, an increase in t_A increases the distortions on the platform's choice, and hence reduces the pre-tax profit V. Hence the second effect is negative. The third term captures the effect on the apportionment key. When the reaction functions are increasing and the direct effect dominates the indirect effect, this effect is also negative. Hence the effect of an increase in t_A on the tax revenues of country A is ambiguous.

Consider next the effect of an increase in t_A on the tax revenues of country B:

$$\frac{\partial R_B}{\partial t_A} = t_B \frac{x_B}{x_A + x_B}\frac{\partial V}{\partial t_A} - \frac{t_B}{(x_A + x_B)^2}(x_B \frac{\partial x_A}{\partial t_A} - x_A \frac{\partial x_B}{\partial t_A})V.$$

There is no price effect, and the only two effects are (i) the negative effect on the tax base and (ii) the positive effect (when the reaction functions are increasing and the direct effect dominates the indirect effect) on the apportionment key.

In order to make progress, we consider two specific models: one where externalities across jurisdictions are absent and interdependence of choice only results from the effect of the apportionment key, and one where countries are symmetric and one country contemplates an increase above a uniform tax rate and a model with linear inverse demands.[5]

4.1 No externalities

Suppose that there are no externalities across jurisdictions, so that the cross-derivative $\frac{\partial V}{\partial x_A}\partial x_B$ is equal to zero. When tax rates are equal, the optimal choice of the number of users in the two jurisdictions are independent. Hence, an increase in the corporate tax rate t_A only affects the optimal number of users through the direct effects. Furthermore, when tax rates are equal, the effect of a change in the tax rate t_A on the tax base is negligible, so that the tax revenues in country B are increasing when t_A increases. We summarize this discussion in the following Proposition.

Proposition 4 *In a situation with no externalities across jurisdictions, a small increase in the tax rate t_A above the uniform tax rate t results in a decrease in the number of users in country A and an increase in the number of users in country B. In addition, an increase in t_A always results in an increase in the tax revenues of country B.*

4.2 A symmetric model

We next consider a model where the two jurisdictions are symmetric, and the inverse demand functions satisfy $P_A(x_A, x_B) = P_B(x_B, x_A)$ for all x_A, x_B. Country A contemplates an increase in the corporate tax rate t_A, starting from a uniform tax rate $t_A = t_B = t$. When the tax rates are equal, the sign of the slope of the reaction functions only depends on the cross-derivative $\frac{\partial^2 \Pi}{\partial x_A \partial x_B}$. The reaction functions are increasing when markets are complements and decreasing when markets are substitutes. In addition, concavity implies that, when markets are complements, the slope of the reaction function is smaller than one, so that the direct effect always dominates the indirect effect. We conclude that an increase in t_A always results in a decrease in x_A and an increase in x_B. In addition, when $t_A = t_B$, the platform's choice of pre-tax profit is optimal, so that a small increase in t_A has a negligible effect on the pre-tax profit. Hence, tax revenues in country B are always increasing when the corporate tax rate in country A increases. The following Proposition summarizes our findings.

Proposition 5 *In the symmetric model, a small increase in the tax rate t_A above the uniform tax rate t results in a decrease in the number of users in country A and an increase in the number of users in country B. In addition, an increase in t_A always results in an increase in the tax revenues of country B.*

[5] Under FA, the situation with one-sided externalities is not fundamentally different from the situation with two-sided externalities. Even if externalities do not flow from B to A, the number of users in market B affects the choice of users in market A because the platform maximizes total profit, taking into account the fact that the allocation key is the ratio of the number of users.

