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Sharing Commitment? Monetary Regimes and Time Consistent Fiscal Policy

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Sharing Commitment?

Monetary Regimes and Time Consistent Fiscal Policy

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Abstract

We study monetary-fiscal interactions when only the central bank has a commitment technology to credibly announce future policy decisions. The analysis focuses on the commitment intensity required to implement either of two policy regimes: one where the central bank follows a simple monetary rule, e.g. inflation targeting, against one where monetary decisions react to fiscal decisions, with the objective to eliminate the incentives of the treasury to deviate from an optimal policy plan. When the central bank *shares* its commitment technology with the treasury, it generally reduces the commitment intensity necessary to sustain monetary promises, in particular in circumstances where central bank's required credibility is high: when outstanding public debt is high and nominal.

Keywords: *Public debt, fiscal-monetary policy, time-consistency, credibility and partial commitment.*
JEL classification: E02, E52, E58, E61, E62

> *"At the present time, the existence of a debt of unprecedented magnitude in all countries of the world makes it desirable to manage this debt as to prevent a material increase in interest rates. It is thus the increase in the public debt (...) that is the precondition of the present-day changes in the principles and technique of central banking."* [Kriz (1948)]

As in 1948, high level of public debt is influencing monetary-fiscal institutional arrangements.[1] Heavily indebted treasuries challenge the independence of monetary institutions by seeking accommodating policies in response to fiscal decisions. Both the institutional mandate and acquired credibility of modern central banks could be at stake. In this paper, we evaluate a refinement of current monetary frameworks: a central bank recognizes that treasuries might compromise its independence and to preserve credibility, it designs

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[1] For a long run perspective on public debt and how it interacts with economic and political developments, see Eichengreen, El-Ganainy, Esteves, and Mitchener (2019).

interventions contingent on fiscal decisions. In other terms, we study how monetary interventions can incentivize a fiscal authority to pursue a specific course of actions, and whether such policies might enhance the credibility of monetary promises.

Surprisingly, several components of existing monetary frameworks already include provisions contingent on fiscal decisions, especially in the Eurozone. Multiple supporting schemes of the European Central Bank (ECB) are conditional on the pursuit of fiscal programs, with the implicit objective to contain the spending bias of treasuries. These conditions are present in Emergency Liquidity Assistance (ELA) programs to banks or in the Outright Monetary Transactions (OMT) of sovereign bonds.[2] The refinancing framework of the ECB includes similar clauses, where an investment grade rating of public debt is a pre-requisite to participate in refinancing operations.[3] The introduction of fiscal contingencies in monetary interventions is controversial, as it might per se compromise the mandate of the independent central bank, and threaten the acquired credibility of monetary policy makers.[4] We take this concern seriously and study formally whether fiscal contingent monetary interventions are effective to discipline fiscal authorities without compromising the pursuit of core monetary policy targets.

To this end, we analyze the conduct of monetary and fiscal policy under asymmetric and limited commitment: only the monetary authority has a technology that allows to announce policies and implement them later in time, while the fiscal authority makes decisions sequentially under discretion.[5] Importantly, the commitment technology of the central bank is restricted: both economic state, e.g. the level of public debt, and fiscal decisions can compromise the implementation of monetary pledges. We use this framework to analyze three specific questions. What is the influence of asymmetric commitment on the conduct of public policy? Can a central bank design policy rules to share the benefits of its commitment technology with a fiscal authority? And finally, does this type of intervention undermine the credibility of monetary policy makers?

Specifically, we contrast the credibility required to sustain two monetary regimes. In the first, the central bank follows a *standard rule*, where consistently with most mandates of independent central banks, monetary decisions do not respond directly to fiscal policy. In an alternative regime, the central bank designs interventions explicitely conditional on fiscal decisions, with the objective to induce an optimal dynamic policy plan in equilibrium. The construction of these *strategic rules* goes as follow: the central bank picks a desired equilibrium path and sets its policy accordingly. Then, it constructs its policy off the equilibrium path to discourage fiscal deviations from the equilibrium path. Under this monetary regime, the central bank "shares" its commitment technology with the fiscal authority, in the sense that it induces an

[2]Conditional monetary assistance is explicit for the OMT program: "A necessary condition for Outright Monetary Transactions is strict and effective conditionality attached to an appropriate European Financial Stability Facility/European Stability Mechanism (EFSF/ESM) program." ELA for Greek banks turned out to be a critical element in the bargaining game between European institutions and the newly elected Greek government in 2015.

[3]For details on the collateral framework policies of the European Central Bank, see Claeys and Goncalves Raposo (2018).

[4]The argument against the OMT program, brought to the German Constitutional Court in 2014, stressed that conditional support to treasuries was beyond the mandate of the ECB, because it was in conflict with the prohibition of monetary financing of member states. In particular, the linkage to the conditionality of a European Stability Mechanism program indicates that it would interfere with economic policies reserved to member states. See Siekmann and Wieland (2014).

[5]Commitment asymmetry is in line with the institutional set-up in most developed economies, where fiscal and monetary policies are determined by independent authorities with their own respective mandates and appointment procedures. Within the model, both policy makers are benevolent and share the same welfare functions, but given the asymmetric commitment technology, they don't solve the same optimization problem.

equilibrium outcome that coincides with the one that would prevail if both authorities had a commitment technology.

Naturally, from a welfare perspective, *strategic rule* dominates *standard rules*, but are they credible? To evaluate the credibility of each monetary regime, we derive and compare the commitment intensity required to sustain each monetary rule in equilibrium. The key result of the analysis is that strategic monetary rules, designed to share the benefits of a commitment technology with the treasury, generally relax the commitment intensity required to sustain a monetary target. This improvment is especially large in the economic states where credibility of *standard* monetary rule is low, i.e. at higher level of public debt.

The analysis rests on the following dynamic game. Initially, at a *constitutional* stage, the central bank discloses its operational framework and commits to a monetary rule, which specifies monetary interventions for every possible state of the economy. Then, at each point in time, first the fiscal authority implements a policy decision, then the monetary authority considers the following alternative: either it sticks to the announced monetary rule and follows the associated policy, or it renounces its promise and implements an alternative decision. Importantly, renouncing the strategy is costly to the central bank. The magnitude of the cost captures the intensity of the commitment technology.[6]

Under this timing, the fiscal authority might have an incentive to challenge the monetary rule, and reap the short term benefits of policy discretion. Still, the central bank enjoys a first mover advantage at the constitutional stage, where it can design threats to discourage undesirable fiscal decisions. In this context, the credibility of a monetary rule is defined as the minimum level of commitment that (i) contains the fiscal incentives to challenge the monetary pledge and (ii) eliminates all monetary incentives to renounce the rule.[7]

We study the equilibrium implications of this game in two different yet related frameworks. First, we consider an extension of the linear quadratic environment of Barro and Gordon (1983), augmented to capture the interactions of policy institutions. In this environment, the policy game is static in the sense that each policy maker plays only once. Absent a commitment technology, both monetary and fiscal authorities are confronted with time inconsistency problems, giving rise to an *inflation bias* and, by analogy, a *fiscal bias*. This parsimonious environment is used to expose the institutional game and the implications of different monetary rules. We contrast two cases: one where the central bank commits unconditionally to an inflation target with one where the monetary authority commits to a strategic rule designed to share its commitment technology with the fiscal authority, which effectively eliminates the *fiscal bias* and achieves the inflation target. In both cases, the credibility of the monetary rule decreases when the relative gains of renouncing the rule are large. Further, contrasting credibility across regimes provides a clear characterization of our central result: the strategic monetary rule is more credible than the unconditional rule precisely when the central bank's credibility is weaker.[8] In other terms, strategic monetary rule requires less commitment intensity to deliver the inflation target, precisely in circumstances when renouncing this objective is the most tempting.

We use this framework to relax the assumption of benevolent policy makers. First, we consider the

[6]The extreme cases where the cost is zero or infinite correspond respectively to no commitment or full commitment.

[7]Accordingly, independently of the nature of the monetary rule, evaluating credibility requires to assess on and off equilibrium paths for both the monetary and fiscal authority.

[8]Using the terminology of the model, the central bank's credibility is at stake when monetary deviations from its policy target are not costly and generate large effects on output, relative to fiscal deviations from its target.

desirability of appointing a conservative central banker, as in Rogoff (1985), in the context of our monetary-fiscal game. As is well understood, the credibility of an unconditional standard rule is improved by the appointment of an inflation conservative policy maker. In contrast, the credibility of strategic monetary rules is at stake when the central banker is too conservative. Indeed, these interventions rely on a threat to offset fiscal deviations: conservatism increases the cost of the threat, endangering the credibility of the monetary regime. Finally, we show that the credibility of strategic monetary rules is not sensitive to fiscal political economy frictions which generally put more emphasis on short term stimulation of output.[9]

The opening quote suggests that public debt is a critical element driving fiscal suasion over monetary policy and the credibility of central bank promises. To analyze this specific dimension of policy interactions, the institutional game is embedded into a cash-credit production economy, where public debt is a dynamic endogenous state variable. This environment brings together common concerns for the conduct of monetary and fiscal policy under lack of commitment. The monetary authority is tempted to generate unexpected inflation to inflate away outstanding debt. The fiscal authority is tempted to manipulate interest rates and the price of newly-issued debt. As in the static game, we contrast equilibrium outcomes under two monetary regimes: one where the central bank commits to a *standard* constant money growth rate rule, and one where the central bank designs *strategic* interventions, with the objective to eliminate the sequential incentives of the fiscal authority to manipulate interest rates.

The analysis yields the following results. Under both regimes, a higher level of outstanding debt increases the commitment intensity required to implement either monetary rule. Second, the nature of public debt is critical, both for the design of monetary rules and associated credibility. If debt is nominal, then simple constant money growth rate policies eliminate the fiscal incentives to manipulate the interest rate.[10] Still, pursuing such an unconditional rule requires a higher level of credibility than under a strategic rule, where monetary interventions are explicitly contingent on fiscal choices. Finally, the commitment intensity to support the strategic monetary rule is lower when debt is real, i.e. indexed to variations in the price level. Indeed, as established in the linear quadratic framework, the commitment intensity required to support a monetary rule depends on the relative benefits of renouncing the rule: under nominal debt, a central bank defaulting on its promise can achieve larger welfare gains by inflating outstanding liabilities, in contrast to the case where debt is real. Overall, the analysis unveils an interesting "paradox": with nominal debt, sharing commitment relies on simple monetary interventions, but requires substantial commitment intensity to be defended against fiscal actions.

Finally, we show that a central bank can improve the credibility of its promises by deviating along the equilibrium path from its optimal policy target. Indeed, when the central bank commits to an inflation target higher than optimal (or equivalently, a money growth rate superior to the optimal steady deflation recommended by the Friedman rule), then the relative benefits to renouncing the rule are lower, improving the credibility of the original promise.

[9]More generally, the following insight applies: a strategic rule should be designed using the preferences of the fiscal authority, while the credibility should be evaluated using central bank's preferences.

[10]Indeed, when the central bank credibly commits to a constant money growth rate, the discretionary incentives of the fiscal authority to manipulate interest rates (and influence the price of newly issued bonds) is offset by a revaluation of outstanding liabilities. This offsetting effect is absent when debt is real/indexed.

Related literature. The time inconsistency of optimal policy plans has been a central issue in the macroeconomic literature since Kydland and Prescott (1977), where in sequential models, ex ante optimal promises could be sub-optimal ex post: the absence of a commitment technology leads to welfare losses. The literature has expanded in various directions to study this tension in different policy environments, with the objective to design institutions that mitigate the lack of credibility of dynamic policy plans. We investigate the implication of this problem in the context of monetary fiscal interactions. Our cash credit economy brings together the dynamic incentives of the fiscal authority to manipulate the real interest rate (as in Lucas and Stokey (1983), Debortoli and Nunes (2013) and Debortoli, Nunes, and Yared (2017)) and the inflation bias of monetary policy, driven by the benefits to inflate nominal claims and collects resources from seignorage. Diaz-Gimenez, Giovannetti, Marimon, and Teles (2008) study extensively the implications of monetary lack of commitment, with a similar emphasis on nominal vs. real debt.

Numerous contributions study optimal monetary and fiscal policy under the assumption that a single and benevolent policy maker chooses all policy instruments, with and without commitment. Our analysis includes a discussion of these arrangements, but our main analysis considers a non cooperative game between the central bank and the treasury, as in Dixit and Lambertini (2003). Our institutional set up features asymmetric commitment, in the sense that only the central bank has the ability to commit and respond to fiscal policy.[11] This is an approach followed by Gnocchi (2013) and Gnocchi and Lambertini (2016): fiscal contingent monetary strategies clearly welfare dominate restricted ones.[12]

Our credibility criteria for guiding policy recommendation is novel. To contrast the credibility of monetary regimes, we consider the case where the commitment technology of the central bank is limited. Our modeling approach is similar to Farhi, Sleet, Werning, and Yeltekin (2012), where policy plans can be revised endogenously against a cost.[13] As they do, we relate the magnitude of the cost to the intensity of the commitment technology. But while they solve for the constrained equilibrium given the commitment intensity, we derive the commitment intensity required to support on equilibrium a given monetary regime.[14]

Finally, our game-theoric construction of *strategic monetary rule* relates to *strategies* studied by Bassetto (2005) or *sophisticated policies* in Atkeson, Chari, and Kehoe (2010), where off equilibrium policies influence the equilibrium outcome. Both apply this idea to environments with multiple equilibria with the objective to implement a unique and superior equilibrium outcome.[15] In contrast, we are interested in the effectiveness of this class of monetary interventions to eliminate the time inconsistency of optimal fiscal policy.

[11]Formally, both the central bank and the treasuries are benevolent, but due to asymmetric commitment, they don't have the same objective function. We also discuss how relaxing the assumption of benevolent policy makers influence our results, considering monetary conservatism, as in Adam and Billi (2008) and Niemann (2011a), or fiscal short termism, as discussed in Alesina and Passalacqua (2016).

[12]The analysis in Gnocchi (2013) is conducted in a New Keynesian environment without debt, with a focus on stabilization vs. provision of public good. Our cash-credit economy with debt allows us to study a different problem, namely the capacity of the central bank to eliminate the incentives of the treasury to manipulate the real interest rate.

[13]Alternative approaches have been proposed to model limited commitment, including stochastic "loose commitment" in Debortoli and Nunes (2013) or "limited time commitment" in Clymo and Lanteri (2018).

[14]Also, in Section 1.5.2, we expose how the modeling of our commitment technology relates to *reputation* or *trigger-type* environment.

[15]For instance, Bassetto (2005) highlights the shortcomings of standard Ramsey analyses for fiscal policy. Atkeson, Chari, and Kehoe (2010) study *sophisticated monetary policy* to implement a unique equilibrium outcome. Also, Camous and Cooper (2014) study how monetary strategies can deter self-fulfilling debt crises.

Plan. The rest of the paper is organized as follows. Section 1 presents the institutional environment in a linear quadratic framework, exposes the construction of monetary strategies and discusses the main results. Section 2 then embeds the policy game in a dynamic cash credit economy and analyzes the influence of public debt on the design of strategic monetary rules and associated credibility. Section 3 concludes. Unless stated otherwise, proofs are relegated to an appendix.

1 Linear Quadratic Framework

This section presents the main ideas in a linear quadratic framework, in the tradition of Barro and Gordon (1983).[16] After presenting the institutional set-up, we follow two objectives: first, to expose the construction of *strategic* monetary interventions designed to share commitment with the fiscal authority, and second, to contrast the credibility of such interventions with a *standard* regime of strict inflation targeting.

1.1 Monetary and Fiscal Policy

1.1.1 Environment

A fiscal authority chooses a tax instrument τ, while a central bank sets π, e.g. inflation. e captures private agents' expectations of policy decisions. Economic outcomes and policy choices are welfare ranked according to the following loss function:

$$\mathcal{L}(e,\tau,\pi) = \frac{1}{2}\left[(\tau-\tau^*)^2 + \lambda(\pi-\pi^*)^2 + \gamma(y-y^*)^2\right]. \tag{1}$$

$\lambda > 0$ captures the welfare cost of monetary deviations from an optimal target π^*, relative to fiscal deviations from τ^*.[17] Similarly, $\gamma > 0$ stands for the temptation to stimulate the economy toward a *first best* level of output $y^* > 0$. A "Phillips curve" captures the influence of policy decisions (τ,π) on output y:

$$y = \tau - \tau^e + \alpha(\pi - \pi^e). \tag{2}$$

As usual, expansionary policy choices (τ,π) can stimulate output beyond its natural level (here $y^n = 0$), only if unanticipated by private agents' expectations (τ^e,π^e).[18] $\alpha > 0$ is the relative efficiency of monetary policy to stimulate the economy. Private agents form rational expectations, i.e. they perfectly anticipate the conduct of public policy:

$$\tau^e = \tau \qquad\qquad \pi^e = \pi. \tag{3}$$

[16]Variants of this environment to study monetary-fiscal interactions have been developed by Dixit and Lambertini (2003) and Dixit (2000) in the context of a monetary union.

[17]τ^* could stand either for an optimal provision of a public good, or supporting a redistributive policy program.

[18]The natural level of output $y^n < y^*$ is inefficiently low due to the presence of distortionary taxation for instance. This gives policy makers a motive to use policy tools to stimulate output.

Given the linear structure of the economy, one can write private agents' expectations as:

$$e = \tau^e + \alpha\pi^e. \tag{4}$$

1.1.2 Benchmark Policy Regimes

Before introducing the ingredients of asymmetric and limited commitment, we consider equilibrium outcomes when the central bank and the treasury cooperate in choosing (τ, π). Given that policy makers are benevolent, this situation is equivalent to the choice of policy intruments by a joint authority, called *the government.* When the government has an unrestricted *commitment* technology, it internalizes how policy choices (τ^c, π^c) influence private agents expectations e^c.[19] Accordingly, it understands it cannot stimulate output, and delivers policy targets (τ^*, π^*) to minimize the welfare loss (1):

$$\tau^c = \tau^* \qquad \pi^c = \pi^* \qquad e^c = \tau^* + \alpha\pi^* \qquad y^c = 0. \tag{5}$$

This benchmark equilibrium outcome is not senstive to preference parameters (λ, γ).

When the government cannot rely on a commitment technology, it takes policy decisions sequentially, i.e. after private agents form expectations. Under this regime of *discretion*, the government is tempted to stimulate output toward y^*. In equilibrium though, private agents anticipate the conduct of public policy, and output is not stimulated:

$$\tau^d = \tau^* + \gamma y^* \qquad \pi^d = \pi^* + \frac{\alpha}{\lambda}\gamma y^* \qquad e^d = \tau^* + \gamma y^* + \alpha\big(\pi^* + \frac{\alpha}{\lambda}\gamma y^*\big) \qquad y^d = 0. \tag{6}$$

Under discretion, policy choices are characterized by a *fiscal bias* and a *monetary bias* that weigh on welfare. The tension between *ex ante* and *ex post* optimal choices implies that *commitment* policies require some technology to be credibly implemented. The monetary bias under discretion is increasing in α, i.e. in the relative efficiency of monetary policy to stimulate output, and decreasing in λ, i.e. in the relative cost of monetary deviation from the target π^*.

1.2 The credibility of monetary policy under asymmetric commitment

We develop an institutional set-up of *asymmetric and limited commitment* to address the core questions of interest. What is the influence of asymmetric commitment on the conduct of public policy? Can a central bank "share" the benefits of its commitment technology with the fiscal authority? Are these interventions credible? To discuss these elements, we first specify the policy game that underlies institutional interactions.

1.2.1 Institutional set-up

We consider the following game. Initially, at a *constitutional stage*, the central bank announces a *policy rule* $\{\pi^k(\mathcal{S})\}$. A rule is a set of monetary interventions conditional on the state $\mathcal{S} = (e, \tau)$, private agents

[19] Formally, under *commitment*, the government moves first and decides upon (τ^c, π^c), then private agents form expectations, and finally the government implements its pre-announced policy. See Appendix A.1 for a formal treatment.

expectations e and fiscal decisions τ. Then, the following sequence of actions takes place:

i. Private agents form expectations $e = (\tau^e, \pi^e)$.

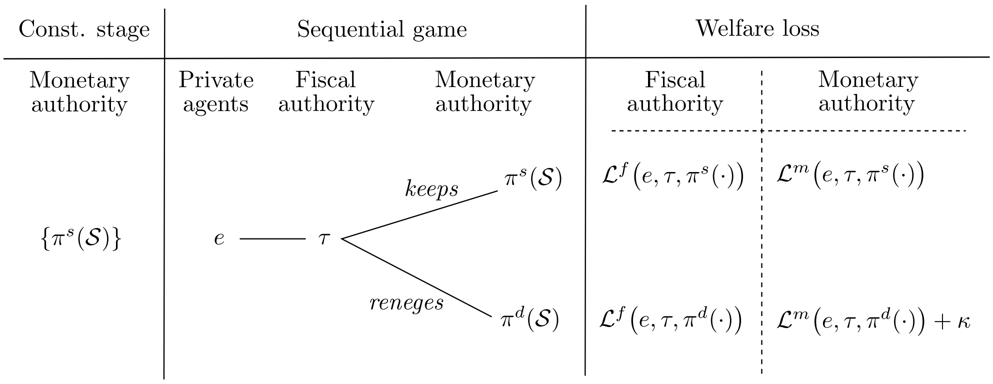
ii. The fiscal authority sets τ.

iii. Given $\mathcal{S} = (e, \tau)$, the central bank,

- either *keeps* its promise and follows its policy rule $\pi^k(\mathcal{S})$,
- or *reneges*, suffers a welfare cost $\kappa > 0$, and implements $\pi^r(\mathcal{S})$:

$$\pi^r(\mathcal{S}) = \text{argmin}_\pi \ \mathcal{L}(e, \tau, \pi).$$

Figure 1: Institutional set-up



This figure displays the sequence of choices and associated welfare to each authority.

Figure 1 graphically represents the sequence of choices given a monetary rule $\{\pi^k(\mathcal{S})\}$ designed at the constitutional stage. During the course of the game, the central bank is a Stackelberg follower w.r.t. to the fiscal authority, as it moves after fiscal choices τ.[20] But at the constitutional stage, the monetary authority enjoys a first mover advantage and can possibly design *threats* and eliminate the incentives of the fiscal authority to challenge the policy rule. The credibility of the rule depends on the magnitude of the welfare cost κ, which captures the intensity of the monetary commitment technology.[21] If both authorities are benevolent and $\kappa = 0$, then no strategy can be credibly implemented and the equilibrium outcome coincides with the one labeled *discretion*.[22] If κ is arbitrarily large, then any strategy can be implemented against any fiscal decision. To form monetary policy recommendations, we contrast the *credibility* of different monetary policy rules.

In figure 1 and the following developments, we index the social loss function $\mathcal{L}(\cdot)$ with the identity of the policymaker and the decision of the central to *keep* or *reneges* its promise: for instance, $\mathcal{L}^{f,k}(\cdot)$ is the

[20]Importantly, the fiscal authority acts under discretion, i.e. given private agents' expectations e, and anticipating the reaction of monetary policy to its decision.

[21]While we do not explicitly micro-found κ, it can be interpreted as a reduced-form reputation cost, born by the central bank for reneging on a promise, see Section 1.5.2 for a formal discussion of this interpretation. A similar modeling approach is followed by Farhi, Sleet, Werning, and Yeltekin (2012).

[22]In this non cooperative game, both fiscal and monetary authorities evaluate policy decisions and economic outcomes with the same loss function. Hence, the equilibrium outcome does not change if policy moves are sequential or simultaneous. See Section 1.4 for a discussion of cases where preferences differ across policy makers.

loss to the fiscal authority conditional on the central bank keeps its promise, and $\mathcal{L}^{m,r}(\cdot)$ is the loss to the monetary authority conditional on reneges. While we preserve the assumption of benevolent policy makers, this notation allows to keep track of the sequential nature of the game. In Section 1.4, we relax the assumption of benevolent policy makers.

1.2.2 Evaluating credibility

The objective is to contrast the commitment intensity required to sustain different monetary policy regimes. In an environment with asymmetric commitment, the fiscal authority might try to induce the central bank to renounce its promise: the treasury strategically chooses a value τ, followed by the central bank reneging on $\pi^k(\mathcal{S})$ and implementing $\pi^r(\mathcal{S})$. Alternatively, the central bank resists the temptation to reoptimize and follows $\pi^k(\mathcal{S})$ if it has enough commitment intensity, i.e. if the institutional cost κ is high enough. Accordingly, deriving the credibility of a monetary strategy requires to study fiscal and monetary decisions on and off equilibrium paths.

To characterize credibility formally, we consider fiscal choices conditional on whether the central bank keeps or reneges on a generic monetary rule $\pi^k(\mathcal{S})$. Then, we derive conditions on the commitment technology κ such that along the equilibrium path, the monetary authority follows its rule $\pi^k(\mathcal{S})$.

If the fiscal authority anticipates that the monetary authority keeps its promise and follow its rule $\pi^k(\mathcal{S})$, it chooses $\tilde{\tau}$ defined as:

$$\tilde{\tau} = \text{argmin}_{\tau}\ \mathcal{L}^{f,k}\big(e, \tau, \pi^k(\mathcal{S})\big). \tag{7}$$

Along this equilibrium path, private agents' expectations satisfy $\tilde{e} = \tilde{\tau} + \alpha\pi^k(\tilde{e}, \tilde{\tau})$.

Alternatively, the treasury anticipates that the commitment technology of the central bank cannot sustain the rule $\pi^k(\mathcal{S})$ against some $\tau \neq \tilde{\tau}$. Importantly, this fiscal decision needs to yield a welfare gain to the fiscal authority. This condition defines a set of *profitable fiscal deviations*, given private agents expectations e:

$$\mathrm{T}(e) = \big\{\tau \ \ \text{s.t.} \ \ \mathcal{L}^{f,r}\big(e, \tau, \pi^r(e, \tau)\big) \leq \mathcal{L}^{f,k}\big(e, \tilde{\tau}, \pi^k(e, \tilde{\tau})\big)\big\}. \tag{8}$$

Combine these elements to formally define the notion of *credible* monetary rule:

Definition 1. *A monetary policy rule $\pi^k(\mathcal{S})$ is credible if for all profitable fiscal decisions $\tau \in \mathrm{T}(\tilde{e})$:*

$$\mathcal{L}^{m,k}\big(\tilde{e}, \tau, \pi^k(\tilde{e}, \tau)\big) \leq \mathcal{L}^{m,r}\big(\tilde{e}, \tau, \pi^r(\tilde{e}, \tau)\big) + \kappa,$$

where $\pi^r(\mathcal{S})$ =argmin$_\pi$ $\mathcal{L}(e, \tau, \pi)$.

In turn, one can define a credibility cut-off, i.e. the minimum level of commitment intensity associated to a credible monetary rule.