5 A comparison between SA and FA

In this section, we use a numerical simulation to compare equilibrium outcomes under Separate Accounting and Formula Apportionment for a peer-to-peer platform. Each user's utility depends on the total number of users in the two jurisdictions, with

$$u_A(x_A, x_B) = u_B(x_A, x_B) = \gamma(x_A + x_B).$$

Valuations are drawn from a uniform distribution over $[0, 1]$ in the two markets, and the markets have the same size, $s = 1$. The inverse demand function is thus given by

$$P_i(x_i, x_j) = 1 - (1 - \gamma)x_i + \gamma x_j.$$

We compute the optimal choices of the platform under Separate Accounting and Formula Apportionment, assuming that $\gamma = 0.2$ and $t_B = 0.2$, and let the tax rate of country A increase from the uniform tax rate $t_A = 0.2$ to 0.4. Figures 3 and 4 show how different economic variables vary with changes in t_A under Separate Accounting (in red) and Formula Apportionment (in blue).[6]

Figure 3 illustrates how the numbers of users and prices in the two counties vary when the tax rate t_A increases. The direct effect dominates the indirect effect in the two régimes, not only at the uniform tax rate (as indicated by Propositions 2 and 5), but for the entire range of tax rates. Hence, an increase in t_A results in an increase in the number of users in country A and a decrease in the number of users in country B under SA, and a decrease in the number of users in country A and an increase in the number of users in country B under FA. As a consequence, prices in jurisdiction A are decreasing in t_A under SA but increasing under FA. Prices in jurisdiction B follow the opposite trend: they are increasing in t_A under SA but decreasing in t_A under FA.

Figure 4 shows how profits and tax revenues are affected by an increase in the corporate tax rate t_A. The left upper panel considers pre-tax profit. Clearly, output distortions in response to the difference in corporate tax rates move the platform away from its optimal pre-tax profit. The effect becomes stronger when the difference in tax rates increases. The computations show that the effect is stronger under FA than under SA. As the number of users affects the apportionment key in addition to profit, output distortions are larger under FA than under SA.

When one considers post-tax profits however, the difference between the two régimes becomes less stark. As shown in the upper right panel, the post-tax profit of the platform falls at a similar rate under SA than under FA. As pre-tax profits fall faster under FA than under SA, this suggests that the total tax bill of the platform is lower under FA than under SA.

To assess the effect of an increase in t_A on the tax bill, we decompose the tax revenues into tax revenues received by country A (lower left panel) and country B (lower right panel). In country A, we find that the positive tax level effect dominates the tax base effect (see the decompositions (5) and (17)), so that tax revenues are increasing in t_A under both régines. In addition, it appears that the tax revenues increase faster under SA than under FA, as the platform reacts more strongly to the increase of the corporate tax rate under FA than under SA. Hence, *the high tax country prefers the régime of Separate Accounting to Formula Apportionment.*

[6]Robustness checks show that similar pictures are obtained for different values of s, γ and t_B.

As indicated by Propositions 2 and 5 (for the uniform tax rate), an increase in t_A always results in an increase in the tax revenues of country B. This increase is of much smaller magnitude than the increase in tax revenues of country A. Interestingly, the increase is larger under FA than under SA, so that *the low-tax country prefers the régime of Formula Apportionment to Separate Accounting.*

Hence, in the taxation of peer-to-peer platforms, the high-tax and low-tax countries have opposite preferences over the two régimes of profit-sharing. However, as most of the tax revenues increase accrues to the high tax country, the sum of tax revenues is higher under SA than under FA, indicating that the two countries could agree on the Separate Accounting régime with appropriate compensations to the low-tax country.

6 Conclusion

This paper analyzes taxation of an internet platform attracting users from different jurisdictions. When corporate income tax rates are different in the two jurisdictions, the platform distorts prices and outputs in order to shift profit to the low tax country. We analyze the comparative statics effects of an increase in the tax rate of one country. When cross effects are present in both countries, the platform has an incentive to increase the number of users in the high tax country and decrease the number of users in the low tax country. When externalities only flow from one market to another, an increase in the corporate tax rate results either in a decrease or an increase in the number of users in both countries depending on the direction of externalities. We compare the baseline regime of separate accounting (SA) with a regime of formula apportionment (FA), where the tax bill is apportioned in proportion to the number of users in the two countries. Under FA, an increase in the corporate tax rate increases the number of users in the low-tax country and decrease the number of users in the high-tax country. We use a numerical simulation to show that the high-tax country prefers SA to FA whereas the low-tax country prefers FA to SA.