Definition 2. *Let $\bar{\kappa}$ be the credibility cut-off of a monetary rule $\pi^k(\mathcal{S})$, defined as:*

$$\bar{\kappa} = \max_{\tau \in \mathrm{T}(\tilde{e})} \; \mathcal{L}^{m,k}(\tilde{e}, \tau, \pi^k(\tilde{e}, \tau)) - \mathcal{L}^{m,r}(\tilde{e}, \tau, \pi^r(\tilde{e}, \tau)).$$

This definition makes clear that the credibility of a monetary rule relies on monetary fiscal interactions: $\bar{\kappa}$ is the minimum welfare cost that eliminates the incentives of the monetary authority to renounce its policy rule for every profitable fiscal deviation $\tau \in \mathrm{T}(\tilde{e})$. In other terms, a monetary rule is credible if and only if $\kappa \geq \bar{\kappa}$. We relate the credibility of a rule to the cut-off value $\bar{\kappa}$: a rule associated with a high credibility cut-off $\bar{\kappa}$ requires substantial commitment to be defended against strategic fiscal decisions, and accordingly be implemented in equilibrium.[23]

Finally, note that in Definitions 1 and 2, private agents' expectations are anchored at $\tilde{e} = \tilde{\tau} + \alpha\pi^k(\mathcal{S})$. When evaluating the credibility of different monetary policy regimes, we investigate the behavior of fiscal and monetary policy decisions on and off equilibrium paths, taking private agents' expectations anchored at the associated equilibrium path. This reflects our interest in understanding how the design of a monetary strategy, and the subsequent fiscal incentives to challenge it, shape the credibility of a given monetary policy regime.[24]

1.3 Contrasting Monetary Policy Regimes

We are now ready to analyze equilibrium outcomes under asymmetric commitment and contrast the credibility required to implement either of the following monetary policy regimes:

- **Monetary Regime 1 -** the central bank follows a *standard* monetary rule, namely to hit the inflation target π^* unconditionally.

- **Monetary Regime 2 -** the central bank designs a *strategic* rule, conditional on fiscal decisions, with the objective to reach its policy target π^* and induce the fiscal authority to implement τ^*.

Comparing the commitment intensity required to sustain these policy regimes is key to understanding whether adopting the second monetary regime, designed to share commitment with the fiscal authority, could compromise the core objective of the central bank to deliver a policy target π^*.

1.3.1 Monetary Regime 1 - Unconditional inflation target

Under this regime, the central bank promises to deliver its inflation target unconditionally:

$$\forall \mathcal{S}, \; \pi^k(\mathcal{S}) = \pi^*. \tag{9}$$

[23] Readers familiar with the notion of dynamic reputational equilibrium might note that $\bar{\kappa}$ relates to the long run reputation of the policy maker that eliminates all short run deviations from a policy plan. See a formal discussion in Section 1.5.2.

[24] One could be worried that in an environment with limited commitment, variations in private agents' expectations are self-fulfilling and multiple equilibria might arise. We discuss in Appendix A.6 a simple version of the Taylor principle in the linear quadratic environment that eliminates these concerns.

If credible, this monetary policy rule induces the following equilibrium outcome:[25]

$$\tau_1 = \tau^* + \gamma y^* \qquad \pi_1 = \pi^* \qquad e_1 = \tau^* + \gamma y^* + \alpha\pi^* \qquad y_1 = 0 \tag{10}$$

The central bank naturally implements its policy target π^*, while the fiscal decision is still characterized by a *fiscal bias*. In equilibrium, expectations reflect policy choices and output is not stimulated beyond its natural level.

This rule requires commitment to be sustained in equilibrium.[26] To evaluate the credibility of (9), one needs to understand the incentives of the fiscal authority to challenge π^*, and the incentives of the central bank to keep or renege on its policy target π^*. Figure 2 provides a graphical illustration of the following elements. First, we characterize the set $\mathrm{T}(e_1)$ of profitable fiscal deviations:

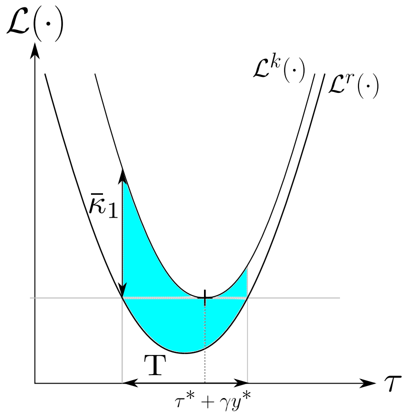
$$\mathrm{T}(e_1) = \big\{\tau \text{ s.t. } \mathcal{L}^{f,r}\big(e_1, \tau, \pi^r(\mathcal{S})\big) \leq \mathcal{L}^{f,k}(e_1, \tau_1, \pi^*)\big\}. \tag{11}$$

Following Definition 1, the inflation target (9) is credible if and only if:

$$\forall \tau \in \mathrm{T}(e_1),\ \kappa \geq \mathcal{L}^{m,k}(e_1, \tau, \pi^*) - \mathcal{L}^{m,r}\big(e_1, \tau, \pi^r(e_1, \tau)\big). \tag{12}$$

Let $\bar{\kappa}_1$ be the credibility cut-off associated with the inflation target regime (9). As illustrated in Figure 2, it

Figure 2: Monetary Regime 1 - standard monetary rule



This figure illustrates the characterization of the credibility cut-off $\bar{\kappa}_1$, when the central bank follows a standard inflation target rule (9). Given the sequential nature of the game, credibility is evaluated for a set $\mathrm{T}(e_1)$ of fiscal choices τ, represented by the blue area. The credibility cut-off $\bar{\kappa}_1$ is then given by the maximum distance within that set between $\mathcal{L}^k(\cdot)$ and $\mathcal{L}^r(\cdot)$, corresponding respectively to the cases where the central bank *keeps* its promise (9) or *reneges* and implements $\pi^r(\mathcal{S})$.

is derived from an off equilibrium path where the fiscal authority implements the most contractionary policy $\tau_l = \min\{\mathrm{T}(e_1)\}$, followed by an expansionary offsetting monetary action $\pi^r(e_1, \tau_l)$. Formally:

Proposition 1. *The unconditional inflation target rule* (9) *is credible if and only if the welfare cost of the*

[25] See formal derivation in Appendix A.2.
[26] Indeed, *after* private agents form expectations, the central bank has an incentive to renounce its promise and reoptimize.

central bank to renounce its promise satisfies:

$$\kappa \geq \bar{\kappa}_1 = \frac{(y^*)^2}{2}(\gamma - \eta)(1+\gamma)\Big(\frac{\sqrt{1+\gamma} + \sqrt{\gamma - \eta}}{1+\eta}\Big)^2,$$

with $\eta = \frac{\lambda\gamma}{\lambda + \gamma\alpha^2}$. *In addition:*

$$\frac{d\bar{\kappa}_1}{d\lambda} < 0 \qquad \text{and} \qquad \frac{d\bar{\kappa}_1}{d\alpha} > 0.$$

Proof. See Appendix A.2 ∎

The sensitivity of the credibility cut-off to parameters (λ, α) makes clear how the credibility of a standard inflation target rule is interwined with the commitment problem of the central bank. If the cost λ of monetary deviations from its target π^* is low, then the monetary choice conditional on *reneging* is associated with a large monetary stimulus, as explicit in (6): because there are large benefits to renouncing the promise, the commitment intensity to sustain (9) is high. The same argument explains the sensitivity of $\bar{\kappa}_1$ to the monetary policy efficiency parameter α: it requires more credibility for the central bank to sustain (9) when α is high, i.e. when it could achieve a large discretionary output stimulus at a low welfare cost.

1.3.2 Monetary Regime 2 - Sharing commitment with the treasury

Can the central bank build on its commitment technology to induce the fiscal authority to follow $\tau = \tau^*$? Does it compromise the core monetary objective to deliver its policy target π^*?

This section derives the existence of *strategic* monetary rules $\pi^k(\mathcal{S})$, explicitely contingent on fiscal decisions, that are effective to eliminate the *fiscal bias.* Second, it infers the minimum credibility cut-off $\bar{\kappa}_2$ that sustains the associated equilibrium outcome:

$$\tau_2 = \tau^* \qquad \pi_2 = \pi^* \qquad e_2 = \tau^* + \alpha\pi^* \qquad y_2 = 0. \tag{13}$$

The central element of this monetary rule is that the central bank eliminates all fiscal incentives to choose $\tau \neq \tau^*$, while delivering the inflation target π^* on equilibrium. To reach this objective, the central bank needs to follow a monetary rule $\pi^k(\mathcal{S})$ that satisfies:[27]

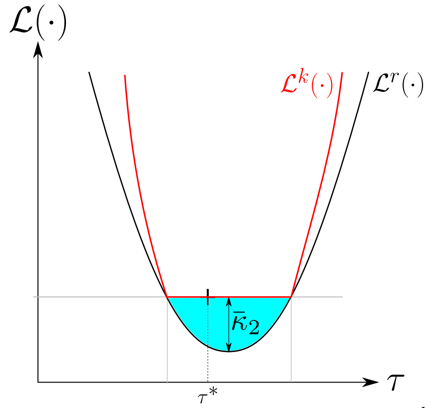
$$\pi^k(e_2, \tau^*) = \pi^*, \tag{14}$$

$$\forall \tau \ \mathcal{L}^{f,k}\big(e_2, \tau, \pi^k(e_2, \tau)\big) \geq \mathcal{L}^{f,k}\big(e_2, \tau^*, \pi^*\big). \tag{15}$$

In (14), the central bank picks the desirable equilibrium and sets its policy consistently. In (15), it structures its policy off equilibrium to discourage fiscal deviations from the equilibrium path. Any monetary rule that satisfies these conditions is *strategic* in the sense that it embeds a threat to "punish" fiscal choices $\tau \neq \tau^*$. Given the capacity of the central bank to set arbitrary π and hit any level of welfare loss for all τ, the existence of strategies $\pi^k(\mathcal{S})$ that satisfy (14) and (15) is natural.

[27] Again, we retain the assumptions that private expectations are anchored at $e_2 = e^*$ while evaluating the credibility of the monetary strategy.

Figure 3: Monetary Regime 2 - strategic monetary rule



This figure represents the construction of a strategic monetary rule $\pi^k(\mathcal{S})$ aimed to implement the equilbrium outcome (13), with a minimum commitment intensity. To induce the fiscal authority to implement τ^*, the central bank designs its rule so that the loss function for all τ is higher evaluated at $(e_2, \tau, \pi^k(\mathcal{S}))$ than at the desirable equilibrium outcome (e_2, τ^*, π^*). To minimize the commitment intensity required to support this rule, monetary interventions offset the welfare implications of fiscal deviations, within the relevant $\mathrm{T}(e_2)$, associated to the blue area on the figure.

We are interested in the design of strategic rules that minimize the level of commitment intensity required to be sustained in equilibrium. Figure 3 illustrates the following argument: given the sequential nature of the policy game, the fiscal authority considers (off equilibrium) deviations $\tau \neq \tau^*$ that would lead to a welfare gain if the central bank were to renounce its promise. This condition determines the set $\mathrm{T}(e_2)$ of profitable fiscal deviations (8):

$$\mathrm{T}(e_2) = \big\{\tau \ \text{ s.t. } \ \mathcal{L}^{f,r}\big(e_2, \tau, \pi^r(e_2, \tau)\big) \leq \mathcal{L}^{f,k}(e_2, \tau^*, \pi^*)\big\}. \tag{16}$$

To minimize the commitment intensity required to sustain (13), the central bank designs $\pi^k(\mathcal{S})$ so as to minimize the relative benefit to renouncing the strategy, while preserving the incentives (15) and (14) of the fiscal authority. This is achieved when the central bank credibly commits to a rule that satisfies:[28]

$$\forall \tau \in \mathrm{T}(e_2), \ \mathcal{L}^{f,k}\big(e_2, \tau, \pi^k(e_2, \tau)\big) = \mathcal{L}^{f,k}\big(e_2, \tau^*, \pi^*\big). \tag{17}$$

The policy of the central bank to eliminate fiscal choices $\tau \neq \tau^*$ is built on threats to the fiscal authority to maintain the welfare loss reached at the desired equilibrium (13). The credibility of the strategy is evaluated against a sequential fiscal deviation followed by the central bank reneges and optimizes. When both institutions are benevolent, it coincides with the discretionary choice (τ, π) under cooperation, given private agents expectations $e_2 = e^*$. Formally, the credibility cut-off $\bar{\kappa}_2$ satisfies

$$\bar{\kappa}_2 = \mathcal{L}^{m,k}(e_2, \tau^*, \pi^*) - \min_{\tau, \pi} \mathcal{L}^{m,r}(e_2, \tau, \pi), \tag{18}$$

and we have the following proposition:

[28]For all $\tau \notin \mathrm{T}(\cdot)$, the monetary rule can specify any $\pi \geq \pi^r(\mathcal{S})$, as the fiscal authority will not consider such a policy path to challenge the strategy of the central bank.

Proposition 2. *A monetary rule $\pi^k(\mathcal{S})$ that satisfies* (14) *and* (17) *credibly implements* (13) *if and only if:*

$$\kappa \geq \bar{\kappa}_2 = \frac{\gamma(y^*)^2}{2}\frac{\gamma(\lambda+\alpha^2)}{\lambda+\gamma\alpha^2+\lambda\gamma}.$$

In addition, we have:

$$\frac{d\bar{\kappa}_2}{d\lambda} < 0 \qquad \textit{and} \qquad \frac{d\bar{\kappa}_2}{d\alpha} > 0.$$

Proof. See Appendix A.3 ∎

The sensitivity of the credibility cut-off $\bar{\kappa}_2$ to parameters (α, λ) reflects the intuition built in Section 1.3.1: the commitment intensity required for the central bank to follow $\pi^k(\mathcal{S})$ is directly influenced by the relative gains to renouncing the strategy and implement $\pi^r(\mathcal{S})$.