While this analysis is a first step in the study of multinational two-sided platforms, we are aware of important limitations that need to be addressed in future research. First, for two-sided platforms, we assume that all users in one jurisdiction are on the same side of the platform. Extending the model to a situation where both types of users are present in both jurisdictions increases the complexity of computations, but should be considered in future work. Second, we limit our analysis to two jurisdictions. While we believe that the main results extend to a model with an arbitrary number of jurisdictions, more computations are needed to test the robustness of the results. Finally, we ignore the role played by intellectual property rights in profit shifting. We do not consider incentives to improve the matching algorithms and the geographical distribution of intellectual property rights. In order to get a better understanding of the location decisions of internet platforms, we need to include these elements in future research.

7 References

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8 Appendix

Proof of Proposition 2: Remember that the sign of $\frac{\partial X_A}{\partial t_A}$ is the same as the sign of

$$A = \frac{\partial V_A}{\partial x_A}\frac{\partial \Pi^2}{\partial x_B^2} - \frac{\partial V_A}{\partial x_B}[(1-t_A)\frac{\partial^2 V_A}{\partial x_A \partial x_B} + (1-t_B)\frac{\partial^2 V_B}{\partial x_A \partial x_B}].$$

Now, suppose that $t_A = t_B = t$. By the first order condition (3),

$$\frac{\partial V_A}{\partial x_A} = -\frac{\partial V_B}{\partial x_A}.$$

and by symmetry,

$$\frac{\partial V_B}{\partial x_A} = \frac{\partial V_A}{\partial x_B},$$

so that

$$\frac{\partial V_B}{\partial x_A} = -\frac{\partial V_A}{\partial x_A}.$$

Note also that, as $t_A = t_B = t$,

$$\begin{aligned}
[(1-t_A)\frac{\partial^2 V_A}{\partial x_A \partial x_B} + (1-t_B)\frac{\partial^2 V_B}{\partial x_A \partial x_B}] &= (1-t)(\frac{\partial^2 V_A}{\partial x_A \partial x_B} + \frac{\partial^2 V_B}{\partial x_A \partial x_B}), \\
&= \frac{\partial^2 \Pi}{\partial x_A \partial x_B}.
\end{aligned}$$

Finally, by concavity

$$\frac{\partial^2 \Pi}{\partial x_A^2}\frac{\partial^2 \Pi}{\partial x_B^2} > (\frac{\partial^2 \Pi}{\partial x_A \partial x_B})^2,$$

and by symmetry $\frac{\partial^2 \Pi}{\partial x_A^2} = \frac{\partial^2 \Pi}{\partial x_B^2}$ so that

$$-\frac{\partial^2 \Pi}{\partial x_B^2} > |\frac{\partial^2 \Pi}{\partial x_A \partial x_B}|.$$

We find that

$$\begin{aligned}
A &= \frac{\partial V_A}{\partial x_A}\frac{\partial \Pi^2}{\partial x_B^2} - \frac{\partial V_B}{\partial x_A}\frac{\partial \Pi^2}{\partial x_A \partial x_B} \\
&= \frac{\partial V_A}{\partial x_A}(\frac{\partial \Pi^2}{\partial x_B^2} + \frac{\partial \Pi^2}{\partial x_A \partial x_B}) \\
&> 0,
\end{aligned}$$

showing that $\frac{\partial X_A}{\partial t_A} > 0$.

Now it is easy to check that $\frac{\partial X_B}{\partial t_A} = -\frac{\partial X_A}{\partial t_A} < 0$. The effect of an increase in t_A on the prices P_A, P_B and the tax revenues R_A and R_B are immediately obtained. ■

Proof of Proposition 3: We have computed the reaction functions in the text. It is convenient to introduce the following functions, which give the reaction functions provided the bounds on the markets are satisfied, and make explicit the dependence with respect to ρ:

$$\chi_A(\rho, x_B) = \frac{1}{2\sigma_A}[1 + (\rho\alpha + \beta)x_B] \tag{18}$$

$$\chi_B(\rho, x_A) = \frac{1}{2\sigma_B}[1 + \frac{1}{\rho}(\rho\alpha + \beta)x_A] \tag{19}$$

From (19), given ρ, $\phi_B(x_A) = \min(\chi_A(\rho, x), s)$ and $\chi_B(x_A) = \min(\chi_B(\rho, x_A), 1)$.