Using Propositions 1 and 2, we can contrast the commitment intensity required to implement Regimes 1 and 2. In other terms, we can derive conditions under which strategic monetary rules implementing (13) relaxes the commitment intensity required to deliver the monetary target π^*. Formally:

Proposition 3. *The commitment intensity $\bar{\kappa}_2$ required to implement a strategic monetary rule that satisfies* (14) *and* (17) *is lower than $\bar{\kappa}_1$, associated to the unconditional inflation target, for relatively low values of λ and high values of α. Formally,*

- *for every $\lambda > 0$, there is a threshold $\bar{\alpha} > 0$ such that for all $\alpha > \bar{\alpha}$, $\bar{\kappa}_1 > \bar{\kappa}_2$.*

- *for every $\alpha > 0$, there is threshold $\bar{\lambda} > 0$ such that for all $\lambda < \bar{\lambda}$, $\bar{\kappa}_1 > \bar{\kappa}_2$.*

Proof. See Appendix A.3 ∎

To understand this result, one needs to contrast costs and benefits of strategic rules. On the one hand, when the central bank shares its commitment technology across the government, it anchors private expectations at e^* and deters fiscal decisions $\tau \neq \tau^*$. On the other hand, the design of strategic rules embeds off-equilibrium threats $\pi^k(\mathcal{S}) \neq \pi^*$ to preserve the fiscal incentives not to deviate from τ^*. Proposition 3 indicates that strategic rules improves the credibility of the monetary target π^* when λ is relatively low and α relatively high, i.e. precisely in circumstances where the commitment problem of the central bank to sustain its target π^* is the most intense, when the temptation to renounce a rule and reoptimize is stronger.

1.4 Monetary credibility and non benevolent policy makers

We have maintained so far the assumption of benevolent monetary and fiscal authorities, despite different commitement technology. How would different preferences over equilibrium outcomes influence the design of strategic monetary rules and the commitment intensity required to implement them? To illustrate these questions, we revisit the classic themes of monetary conservatism and short-sighted fiscal authority.

Despite possible diverse preferences, the construction of monetary rules and evaluation of credibility follows the exposition in Section 1.3. For instance, a strategic rule is designed to provide incentives to the fiscal authority, as in (17):

$$\forall \tau \in \mathrm{T}(e^*),\ \mathcal{L}^{f,k}\big(e^*, \tau, \pi^k(\mathcal{S})\big) = \mathcal{L}^{f,k}(e^*, \tau^*, \pi^*), \tag{19}$$

where $\mathrm{T}(e^*)$ is defined as in (16):

$$\mathrm{T}(e^*) = \big\{\tau \ \text{ s.t. } \ \mathcal{L}^{f,r}\big(e^*, \tau, \pi^r(\cdot)\big) \leq \mathcal{L}^{f,k}(e^*, \tau^*, \pi^*)\big\}. \tag{20}$$

and $\pi^r(\mathcal{S}) = \text{argmin}_\pi \mathcal{L}^{m,r}(e^*, \tau, \pi)$. The evaluation of the required commitment intensity relies on the preferences of the central bank, as in (18):

$$\bar{\kappa}_2 = \max_{\tau \in \mathrm{T}(e^*)} \mathcal{L}^{m,k}\big(e^*, \tau, \pi^k(\mathcal{S})\big) - \mathcal{L}^{m,r}\big(e^*, \tau, \pi^r\big(\mathcal{S}\big)\big). \tag{21}$$

Credibility of monetary conservatism. Consider the desirability of appointing a conservative central banker in the context of asymmetric commitment.[29] Note $\tilde{\lambda}$ the welfare weight to the conservative central bank of inflation deviation from π^*, while the fiscal authority is benevolent and evaluates policy outcomes with parameters (λ, γ). The objective of the central bank is to implement either (10) under Regime 1 or (13) under Regime 2. We evaluate the sensitivity of credibility cut-offs $\bar{\kappa}_1$ and $\bar{\kappa}_2$ to the relative conservatism $\tilde{\lambda}/\lambda$ of the central bank.[30]

As is well understood, an increase in conservatism $\tilde{\lambda} > \lambda$ decreases the commitment intensity $\bar{\kappa}_1$ necessary to sustain an unconditional monetary target π^*: monetary conservatism reduces the gain to *renege* on the policy target π^* and accordingly $\bar{\kappa}_1$. When the central bank follows a strategic rule that satisfies (19) and (21), then an increase in monetary conservatism reduces the relative gains to renouncing the strategy, as in Regime 1. However, an increase in $\tilde{\lambda}$ also increases the cost associated with implementing the threat $\pi^k(\mathcal{S})$ when fiscal choices deviate from τ^*. Hence, the credibility cut-off $\bar{\kappa}_2$ is U-shaped w.r.t. monetary conservatism. These elements are illustrated in Figure 4, panel (a).

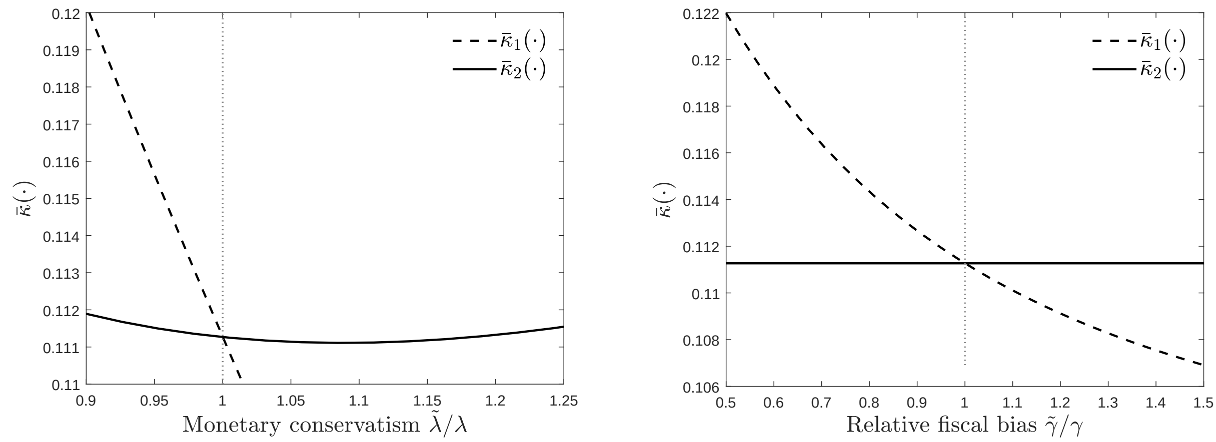
Short-sighted treasury. Another common departure from benevolent policy making is to consider fiscal preferences that exacerbate the short term temptation to stimulate the economy.[31] In our set-up, we study this possibility and assume that the fiscal weight on output deviation from y^* is $\tilde{\gamma} > \gamma$, while the central bank is benevolent. Does this situation influence the credibility of implementing (13) with strategic monetary rules? We derive the credibility cut-off $\bar{\kappa}_2$ and discuss its sensitivity to $\tilde{\gamma}$. As explicit in (19), the strategic rule $\pi^k(\mathcal{S})$ that implements (13) is sensitive to fiscal preference $\tilde{\gamma}$. The credibility of the strategic rule is

[29] The benefits of appointing an inflation conservative central banker have been discussed first by Rogoff (1985). Recent contributions, including Adam and Billi (2008) and Niemann (2011b), discuss the desirability of appointing a conservative central banker in the context of monetary and fiscal policy discretion. Here, our approach is to contrast the credibility of monetary regimes 1 and 2 for different degrees of monetary conservatism under asymmetric commitment.

[30] Formal derivations are provided in Appendix A.4.1.

[31] Political economy frictions can fuel this type of fiscal behavior, incl. electoral constraints or budgetary processes. For a review, see Alesina and Passalacqua (2016).

Figure 4: Credibility cut-off for non benevolent policy makers



(a) Conservative Central Bank

(b) Short Sighted Fiscal Treasury

This figure represents commitment intensity $\bar{\kappa}$ for each monetary regimes. The left panel compares the credibility of Regime 1 against Regime 2 for different degree of central bank conservatism. Panel (b) compares the credibility across regimes for different intensity of fiscal bias. (Numerical values are set so that $\bar{\kappa}_1 = \bar{\kappa}_2$ when authorities have the same social preferences: $\lambda = 3/2$, $\gamma = 1/2$, $\alpha = 0.952$.)

evaluated with the preferences to the monetary authority:

$$\bar{\kappa}_2 = \max_{\tau \in \mathrm{T}(e^*)} \mathcal{L}^{m,k}\big(e^*, \tau^*, \pi^*\big) - \mathcal{L}^{m,r}\big(e^*, \tau, \pi^r(\cdot)\big), \tag{22}$$

Accordingly, the credibility cut-off $\bar{\kappa}_2$ is not sensitive to the intensity of the fiscal bias $\tilde{\gamma}$, and satisfies the expression derived in Proposition 2.[32] This example makes clear that the credibility of strategic monetary rules is insulated against political frictions influencing treasury's decisions. As illustrated in Figure 4, panel (b), this differs with the credibility required to sustain the inflation target under Regime 1, where both on and off equilibrium policy choices depend on the wedge $\tilde{\gamma}/\gamma$.

1.5 Extensions

The linear-quadratic framework allows to investigate an additional set of related questions. We choose to report two of them.

1.5.1 Inflation target and the credibility of strategic rule

Consider a scenario where the commitment technology of the central bank κ is lower than the credibility cut-off $\bar{\kappa}_2$ required to implement the equilibrium outcome (13). Can the central bank adjust its policy framework to mitigate this lack of credibility? One possibility is to target an equilibrium inflation rate $\pi^s > \pi^*$ and

[32] The set of profitable deviations to the fiscal authority $\mathrm{T}(e^*)$ does depend on $\tilde{\gamma}$, but the most tempting deviation to the monetary authority is not sensitive to $\tilde{\gamma} \geq \gamma$. Appendix A.4.2 details the underlying derivations.

design a strategy $\pi^k(\mathcal{S})$ that implements the following equilibrium outcome:[33]

$$\tau_2 = \tau^* \qquad \pi_2 = \pi^s > \pi^* \qquad e_2 = \tau^* + \alpha\pi^s \qquad y_2 = 0. \tag{23}$$

Following the exposition in Section 1.3.2, such a rule is designed to provide incentives to the fiscal authority:

$$\pi^k(e_2, \tau^*) = \pi^s \tag{24}$$

$$\forall \tau \in \mathrm{T}(e_2),\ \mathcal{L}^{f,k}\big(e_2, \tau, \pi^k(e_2, \tau)\big) = \mathcal{L}^{f,k}\big(e_2, \tau^*, \pi^s\big), \tag{25}$$

where $\mathrm{T}(e_2)$ is defined as in (16), adjusted for the equilibrium inflation target π^s.

The equilibrium monetary target $\pi^s > \pi^*$ generates a systematic welfare loss, but lowers the commitment intensity $\bar{\kappa}_2(\pi^s)$ required to eliminate the fiscal bias and implement (23). Formally,[34]

$$\frac{d\bar{\kappa}_2(\cdot)}{d\pi^s}\bigg|_{\pi^s=\pi^*} < 0. \tag{26}$$

The intuition is straightforward: by targeting a policy rate higher than π^*, the central bank reduces the relative gains of renouncing $\pi^k(\mathcal{S})$, which in turn reduces the commitment intensity required to support this strategy.

1.5.2 Credibility and reputation.

Reputation or trigger type equilibria are a common construction for commitment technologies in repeated policy games.[35] To make the mapping between our modeling choices with these concepts explicit, consider an infinite repetition of the game described before. The objective of the central bank is to implement (13) by following a strategic rule $\pi^k(\mathcal{S})$ of the type derived in Section 1.3.2, without the presence of exogenous cost κ in case of renouncment to the rule. In contrast, the commitment technology is derived from the long run consequences of renouncing the rule, namely the infinite repetition of the discretionary equilibrium outcome (6).

Formally, let $\mathrm{T}(e^*)$ be the set of profitable deviations, as in (16). The central bank follows $\pi^k(e, \tau)$ given by (14) and (17) for all $\tau \in \mathrm{T}(e^*)$ if $e = e^*$. Otherwise, if $e \neq e^*$ or $\tau \neq \mathrm{T}(e^*)$, it implements $\pi^r(e, \tau)$. In turn, the process for private households expectations keep track of the history of central banks decisions, with :

$$\begin{aligned} e &= e^* \quad \text{if} \quad \pi_{-1} = \pi^k(e_{-1}, \tau_{-1}) \\ e &= e^d \quad \text{if} \quad \pi_{-1} = \pi^r(e_{-1}, \tau_{-1}) \end{aligned} \tag{27}$$

In other terms, if the central bank implemented $\pi^r(\mathcal{S})$ once in the past, private agent expectations carry

[33] The optimal policy mix (π, τ) under restricted commitment would require a more comprehensive analysis of policy trade-offs. We do not pursue this route, as we are not interested in optimal constrained equilibria per se. We rather anchor our discussion in the design of the monetary policy framework.

[34] See formal derivations in Appendix A.5.

[35] Various versions of this construction are discussed in Barro and Gordon (1983), Stokey (1989) or Chari and Kehoe (1990).

this information.[36] The strategic rule $\pi^k(\mathcal{S})$ supports the equilibrium outcome (13) if and only if:

$$\forall \tau \in \mathrm{T}(e^*)\ \Delta(\tau) = \underbrace{\mathcal{L}\big(e^*, \tau, \pi^k(\mathcal{S})\big) - \mathcal{L}\big(e^*, \tau, \pi^r(\mathcal{S})\big)}_{\text{Short Term Gain}} - \underbrace{\frac{\beta}{1-\beta}\Big[\mathcal{L}\big(e^*, \tau^*, \pi^*\big) - \mathcal{L}\big(e^d, \tau^d, \pi^d\big)\Big]}_{\text{Long Term Loss}} \leq 0, \tag{28}$$

where β is a discount factor. Under this construction, deviating from the rule yields a short term gain, which is evaluated against the cost of an infinite repetition of the discretionary equilibrium outcome (6). This term maps precisely into κ, the welfare cost to the central bank of breaking a promise. It must be high enough to deter the short run incentives to renounce the rule, i.e. it must be higher than the cut-off value $\bar{\kappa}$, associated to the short term gain in (28).

Importantly, the reputation construction requires to consider explicitly the strategic decisions of private households within our game, without adding economic insight to our discussion of monetary-fiscal interactions. Given the intuitive interpretation of our reduced form reputation cost κ, we have chosen to abstract from these elements.

2 Dynamic Cash-Credit Economy

The institutional environment developed in Section 1 is now introduced in a dynamic cash-credit economy with public debt, as in Lucas and Stokey (1987). The optimal policy plan is time inconsistent, as both policy institutions have the incentives to alleviate the burden of outstanding public debt.[37] This section studies how the nature and level of public debt influence the design and credibility of monetary rules, in particular when the objective of the central bank is to induce the fiscal authority to follow the optimal policy plan. Formally, with a dynamic and endogenous state variable, the credibility of monetary rules is evaluated against the level of newly issued debt, both on and off equilibrium path.

2.1 Economic Environment

Time is discrete and each period is indexed with $t \geq 0$. The economy is populated by a representative agent and a government. The resource constraint of the economy is

$$c_t + d_t + g = 1 - l_t, \tag{29}$$

where c_t is private consumption of a *credit good*, d_t is private consumption of a *cash good*, $g > 0$ is exogenous and constant public consumption, l_t is leisure, while production is linear in labor $y_t = 1 - l_t$.

[36] Finite forms of punishment are also possible.

[37] The central bank has the incentives to generate "surprise" inflation, collect resources from seignorage and reduce the real value of nominal debt. The fiscal authority is tempted to modify the sequence of tax rates, manipulate the interest rate and the price of newly issued debt. See details in Section 2.2.

Private agents. A representative household enjoys utility from private consumption and leisure:

$$\sum_{t=0}^{\infty} \beta^t U(c_t, d_t, l_t) = \sum_{t=0}^{\infty} \beta^t \big(\alpha \log c_t + (1-\alpha) \log d_t + \gamma l_t\big), \tag{30}$$

where $\beta \in (0,1)$ is the time discount factor.[38]

The household supplies labor to competitive firms that produce both cash and credit goods. The household earns a real wage equal to the (unitary) marginal product of labor and is taxed at a linear rate τ_t. She can buy (sell) nominal one-period risk-free government bonds, B_t^h, at a price q_t in period t. The flow budget constraint in nominal terms in period t reads:

$$P_t c_t + P_t d_t + q_t B_t^h + M_t^h = P_t(1-\tau_t)(1-l_t) + B_{t-1}^h + M_{t-1}^h, \tag{31}$$

where P_t is the price level, and M_t^h is the stock of money, carried over from period t into next period. The purchase of cash good d_t is subject to a cash-in-advance constraint, where the beginning-of-period stock of money M_{t-1}^h sets a cap on expenses:[39]

$$P_t d_t \leq M_{t-1}^h. \tag{32}$$

Finally, exogenous debt limits are in place to prevent Ponzi schemes but they do not bind in equilibrium.

Government. The government consists of a fiscal and a monetary authority. The treasury controls the tax rate τ_t on labor income, and the supply of government bonds B_t. The central bank controls the growth rate of money supply:

$$\sigma_t = M_t / M_{t-1} - 1. \tag{33}$$

Every period t, policies satisfy the budget constraint of the government, which in nominal terms reads:

$$q_t B_t + M_t + P_t \tau_t (1-l_t) = P_t g + B_{t-1} + M_{t-1}. \tag{34}$$

Initial outstanding debt, $B_{-1} = B_{-1}^h$, and stock of money, $M_{-1} = M_{-1}^h$, are exogenous and assumed to be nonnegative.

Real Debt. In the analysis, we contrast the nominal bond economy with one where government bonds are indexed to inflation. These bonds, labelled b_t, are effectively a promise to deliver real payoffs. The budget constraint of the government with inflation-indexed bonds reads:

$$q_t P_{t+1} b_t + M_t + P_t \tau_t (1-l_t) = P_t g + P_t b_{t-1} + M_{t-1}. \tag{35}$$

[38] This particular preference specification allows to derive analytical results. Critical assumptions are separability and log utility from consumption of cash good.

[39] The timing is induced by a segmented market assumption, as in Svensson (1985), Nicolini (1998) or Diaz-Gimenez, Giovannetti, Marimon, and Teles (2008): the market for cash good opens before asset market.

Naturally, in that case, the budget constraint of the household is adjusted similarly.[40]

Competitive equilibrium. Our analysis considers competitive equilibria that arise in this economy. We first provide a formal definition of such an equilibrium and then provide a characterization that is used to study the choice of government policies under different institutional arrangements.

Definition 3. *A competitive equilibrium in an economy with nominal debt consists of a price system* $\{P_t, q_t\}_{t=0}^{\infty}$*, a private sector allocation* $\{c_t, d_t, l_t, M_t^h, B_t^h\}_{t=0}^{\infty}$*, and a government policy* $\{M_t, B_t, \tau_t\}_{t=0}^{\infty}$ *s.t.:*

- *Given initial asset positions* $\{M_{-1}^h, B_{-1}^h\}$ *as well as the price system and the policy, the allocation solves the maximization program of the representative household* (30) *subject to the sequence of household budget constraints* (31)*, the cash-in-advance constraints* (32)*, and the exogenous debt limits.*

- *Given initial liabilities* $\{M_{-1}, B_{-1}\}$ *as well as the allocation and the price system, the policy satisfies the sequence of government budget constraints* (34)*.*

- *All markets clear, hence at all times* $M_t^h = M_t$*,* $B_t^h = B_t$*, and the resource constraint* (29) *holds.*

Naturally, there are multiple competitive equilibria, indexed by different government policies, and this creates a scope for the analysis of policy choice. The following expressions characterize household choice given government policy:

$$U_{l,t} = U_{c,t}(1-\tau_t) \quad \Leftrightarrow \quad (1-\tau_t) = \frac{\gamma}{\alpha} c_t, \tag{36}$$

$$U_{c,t} = \beta \frac{P_t}{P_{t+1}} U_{d,t+1} \quad \Leftrightarrow \quad P_{t+1} d_{t+1} = \frac{\beta(1-\alpha)}{\gamma} P_t (1-\tau_t), \tag{37}$$

$$U_{c,t} = \beta \frac{1}{q_t} \frac{P_t}{P_{t+1}} U_{c,t+1} \quad \Leftrightarrow \quad P_{t+1} c_{t+1} = \frac{\beta}{q_t} P_t c_t. \tag{38}$$

These expressions highlight the influence of public policy on household choices. Equation (36) states that a non zero tax rate drives a wedge on the marginal rate of substitution in leisure-consumption. The second equation maps current consumption in credit good c_t with next period consumption in cash good d_{t+1}, where the wedge is driven by variations in the price level, $\frac{P_t}{P_{t+1}}$, i.e. the real return to holding money M_t. Equation (38) is a standard Euler equation, where the intertemporal allocation in credit good is driven by the inverse real interest rate $\tilde{q}_t$:

$$\tilde{q}_t \equiv q_t \frac{P_{t+1}}{P_t} = \beta \frac{U_{c,t+1}}{U_{c,t}}. \tag{39}$$

Finally, the inequality $(1-\alpha)c_t \geq \alpha d_t$ is a complementary slackness condition due to the cash-in-advance constraint (32).[41] Moreover, this constraint imposes an upper bound on nominal interest rate, $q_t \leq 1$, which

[40] The exposition focuses on the economy with nominal bonds. We only highlight differences with real bonds when relevant, otherwise only notations need to be adjusted.

[41] The optimality conditions (36) – (38) remain the same if government debt is real instead of nominal. Hence if nominal and real bond economies are characterized by different equilibria, this is driven by government choices and its effect on household choices, and not directly by a change in household choices.

in turn imposes the following restriction on the monetary growth rate $(1+\sigma_t) \geq \beta$.[42]

A convenient way to characterize a competitive equilibrium is to substitute away price system and allocation using the equilibrium conditions to obtain an implementability constraint in terms of the sequence of policy instruments.[43]

Lemma 1. *A sequence of tax rates and money growth rates, $\{\tau_t, \sigma_t\}_{t=0}^{\infty}$, supports a competitive equilibrium when government debt is nominal if and only if the following constraint is satisfied for all $t \geq 0$ given z_{-1}:*

$$\beta\left[\frac{(1-\alpha)\beta}{1+\sigma_{t+1}}\right] z_t - \alpha(1-\tau_t) - (1-\alpha)\beta\frac{(1-\tau_t)}{(1+\sigma_t)} + \Phi = \left[\frac{(1-\alpha)\beta}{1+\sigma_t}\right] z_{t-1}, \tag{40}$$

where $\Phi \equiv \big(\beta(1-\alpha)+\alpha-\gamma g\big)$, $z_t \equiv B_t/M_t$, and Ponzi-schemes are ruled out by the exogenous debt limits.

Proof. See Appendix B.1 ∎

In (40), $z_t = \frac{B_t}{M_t}$ is the bond-to-money ratio, the relevant nominal state variable to measure outstanding government debt. The analogous constraint for the economy with real debt b_t is:

$$\beta\left[\frac{\gamma}{(1-\tau_{t+1})}\right] b_t - \alpha(1-\tau_t) - (1-\alpha)\beta\frac{(1-\tau_t)}{(1+\sigma_t)} + \Phi = \left[\frac{\gamma}{(1-\tau_t)}\right] b_{t-1}, \tag{41}$$

Unless the distinction between economies with different types of government debt is of essence, we use the following general way of writing constraints (40) and (41) in discussions below:

$$0 = f(s_t, s_{t-1}, \sigma_{t+1}, \sigma_t, \tau_{t+1}, \tau_t), \tag{42}$$

where $s_t \in \{z_t, b_t\}$ refers to the state variable of the nominal or real debt economy. Given our characterization of equilibria in terms of the sequence of policy instruments, we further assess welfare via an indirect flow utility function.

Lemma 2. *In the competitive equilibrium induced by the policy $\{\tau_t, \sigma_t\}_{t=0}^{\infty}$, the flow utility of the representative household is given by the following indirect utility function:*

$$U(\tau_t, \sigma_t) = \alpha\big[\log(1-\tau_t) - (1-\tau_t)\big] + (1-\alpha)\left[\log\left(\beta\frac{(1-\tau_t)}{(1+\sigma_t)}\right) - \beta\frac{(1-\tau_t)}{(1+\sigma_t)}\right]. \tag{43}$$

Proof. See Appendix B.1 ∎

Illustrative callibration. The analytical results are illustrated with numerical simulations of the model. We use a standard calibration for an annualized model that matches some key statistics and long-run ratios, see Table 1.

[42] When $q_t = 1$, we assume that the household keeps the minimum amount of money required to purchase goods, hence the cash-in-advance constraint always holds with equality.

[43] Following Lucas and Stokey (1983), the optimal policy literature often uses the *primal approach*, whereby one substitutes away the price system and policy instruments using the equilibrium conditions to obtain an implementability constraint in terms of the allocation. We are reversing this approach to keep the focus on the interaction of policy makers with different policy instruments.

Table 1: Parameter values

Parameter	Description	Value
β	Discount factor	0.96
γ	Preference leisure weight	5
α	Preference credit good weight	0.5
g	Public spending	0.05

Parameters are set to target moments of the first best allocation - which is the solution to (30) subject to the sequence of resource constraints (29). The implied moments are $g/(c+d) = 0.25$, the fraction of time devated to leisure $l = 0.75$ and an equal consumption of cash and credit good.

Importantly, to compare allocations and policy outcomes across economies with nominal or indexed debt, we use a correspondence introduced in Diaz-Gimenez, Giovannetti, Marimon, and Teles (2008): at $t \geq 0$, an economy with nominal debt and initial liabilities z_{t-1}, where the government policy results in a choice of cash good d_t, has the same period t, ex post real liabilities as an indexed economy with initial, but predetermined, real liabilities $b_{t-1} = z_{t-1}d_t$.[44]

Next, we characterize policies in an environment where policy makers cooperate over the choice of policy instruments. It provides the basis for understanding the sources of time inconsistency and credibility issues, a pre-requisite to the study of monetary interventions under asymmetric and limited commitment.

2.2 Cooperative Optimal Policy

When monetary and fiscal authorities cooperate over the choice of policy instruments, policy choices are in effect under the control of a single government entity. We contrast policy choices with and without commitment. These cases highlight the properties of the optimal policy plan, and the incentives to deviate from it.