We first look for interior solutions of the optimization problem, when the market coverage constraints are not binding. The quantities are given by (10) and (11) when they are positive and smaller than s and 1 respectively. The quantities are positive if $1 - \frac{(\rho\alpha+\beta)^2}{4\rho\sigma_A\sigma_B} > 0$, which is equivalent to the concavity of profit. The inequality can be written as requiring that a quadratic function in ρ is negative. This quadratic function has a positive coefficient in ρ^2 and is negative at $\rho = 1$ by assumption. Thus there is value $\rho_{min} > 1$ such that the profit is concave if and only if $\rho < \rho_{min}$.

Assuming $\rho < \rho_{min}$, consider $X_A(\rho)$ and $X_B(\rho)$. The inequality $X_A(\rho) < s$ can be written as requiring that a quadratic function in ρ with a positive coefficient in ρ^2 is negative. By a similar argument as above, there is unique value ρ_A, $1 < \rho_A < \rho_{min}$, such that $X_A(\rho) \leq s$ for $\rho < \rho_{min}$ if and only if $\rho \leq \rho_A$. The same argument can be used for the function $X_B(\rho)$ and provides the existence of a unique value ρ_B, $1 < \rho_B < 1$, such that $X_B(\rho_B) \leq 1$ for $\rho < \rho_{min}$ if and only if $\rho \leq \rho_B$.

This proves the first part of Proposition 3: the users' numbers $(X_A(\rho), X_B(\rho))$ are the optimal platform choices for $\rho < \min(\rho_A, \rho_B)$.

We now consider the situation where the optimal solution is not interior.

Case 1: $\rho_A < \rho_B$. When ρ increases, the market coverage constraint binds first for A is but not for B. At $\rho = \rho_A$, $\chi_A(\rho_A, x_B) = s$ and $x_B = \chi_B(\rho_A, s) < 1$. Consider $\rho > \rho_A$. Since $\chi_B(\rho, s)$ is decreasing in ρ, $x_B = \chi_B(\rho, s) < 1$ holds. As for $x_A = \chi_A(\rho, x_B)$, x_A is larger than s: if not, x_A, x_B would be an interior solution, in contradiction with $\rho > \rho_A$: Market A is covered but not B, with quantities given by (12).

Case 2:$\rho_A > \rho_B$. When ρ increases, the market coverage constraint binds first for B is but not for A. At $\rho = \rho_B$, $\chi_A(\rho_B, 1) < s$ and $x_B = \chi_B(\rho_B, x_A) = 1$.

Consider $\rho > \rho_B$. Since $\chi_A(\rho, 1)$ is increasing in ρ, there is a value $\widehat{\rho}$ such that $x_A = \chi_A(\rho, 1) < s$ holds for $\rho \in [\rho_B, \widehat{\rho}[$ and $\chi_A(\widehat{\rho}, 1) = s$. The inequality $\chi_A(\rho, 1) < s$ for $\rho \in [\rho_B, \widehat{\rho}[$ implies $\widehat{\rho} < \rho_A$.

Let $\rho \in [\rho_B, \widehat{\rho}[$. We surely have $\chi_B(\rho, x_A) \geq 1$ at $x_A = \chi_A(\rho, 1)$ because otherwise (x_A, x_B) would be an interior solution, in contradiction with $\rho > \rho_B$. This also implies that $\widehat{\rho} < \rho_A$. Thus for $\rho \in [\rho_B, \widehat{\rho}[$ market B is covered but not A, with quantities given by (13).