Cooperation with commitment. Consider first the case of a government with a commitment technology to implement a dynamic policy plan $\{\tau_t, \sigma_t\}$ defined at $t = 0$. This commonly studied *Ramsey plan* is useful to understand whether the associated policy is time-consistent, and when it is not, where the sequential incentives to deviate stem from.[45]

Proposition 4. *Given $s_{-1} > 0$, the Ramsey plan has the following characteristics:*

- *"timeless optimal allocation": for all $t \geq 1$, constant path of consumption and leisure. Similarly, tax rates and public liability s_{t-1}, are constant over time, while the money printing rate follows the Friedman rule: $\sigma_t = \beta - 1$.*

- *"incentives to deviate": at $t = 0$, allocation and policy choices differ from the "timeless" levels. In particular, $\tau_0 < \tau_1$ and $\sigma_0 > \sigma_1$. Debt dynamics satisfies:*

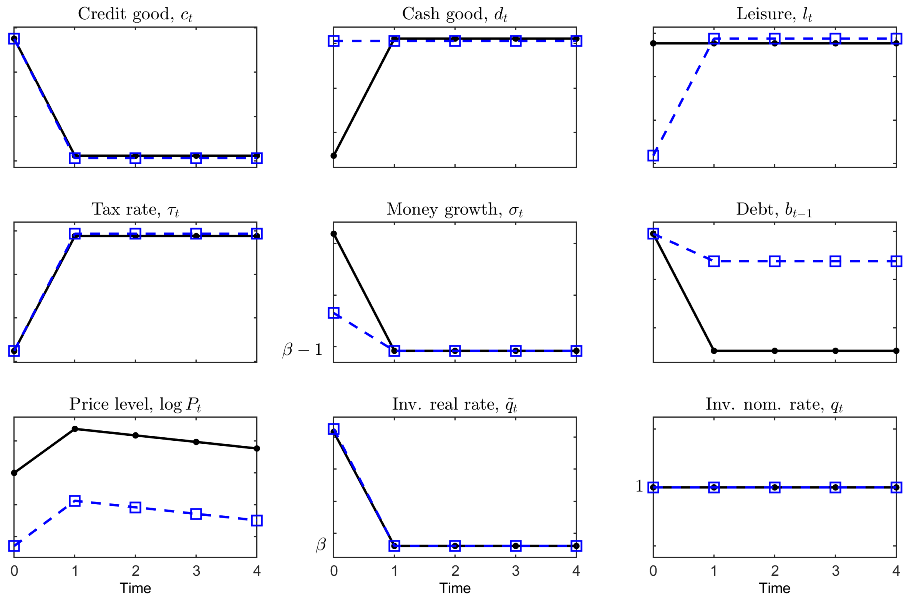
[44] This correspondence makes clear that the ex post real value of nominal debt $z_{t-1}d_t$ is directly influenced by central bank policies, where cash good consumption d_t is related to the effect of the money printing rate on the price level.

[45] *"If the continuation plan of a Ramsey plan is not a Ramsey plan, then the Ramsey plan is time-inconsistent",* Diaz-Gimenez, Giovannetti, Marimon, and Teles (2008).

- *for all $z_{-1} > 0$, newly issued nominal debt z_0 satisfies: $z_0 < z_{-1}$.*
- *in the case of real debt $b_{-1} > 0$, there is $\hat{b} > 0$ s.t. $b_0 < b_{-1}$ if and only if $b_{-1} > \hat{b}$.*

Proof. See Appendix B.2 ∎

Figure 5: Ramsey equilibrium: joint government with commitment



This figure represents allocation, policy instrument and prices when policy is conducted under cooperation and commitment. The dark line refers to the case of nominal debt, the blue line to real debt. The *"timeless"* optimal allocation from $t = 1$ on reflects the tax-smoothing structure of the model, while policy choices at $t = 0$ highlights the incentives to deviate from this stationary policy plan. Nominal and real debt economies are made comparable using the correspondence $b_{-1} = z_{-1}d_0$.

Figure 5 represents graphically the dynamic path of allocations, policy decisions and prices in such a Ramsey equilibrium, both when debt is nominal and real. In both cases, the allocation for $t \geq 1$ reflects the tax-smoothing structure of the model: the consumption bundle $\{c_t, d_t, l_t\}$ is constant over time. Similarly, taxes and the bond-to-money ratio (or real debt level) are constant over time. Finally, the Friedman rule applies: the nominal interest is one, i.e. $q_t = 1$, and the stock of money decreases at a constant rate $\sigma_t = \beta - 1$.[46] We refer to this stationary economic outcome as the *"timeless"* optimal allocation and policy decisions.[47] In the rest of the analysis, and essentially for exposition clarity, we take the *timeless optimal allocation* as our normative benchmark. Given outstanding liabilities s_{-1}, note $W^{ta}(\cdot)$ the associated lifetime welfare of the representative household:

$$W^{ta}(s_{-1}) = \frac{1}{1-\beta} U\big(\tau^*(s_{-1}), \sigma^*\big), \tag{44}$$

where $\tau^*(s_{-1})$ solves the implementability condition (42) under stationary level of debt $s = s_{-1}$ and Friedman deflation $\sigma^* = \beta - 1$.

[46]The intuition behind the Friedman rule is that the government should compensate the household for the friction induced by the cash-in-advance constraint, i.e. for the utility cost of time β.

[47]The *"timeless perspective"* of optimal policy plans is an equilibrium concept suggested by Woodford (1999): optimal policy and allocations should be characterized ignoring initial conditions, as if they were derived in the distant past.

The allocation at $t = 0$ differs when $B_{-1} > 0$ ($b_{-1} > 0$ if debt is real): public debt is critical to induce policy makers to deviate from a pre-announced policy plan. These incentives are multiple, as the model captures both monetary and fiscal policy. As studied in Diaz-Gimenez, Giovannetti, Marimon, and Teles (2008), the money printing rate deviates from the Friedman rate to inflate the real value of outstanding debt (if nominal), and collect tax revenue from the inelastic money tax base.[48] Further, the fiscal deviation is characterized by a tax cut $\tau_0 < \tau_1$, to influence the real interest rate $1/\tilde{q}_0$ and households' willingness to buy newly issued government debt.[49]

Given this characterization, the welfare under the Ramsey plan is simply :

$$W^{rp}(s_{-1}) = \max_{\tau,\sigma,s} U(\tau, \sigma) + \beta W^{ta}(s). \tag{45}$$

Cooperation without commitment. When the government does not have a commitment technology to credibly implement a dynamic policy plan, then arise a dynamic game across policy makers tempted to reap short term welfare gain with systematic *deviations*. Figure 6 represents the dynamic path in a Markov-perfect equilibrium.[50] At each point in time, the government wants to achieve some welfare gain with strategic money printing or tax policy. But it anticipates the future incentives of policy makers to behave similarly. As the incentives to *deviate* increase with the outstanding level of debt, debt left to future policy makers decreases gradually, up to the point where there is no incentives to deviate from the stationary optimal policy plan, i.e. when $B_t = 0$ or $b_t = 0$.

2.3 Non-cooperative Policy Game with Asymmetric Commitment

The institutional set-up presented in Section 1.2 is now applied to the dynamic cash-credit good economy to study the influence of asymmetric and limited commitment on the conduct of public policy. To this end, we consider the following dynamic game where monetary and fiscal policy makers act strategically. Our focus being on the interactions across policy authorities, we treat households as non strategic.[51]

Timing and decisions. At a constitutional stage, before time $t = 0$, the central bank announces a monetary rule ϱ^{Mk} for setting money growth rates, σ_t, at all times in the future.[52] After the constitutional stage, the game unfolds dynamically. In every period $t \geq 0$, the fiscal authority moves first and chooses the tax rate, τ_t. The central bank moves second and chooses the money growth rate, σ_t. The central bank can *keep* the promise and follow the pre-announced rule ϱ^{Mk}. Alternatively, the central bank can *renege*

[48] The incentives to print money are higher under nominal rather than indexed debt, as in the latter case there is no possibility to inflate the real value of debt.

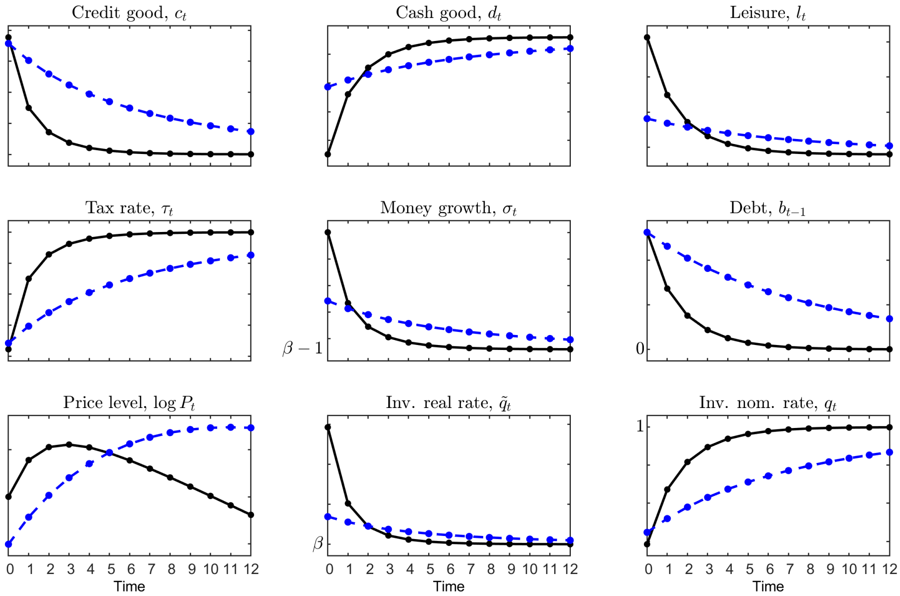
[49] The fiscal incentives to manipulate the real interest rate has been studied in Debortoli and Nunes (2013) and Debortoli, Nunes, and Yared (2017). In particular, the higher willingness of households to buy public debt when $\tau_0 < \tau_1$ and $c_0 > c_1$ is driven by intertemporal substitution effects.

[50] Appendix B.3 defines formally the equilibrium concept, characterizes the long-run level of debt and details the numerical solution. As discussed in the Appendix, we report one specific equilibrium, as is common in the literature, see for instance Debortoli and Nunes (2013).

[51] The definition of the game in Atkeson, Chari, and Kehoe (2010) requires that for all histories, the continuation outcome constitute a competitive equilibrium. This is not a concern in our environment, where the two players of the game operates within the set of competitive equilibria, as defined by the implementatbility constraint (42).

[52] M stands for *monetary* authority and as made clear below, k indicates that the central bank *keeps* its promise when it decides to follow this rule along the game.

Figure 6: Markov equilibrium: joint government without commitment



This figure represents the dynamic allocation induced by cooperative policy makers without commitment. The dark line refers to the case of nominal debt, the blue line to real debt. In both cases, the sequential game induces a gradual reduction in the level of debt, down to $z_t = 0$ or $b_t = 0$, i.e. to the point where policy makers no longer have the incentives to deviate from the optimal stationary policy plan. Nominal and real debt economies are made comparable using the correspondence $b_{-1} = z_{-1}d_0$.

on the promise and deviates from setting the money growth rate prescribed by the rule. Given policy choices, household choose consumption, leisure and savings. The resulting dynamic transition of government liabilities is represented by a function ϱ^s such that $s_t = \varrho^s(s_{t-1}, \tau_t, \sigma_t)$.[53]

Policy objectives and commitment technology. Policy makers are benevolent but differ in their commitment technology. Every period, the fiscal authority sets the tax rate so as to maximize the discounted utility of the houshehold. The central bank then implements its rule ϱ^{Mk} unless it finds it profitable to renounce on its promise. The central bank reneges if it improves the discounted utility of the representative household starting from this period net of an institutional loss $\kappa \geq 0$. This cost $\kappa \geq 0$ born by the central bank if it renounces the rule captures the commitment technology of the monetary authority.[54]

Policy rules and strategies. We study monetary rules ϱ^{Mk} that set the money growth rate as a function of outstanding liabilities s_{t-1}, determined by the history of past play, and the tax rate τ_t set by the fiscal authority. The sequential actions of all the parties involved in the game are described by stationary Markovian strategies. First, the fiscal authority sets the tax rate as a function of outstanding liabilities, $\tau_t = \varrho^F(s_{t-1})$. Second, the decision of the central bank of whether to follow the pre-announced rule is described by an indicator function, $\mathbb{I}^r(s_{t-1}, \tau_t)$, equal to one when the central bank reneges on the promise and zero otherwise.

[53]The transition function captures the conditions associated to the competitive equilibrium Definition 3 and characterized in Lemma 1.

[54]The polar cases where $\kappa = +\infty$ and $\kappa = 0$ correspond respectively to full monetary commitment and no monetary commitment.