Consider now $\rho \geq \widehat{\rho} = \frac{1}{\alpha}[2\sigma_A s - \beta - 1]$. We have $1 < \widehat{\rho} < \rho_A$. At $\rho = \widehat{\rho}$, $\chi_A(\rho, 1) = s$ and $\chi_B(\rho, x_A) \geq 1$ so that both markets are covered. Increasing ρ, both markets are covered as long as $\chi_A(\rho, 1) > s$ and $x_B = \chi_B(\rho, s) > 1$. The first inequality surely holds since χ_A is increasing in ρ. As for the second inequality, $\chi_B(\rho, s)$ is decreasing with ρ with limit $\frac{1}{2\sigma_B}[1 + \frac{1}{\rho}(\rho\alpha + \beta)s]$. Thus $x_B = \chi_B(\rho, s) > 1$ always holds if the limit is at least one; otherwise, $2\sigma_B < 1 + \alpha s$. it becomes optimal to decrease the number of users at the value ρ for which $\chi_B(\rho, s) = 1$ with an optimal number of users is given by $\chi_B(\rho, s)$.

We have worked by increasing ρ, analyzing the solutions to the first order conditions. Since the profit is not always concave, it remains to check that there are no multiple solutions. We know that an interior solution is the unique optimum. We thus need to check that we cannot have two solutions, each with at least one covered market. This is proved as follows. Let a solution with B covered. Assume A is covered as well: covering a market is the best response to the other being covered, hence it is the unique solution with one covered market at least. Assume now that A is not covered. An alternative solution could be that A is covered but not B, which implies $\chi_A(\rho, x_B) \geq s$ with $x_B = \chi_A(\rho, s) < 1$. As $\chi_A(\rho, x_B)$ is increasing in x_B, we must have $\chi_A(\rho, 1) > s$: it is optimal to cover A if B is covered, in contradiction with the initial assumption. ■

Proof of Proposition 5: When the countries are symmetric and $t_A = t_B$, then $x_A = x_B$ and $\frac{\partial V}{\partial x_A} = \frac{\partial V}{\partial x_B} = 0$. This implies that

$$\frac{\partial \psi_A}{\partial x_B} = -\frac{\frac{\partial^2 \Pi}{\partial x_A \partial x_B}}{\frac{\partial^2 \Pi}{\partial x_A^2}},$$

$$\frac{\partial \psi_B}{\partial x_A} = -\frac{\frac{\partial^2 \Pi}{\partial x_A \partial x_B}}{\frac{\partial^2 \Pi}{\partial x_B^2}}.$$

Hence

$$\frac{\partial X_A}{\partial t_A} = \frac{\frac{V x_B}{(x_A + x_B)^2}}{\frac{\partial^2 \Pi}{\partial x_A^2}} + \frac{\frac{V x_A}{(x_A + x_B)^2}}{\frac{\partial^2 \Pi}{\partial x_B^2}} \frac{\frac{\partial^2 \Pi}{\partial x_A \partial x_B}}{\frac{\partial^2 \Pi}{\partial x_A^2}}.$$

Now by symmetry $\frac{\partial^2 \Pi}{\partial x_A^2} = \frac{\partial^2 \Pi}{\partial x_B^2}$ and, by concavity,

$$(\frac{\partial^2 \Pi}{\partial x_A^2})^2 - (\frac{\partial^2 \Pi}{\partial x_A \partial x_B})^2 > 0.$$

Hence

$$\left|\frac{\frac{\partial^2\Pi}{\partial x_A \partial x_B}}{\frac{\partial^2\Pi}{\partial x_A^2}}\right| < 1,$$

which guarantees that

$$\begin{aligned}\frac{\partial X_A}{\partial t_A} &< \frac{\frac{V x_B}{(x_A+x_B)^2}}{\frac{\partial^2\Pi}{\partial x_A^2}} - \frac{\frac{V x_B}{(x_A+x_B)^2}}{\frac{\partial^2\Pi}{\partial x_A^2}},\\ &< 0,\end{aligned}$$

so that X_A is decreasing in t_A. A similar computation shows that x_B is decreasing in t_A. Next observe that $\frac{\partial V}{\partial t_A} = 0$ at $t_A = t_B = t$, so that an increase in the corporate tax rate always increases the tax revenues of country B, concluding the proof. ■