When reneging, the choice of the money growth rate by the central bank is described by a policy strategy $\varrho^{Mr}(s_{t-1}, \tau_t)$. Formally, an equilibrium of this game is

Definition 4. *Given a monetary rule ϱ^{Mk}, a Markov-perfect equilibrium of the policy game consists of a transition function ϱ^s, policy strategies $\left(\varrho^{Mr}, \mathbb{I}^r\right)$ and ϱ^F, and value functions $V^M(\cdot), V^F(\cdot)$ such that:*

(i) Given $\left(\varrho^s, \varrho^{Mr}, \mathbb{I}^r\right)$, the fiscal policy ϱ^F and value function V^F solve the following Bellman equation:

$$V^F(s_{t-1}) = \max_{\tau_t} \; U(\tau_t, \sigma_t) + \beta V^F(\varrho^s(s_{t-1}, \tau_t, \sigma_t)),$$

where

$$\sigma_t = [1 - \mathbb{I}^r(s_{t-1}, \tau_t)]\varrho^{Mk}(s_{t-1}, \tau_t) + \mathbb{I}^r(s_{t-1}, \tau_t)\varrho^{Mr}(s_{t-1}, \tau_t).$$

(ii) Given $\left(\varrho^s, \varrho^F\right)$, the monetary policies $\left(\varrho^{Mr}, \mathbb{I}^r\right)$ and value function V^M solve the following Bellman equation:

$$V^M(s_{t-1}, \tau_t) = \max_{\mathbb{I}_t^r \in \{0,1\}} \; \left[1 - \mathbb{I}_t^r\right] V^{Mk}(s_{t-1}, \tau_t) + \mathbb{I}_t^r \left[V^{Mr}(s_{t-1}, \tau_t) - \kappa\right],$$

where the value functions V^{Mk} and V^{Mr}, corresponding respectively to the central bank keeping or reneging on its rule ϱ^{Mk}:

$$V^{Mk}(s_{t-1}, \tau_t) = U(\tau_t, \varrho^{Mk}(s_{t-1}, \tau_t)) + \beta V^M\Big(\underbrace{\varrho^s\left(s_{t-1}, \tau_t, \varrho^{Mk}(s_{t-1}, \tau_t)\right)}_{s_t}, \underbrace{\varrho^F\left(\varrho^s(s_{t-1}, \tau_t, \varrho^{Mk}(s_{t-1}, \tau_t))\right)}_{\tau_{t+1}}\Big),$$

$$V^{Mr}(s_{t-1}, \tau_t) = \max_{\sigma_t} \; U(\tau_t, \sigma_t) + \beta V^M\left(\varrho^s\left(s_{t-1}, \tau_t, \sigma_t\right), \varrho^F\left(\varrho_t^s(s_{t-1}, \tau_t, \sigma_t)\right)\right).$$

(iii) Given $\left(\varrho^F, \varrho^{Mr}, \mathbb{I}^r\right)$, the transition function ϱ^s induces a competitive equilibrium for any $(s_{t-1}, \tau_t, \sigma_t)$:

$$0 = f(s_t, s_{t-1}, \sigma_{t+1}, \sigma_t, \varrho^F(s_t), \tau_t),$$

where

$$\sigma_{t+1} = \mathbb{I}^r(s_t, \varrho^F(s_t))\varrho^{Mr}(s_t, \varrho^F(s_t)) + [1 - \mathbb{I}^r(s_t, \varrho^F(s_t))]\varrho^{Mk}(s_t, \varrho^F(s_t)).$$

The definition highlights the sequential nature of policy moves within periods. The intraperiod leadership structure of policy interactions is explicit, given differences in relevant state variables for each authority. The fiscal authority is a Stackelberg leader within period, and can thus challenge the pre-announced rule ϱ^{Mk} of the central bank. On the other hand, the central bank retains a first mover advantage at the constitutional stage, with the possibility to design threats in reaction to fiscal deviations from a desirable policy plan.

2.3.1 Credibility of the Central Bank

An equilibrium path of this game is the outcome of non-cooperative actions by policy makers. Both are interested in minimizing tax distortions and intertemporal losses, but only the central bank is endowed with a commitment technology, parametrized by the cost κ of renouncing a promise. Intuitively, a monetary rule ϱ^{Mk} announced at the constitutional stage is credible if the central bank keeps the promise along the equilibrium path.

Definition 5. *Let s_{-1} be initial government liability and $\{\tilde{\sigma}_t, \tilde{\tau}_t, \tilde{s}_t\}_{t=0}^{\infty}$ an equilibrium outcome of the policy game given a monetary rule ϱ^{Mk}. The rule ϱ^{Mk} is credible given s_{-1} if $\tilde{\sigma}_t = \varrho^{Mk}(\tilde{s}_{t-1}, \tilde{\tau}_t)$ $\forall t \geq 0$.*

To characterize the commitment intensity required to ensure the credibility of a monetary rule, we study the sequential incentives of fiscal and monetary authorities on and off the equilibrium path.

Start with the fiscal authority and let $V^{Fk}(s_{t-1}, \tau_t)$ be the value to the fiscal authority of choosing a tax rate τ_t, conditional on the monetary policy *keeps* its promise to follow ϱ^{Mk}:

$$V^{Fk}(s_{t-1}, \tau_t) = U\big(\tau_t, \varrho^{Mk}(s_{t-1}, \tau_t)\big) + \beta V^F\big(\varrho^s\big(s_{t-1}, \tau_t, \varrho^{Mk}(s_{t-1}, \tau_t)\big)\big). \tag{46}$$

Similarly, let $V^{Fr}(s_{t-1}, \tau_t)$ be the value function of the fiscal authority when setting a tax rate τ_t conditional on monetary policy *renouncing* the pre-announced rule:

$$V^{Fr}(s_{t-1}, \tau_t) = U\big(\tau_t, \varrho^{Mr}(s_{t-1}, \tau_t)\big) + \beta V^F\big(\varrho^s\big(s_{t-1}, \tau_t, \varrho^{Mr}(s_{t-1}, \tau_t)\big)\big). \tag{47}$$

Consider an equilibrium path $\{\tilde{\sigma}_t, \tilde{\tau}_t, \tilde{s}_t\}_{t=0}^{\infty}$ under a credible monetary monetary strategy. Along this path, the choices of the fiscal authority satisfy:

$$\tilde{\tau}_t = \text{argmax}_{\tau_t}\ V^{Fk}(\tilde{s}_{t-1}, \tau_t) \qquad \text{and} \qquad V^F(\tilde{s}_{t-1}) = V^{Fk}\big(\tilde{s}_{t-1}, \tilde{\tau}_t\big), \tag{48}$$

where the second expression captures the monetary decision to follow its rule along the equilibrium path.

At $\tilde{s}_{t-1}$, the fiscal authority would deviate from $\tilde{\tau}_t$ and set a tax rate $\tau_t \neq \tilde{\tau}_t$ if it were to lead to a welfare improvement, conditional on the central bank renouncing its promise. Define accordingly $\mathrm{T}(\tilde{s}_{t-1})$, the set of *profitable fiscal deviations* at $\tilde{s}_{t-1}$:

$$\mathrm{T}(\tilde{s}_{t-1}) = \{\tau_t \mid V^{Fr}(\tilde{s}_{t-1}, \tau_t) \geq V^F\big(\tilde{s}_{t-1}\big)\}. \tag{49}$$

Consider now the central bank: its incentives to *renounce* its promise at $(\tilde{s}_{t-1}, \tau_t)$ are evaluated against the value to *keep* its promise and follow the rule. Define the *monetary temptation wedge* $\Delta(\tilde{s}_{t-1}, \tau_t)$ as:

$$\Delta(\tilde{s}_{t-1}, \tau_t) = V^{Mr}(\tilde{s}_{t-1}, \tau_t) - V^{Mk}(\tilde{s}_{t-1}, \tau_t). \tag{50}$$

The central bank follows the rule at $(\tilde{s}_{t-1}, \tau_t)$ if $\kappa \geq \Delta(\tilde{s}_{t-1}, \tau_t)$. The rule eliminates the fiscal incentives to deviate at $\tilde{s}_{t-1}$ from $\tilde{\tau}_t$ if $\kappa \geq \Delta(\tilde{s}_{t-1}, \tau_t)$ for all $\tau_t \in \mathrm{T}(\tilde{s}_{t-1})$. In other terms, the rule is implemented at $\tilde{s}_{t-1}$ against any fiscal decisions if the commitment intensity is high enough to deter the incentives to renounce the rule when it would yield the highest welfare gain to the monetary authority:[55]

$$\kappa \geq \Delta(\tilde{s}_{t-1}) = \max_{\tau_t \in \mathrm{T}(\tilde{s}_{t-1})} \left\{ V^{Mr}(\tilde{s}_{t-1}, \tau_t) - V^{Mk}(\tilde{s}_{t-1}, \tau_t) \right\}. \tag{51}$$

We can formally define a credibility cut-off $\bar{\kappa}(s_{-1})$ as the minimum commitment intensity that supports a credible implementation of the rule, given initial liabilities s_{-1}.

Definition 6. *Given initial liabilities s_{-1}, the credibility cut-off of a monetary rule ϱ^{Mk} is defined as*

$$\bar{\kappa}(s_{-1}) = \min\{\kappa \mid \kappa \geq \Delta(\tilde{s}_{t-1}) \ \forall t \geq 0\} \tag{52}$$

Because the evaluation of credibility involves considering off equilibrium paths, the maximum temptation wedge $\Delta = \max_{\tilde{s}_{t-1}} \Delta(\tilde{s}_{t-1})$ might be a function of the commitment intensity κ. If it is the case, provided the dependence is montotone, the credibility cut-off $\bar{\kappa}$ is a fixed point of $\Delta(\kappa)$. Otherwise, the credibility cut-off is simply equal to the highest realization of the temptation wedge, $\bar{\kappa} = \Delta$.

2.4 Policy Game Analysis

In this section, we contrast the policy implications of two classes of monetary rules ϱ^{Mk}. We first describe these rules and discuss their key properties. We then close this section by comparing the credibility cut-offs associated with these strategies.

2.4.1 Monetary Regime 1 – Standard (unconditional) rule

Recall that the optimal *timeless* policy plan characterized in Proposition 4 prescribes the central bank to set a constant negative money growth rate $\sigma^* = \beta - 1$. We study the effect of a similar monetary rule in the context of the policy game. At the constitutional stage the central bank announces a rule from the following class:

$$\varrho^{Mk}(s_{t-1}, \tau_t) = \sigma \geq \beta - 1, \ \forall s_{t-1}, \tau_t. \tag{53}$$

In this case, the central bank promises to follow a policy independent of economic conditions, and in particular fiscal policy decisions.[56] We label this class of rule as *standard* because it prescribes monetary decisions unconditional of fiscal choices. We have the following result.

Proposition 5. *Let κ be arbitrarily large, and the central bank commits to a rule of type* (53). *If debt is nominal, then the policy game results in a stationary allocation and choice of policy instruments, i.e. for all $t \geq 0$, $z_t = z_{-1}$.*

Proof. See Appendix B.4 ∎

[55] This characterization of credibility makes clear that a credible rule eliminates all the fiscal incentives to challenge the rule and the monetary incentives to renounce the rule.

[56] Formally, this class of monetary interventions can be related to Friedman's k-percent rule, a proposal that money supply should be increased by a constant percentage rate every year, irrespective of other economic considerations.

In turn, this proposition has stark implications for the conduct of public policy under asymmetric commitment. Under the same conditions, if the central bank commits unconditionally to the money growth rate prescribed by the Friedman rule $\sigma = \beta - 1$, then the implied allocation coincides with the *timeless* optimal allocation charaterized in Proposition 4.

Corollary 1. *Let κ be arbitrarily large and debt be nominal. If $\sigma = \beta - 1$, then the induced allocation and policy choices coincide with the timeless optimal allocation.*

Proof. See Appendix B.4 ∎

These results deserve some comments. Only when debt is nominal (in contrast to real), and if the central bank is endowed with a technology that eliminates unanticipated money printing, then the fiscal authority has no longer the incentive to manipulate the interest rate to influence the price of newly issued bonds. In other terms, nominal debt provides an opportunity to share commitment across the government with unconditional monetary interventions. The intuition is the following: in contrast to real debt, nominal bonds are subject to de/revaluation. Hence, when the central bank is committed not to deviate from a constant money growth rate, it deters the fiscal incentives to manipulate the interest rate: a variation in the tax rate from the associated stationary optimal allocation generates an offsetting effect on outstanding debt.[57]

When debt is nominal and the central bank follows the constant deflation rule prescribed by the Friedman rule, the monetary authority in effect shares its commitment technology with the treasury, and induces the equilbrium implementation of the timeless allocation. Still, this rule might require a high commitment intensity to be credible. Thus next section derives the existence of strategic rules - contingent on fiscal decisions - which allow to share commitment independently of the nature of public debt, and with the explicit objective to minimize the associated commitment intensity.[58]

2.4.2 Monetary Regime 2 – Strategic Rule

We now consider a class of monetary rules designed to induce the timeless optimal allocation and minimize the required commitment intensity to implement them. Following the construction presented Section 1.3.2, these *strategic rules* are built to offset the welfare effects of fiscal deviations. They rely upon the second mover advantage of the central bank, which can pledge punishment off equilibrium to maintain the appropriate fiscal incentives to follow the *timeless* policy path on equilibrium.

The construction of such strategic rule goes as follow. Let $\tau^*(s_{t-1})$ denote the tax rate required to sustain the *timeless* allocation, given s_{t-1} at any $t \geq 0$. The central bank designs a monetary rule ϱ^{Mk} that satisfies the following properties:

(p.1) if $\tau_t = \tau^*(s_{t-1})$, then $\varrho^{Mk}(s_{t-1}, \tau_t) = \beta - 1$,

[57] Alternatively, the fiscal authority could be tempted to use its tax instruments to influence the real value of outstanding nominal debt, but this would generate an offsetting effect on the price of newly issued bonds.

[58] In Section 2.4.3, we compare numerically the credibility of *standard* and *strategic* rules.

(p.2) if $\tau_t \neq \tau^*(s_{t-1})$, then $\varrho^{Mk}(s_{t-1}, \tau_t) = \sigma$ such that:

$$V^{Fk}(s_{t-1}, \tau_t) \leq W^{ta}(s_{t-1}), \tag{54}$$

where $V^{Fk}(s_{t-1}, \tau_t)$ given by (46) is the value to the fiscal authority to set τ_t.

These properties have the following interpretation. The central bank selects the timeless allocation as an equilibrium path at every s_{t-1}, and sets its policy accordingly. If the strategic rule is credible, given initial liabilities s_{-1}, the induced equilibrium outcome is the timeless allocation. In that case, the welfare to both authorities is $W^{ta}(s_{-1})$, as defined in (44).

The second property is a threat to the fiscal authority if it were to deviate from the timeless policy plan. Intuitively, the credibility of a monetary rule is interwined with the nature and magnitude of these threats, where the larger the punishment associated to a threat, the larger the commitment intensity to implement the strategy.