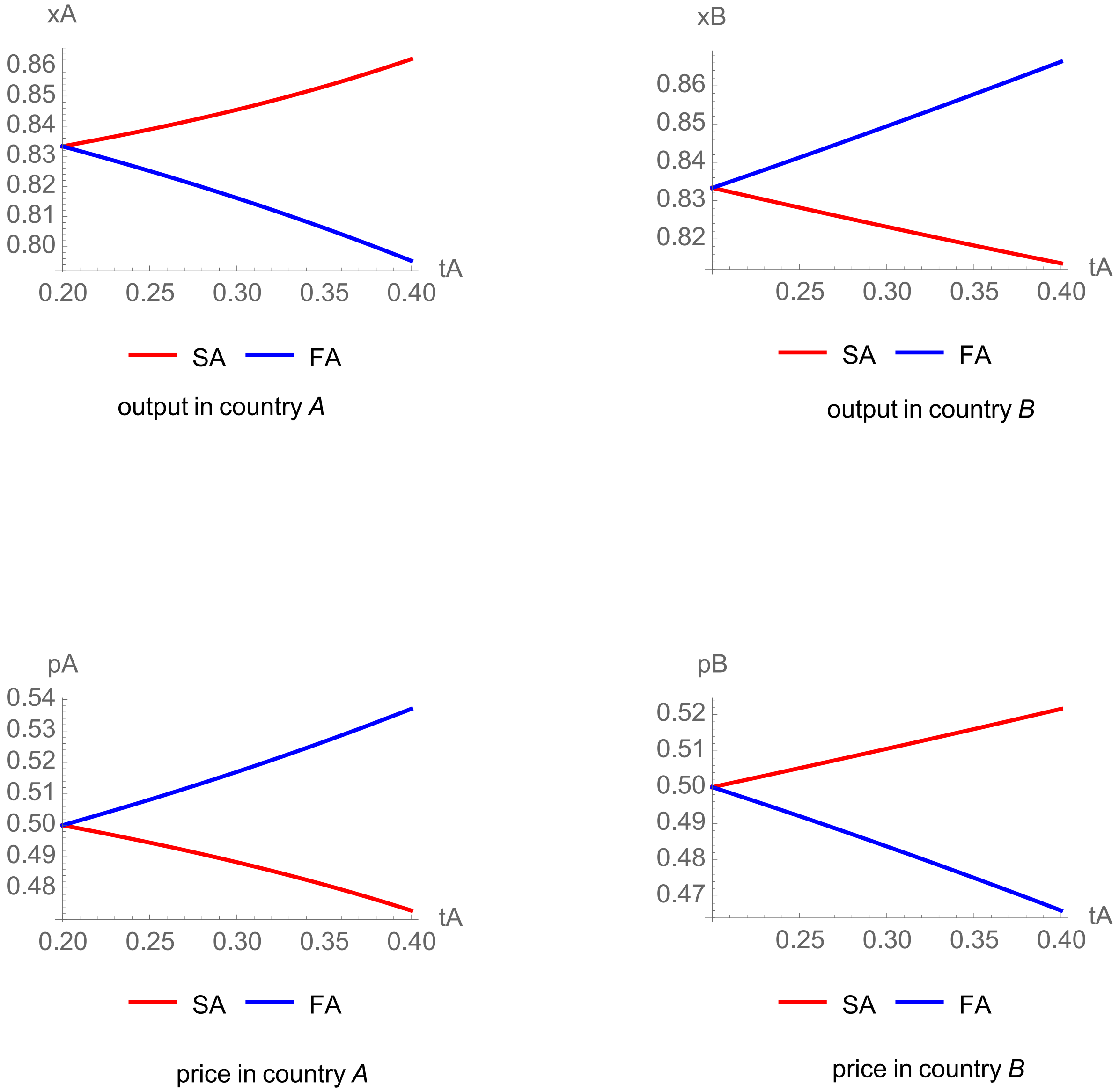

Figure 3: Outputs and prices

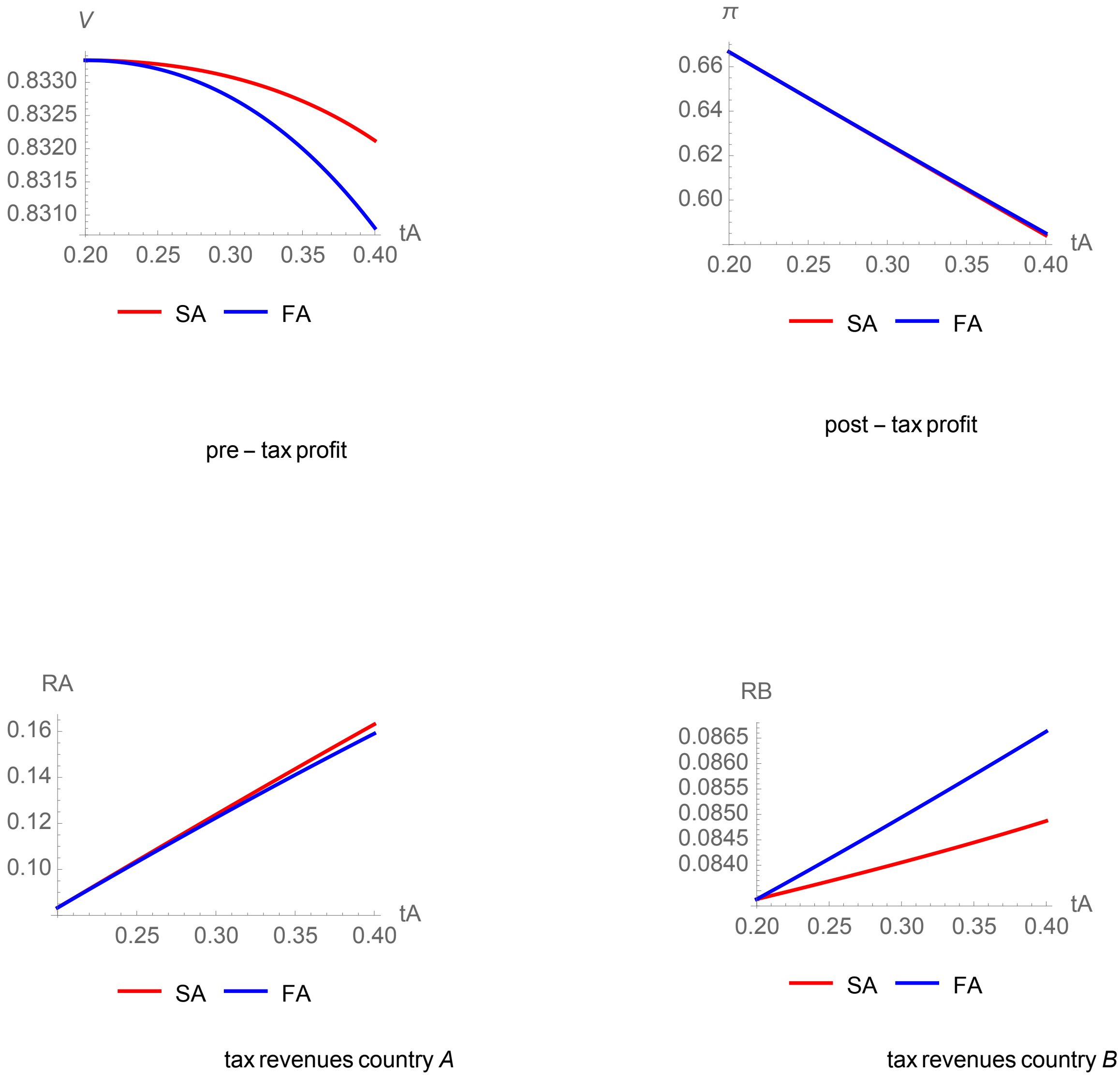

Figure 4: Profits and revenues