Accordingly, we restrict our characterization to strategic rules that satisfy (p.1) and (p.2) and minimize the commitment intensity required to implement them in equilibrium.[59] Given the sequential nature of the game, the fiscal authority considers (off equilibrium) deviations $\tau \neq \tau^*(s_{t-1})$ that would lead to a welfare gain if the central bank were to renounce its promise. The set of profitable fiscal deviations at s_{t-1} reads:

$$\mathrm{T}(s_{t-1}) = \big\{\tau \; | V^{Fr}(s_{t-1}, \tau) \geq V^{Fk}\big(s_{t-1}, \tau^*(s_{t-1})\big) = W^{ta}(s_{t-1})\big\}. \tag{55}$$

To minimize the commitment intensity required to sustain the rule, the central bank calibrates its off equilibrium reaction to offset the incentives of the fiscal authority to deviate from the equilibrium path. Formally,

(p.2b) if $\tau_t \in \mathrm{T}(s_{t-1})$ and $\tau_t \neq \tau^*(s_{t-1})$, then $\varrho^{Mk}(s_{t-1}, \tau_t) = \sigma$ such that:

$$V^{Fk}(s_{t-1}, \tau_t) = W^{ta}(s_{t-1}). \tag{56}$$

Given s_{-1}, a strategic rule minimizes the commitment intensity to sustain the timeless allocation if the credibility of the central bank is sufficient to deter the most profitable deviation within $\mathrm{T}(s_{-1})$.[60] The following proposition characterizes the credibility cut-off required to implement such strategic rules when debt is nominal.[61]

Proposition 6. *Given initial liabilities $z_{-1} > 0$, there is a strategic monetary rule ϱ^{Mk} which implements the timeless allocation as an outcome of the game, if and only if the commitment intensity κ of the central bank satisfies:*

$$\kappa \geq \bar{\kappa}_2(z_{-1}) = W^{rp}(z_{-1}) - W^{ta}(z_{-1}), \tag{57}$$

[59] In particular, as shown in Appendix B.5, the existence of these strategies does not rely on the central bank imposing arbitrarily large inflation level to the economy.

[60] For $\tau \notin \mathrm{T}(s_{-1})$, the central bank rule can specify arbitrary σ, since the payoff to the fiscal authority is strictly lower than to abide by the stationary allocation for any σ.

[61] We provide a characterization of credibility for real debt in Section 2.4.3.

with $\frac{d\bar{\kappa}_2(z_{-1})}{dz_{-1}} > 0$.

Proof. See Appendix B.5. ∎

The credibility of the strategic rule that implements the *timeless allocation* with welfare $W^{ta}(s_{-1})$ is evaluated against the most profitable fiscal deviation $\tau \in \mathrm{T}(s_{-1})$, followed by the central bank reneges on its rule, and the government issues debt z. When debt is nominal, this sequential policy path coincides with the cooperative choice under commitment - the *Ramsey plan* - associated with welfare $W^{rp}(z_{-1})$ and newly issued debt $z \leq z_{-1}$, see Proposition 4. This simple characterization arises precisely because the level of debt z issued under the Ramsey plan satisfies $z \leq z_{-1}$, for all $z_{-1} \geq 0$, and the credibility cut-off is increasing in outstanding debt. In our institutional game, if monetary threats are credible to eliminate today the incentives of the fiscal authority to deviate from $\tau^*(z_{-1})$, then the monetary authority would as well deters (off equilibrium) fiscal incentives to deviate from $\tau^*(z)$ tomorrow for all $z \leq z_{-1}$. Overall, the strategic rule is credible if and only if the commitment intensity of the central bank can eliminate the temptation to reoptimize and implement the Ramsey plan.

Figure 7 panels (a) illustrates Propositions 5 and 6, i.e. the credibility challenge associated to each type of monetary rules under nominal debt.[62] Given a level of debt s_{-1}, for both standard and strategic rules, if the commitment intensity κ is high enough to implement the rule in equilibrium, then the equilibrium path is the timeless allocation. But the unconditional rule requires more commitment intensity to be credible than a strategic rule does, since the latter is precisely designed to minimized the required credibility.

Note how the credibility of monetary rules is directly linked to the level of debt z_{-1}: the larger the level of outstanding liabilities, the higher the relative gains to renounce the rule, hence the higher the credibility required to deter fiscal deviations from the timeless allocation. In particular, in the extreme case where $z_{-1} = 0$, there is no policy incentives to manipulate the interest rate or generate inflation beyond Friedman deflation $\sigma = \beta - 1$, hence $\bar{\kappa}_i(0) = 0$. This positive dependence between credibility and relative gains to the monetary authority to renouncing the rule is further illustrated in Panel (b), which represents the credibility cut-off $\bar{\kappa}_2(z_{-1})$ of strategic rules for different utility cost of inflation α. For higher cost of inflation α, the relative gains to renouncing the rule is lower, the required commitment intensity to implement the strategic rule is lower.[63]

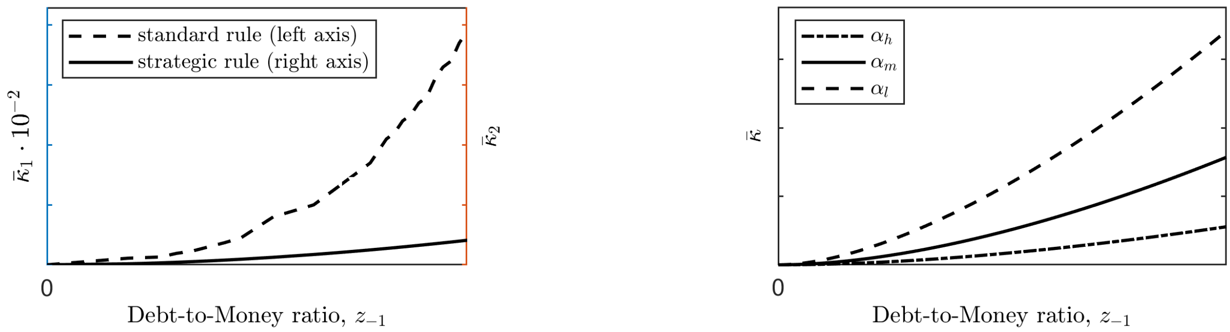
2.4.3 Economic Environment and Credibility

Real and Nominal Debt. As established, the credibility of monetary rule is essentially influenced by the relative gain to renouncing the rule. A similar intuition explains that strategic rules require less commitment intensity to be implemented when debt is indexed instead of nominal, as illustrated in Figure 9. Indeed, when debt is real, there is no possibility to inflate outstanding debt, so that the relative gains to renouncing the rule are lower. Formally, when debt is real, a similar expression to (57) provides an upper bound on the

[62] The characterization of the credibility cut-off for *standard* rules $\bar{\kappa}_1(z_{-1})$ is numerical, since the off equilibrium paths at which credibility is evaluated are associated with spells of central bank playing *reneges* in future periods. Appendix B.6 provides elements related to the solution of the Markov game and characterization of $\bar{\kappa}_1(z_{-1})$.

[63] This result is formally established in Section 1 - Proposition 2 for the linear quadratic game.

Figure 7: Credibility Cut-Offs - Nominal Debt



(a) Standard vs. Strategic Rules

(b) Strategic Rules and Cost of inflation

These figures represent credibility cut-offs for monetary rules under nominal debt. Panel (a) contrasts the commitment intensity required to sustain an unconditional constant money growth rate $\sigma = \beta - 1$ with a strategic rule. Panel (b) displays the sensitivity of credibility cut-offs to variations in the utility cost of inflation $\alpha_h > \alpha_m > \alpha_l$.

credibility cut-off $\bar{\kappa}_2(b_{-1})$:[64]

$$\begin{aligned} \bar{\kappa}_2(b_{-1}) &= W^{rp}(b_{-1}) - W^{ta}(b_{-1}) \quad \text{if } \ b_{-1} \geq \hat{b} \\ &< W^{rp}(b_{-1}) - W^{ta}(b_{-1}) \quad \text{if } \ b_{-1} < \hat{b}. \end{aligned} \tag{58}$$

As seen in Proposition 4, there is a cut-off level of debt $\hat{b}$ such that for higher level of debt $b_{-1} \geq \hat{b}$, newly issued debt under a Ramsey plan satisfies $b \leq b_{-1}$. If the commitment intensity of the central bank is high enough to deter fiscal deviation today at b_{-1}, it is enough to implement (off equilbrium) the timeless allocation tomorrow at lower level b. Accordingly, the inequality (58) is binding.

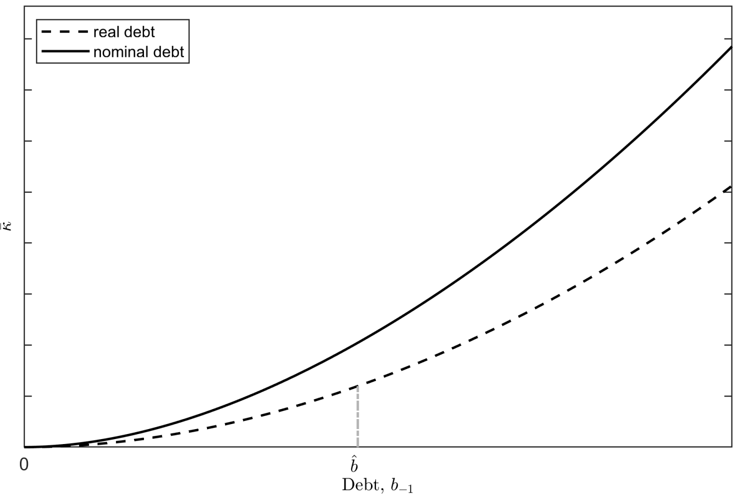
For lower levels of debt $b_{-1} \leq \hat{b}$, a Ramsey planner issues a higher level of debt $b > b_{-1}$. Accordingly, the continuation utility associated with any deviation (τ, σ) is lower than $W^{rp}(b_{-1})$, since it might not be possible to sustain (off equilibrium) the timeless allocation tomorrow at higher level of debt b. Hence, the most profitable deviation within $\mathrm{T}(b_{-1})$ yields a lower relative gain, which reduces the commitment intensity required to defend the strategic rule against fiscal deviations from the timeless allocation. The inequality (58) is strict.

Equilibrium Money Growth Rate and Credibility. Can the central bank adjust its policy framework to a situation where its commitment technology κ is lower than the credibility cut-off $\bar{\kappa}_2(z_{-1})$ required to implement a strategic rule? Consider a case where the central bank designs a strategic rule of the type presented in Section 2.4.2, with an equilibrium money growth rate $\tilde{\sigma} \geq \beta - 1$.[65] By Proposition 5, if the rule is credible, then the equilibrium allocation is stationary at all level of debt z_{t-1}. Note $\tilde{\tau}(z_{t-1}, \tilde{\sigma})$ the tax

[64] Appendix B.5 derives formally this expression.

[65] This analysis is similar to Section 1.5.1 in the linear quadratic game.

Figure 8: Credibility of Strategic Rules: Nominal vs. Real Debt



This figure contrasts the commitment intensity required to sustain the strategic rule under nominal and real debt. $\hat{b}$ corresponds to the level of debt such that the dashed line is an upper bound on $\bar{\kappa}_2(b_{-1})$ for $b_{-1} \leq \hat{b}$. Nominal and real debt economies are made comparable using the correspondence $b_{-1} = z_{-1}d_0$.

rate associated to this stationary allocation. The welfare to both authorities is then:

$$W^s(z_{-1}, \tilde{\sigma}) = \frac{\beta}{1-\beta} U\big(\tilde{\tau}(z_{-1}, \tilde{\sigma}), \tilde{\sigma}\big), \tag{59}$$

where the supercript z stands for *stationary*.[66] The fiscal authority considers the following set of deviations from $\tilde{\tau}(z_{t-1}, \tilde{\sigma})$ at z_{t-1}:

$$\mathrm{T}(z_{t-1}, \tilde{\sigma}) = \big\{\tau \ |V^{Fr}(z_{t-1}, \tau) \geq W^s(z_{-1}, \tilde{\sigma})\big\}. \tag{60}$$

The strategic rule is formally defined as:

(p.1) if $\tau_t = \tilde{\tau}(z_{t-1}, \tilde{\sigma})$, then $\varrho^{Mk}(s_{t-1}, \tau_t) = \tilde{\sigma}$,

(p.2b) if $\tau_t \in \mathrm{T}(z_{t-1}, \tilde{\sigma})$ and $\tau_t \neq \tilde{\tau}(z_{t-1}, \tilde{\sigma})$, then $\varrho^{Mk}(z_{t-1}, \tau_t) = \sigma$ such that:

$$V^{Fk}(z_{t-1}, \tau_t) = W^s(z_{t-1}, \tilde{\sigma}). \tag{61}$$

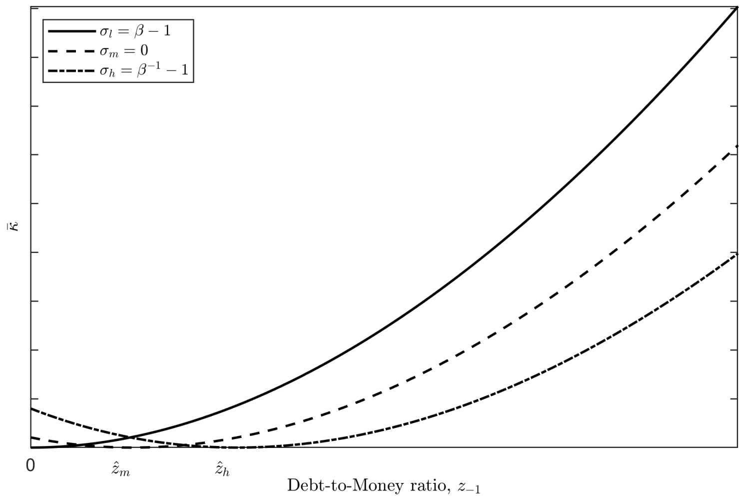
Figure 9 represents the credibility cut-offs $\bar{\kappa}_2(z_{-1}, \tilde{\sigma})$ required to implement strategic rules for different $\tilde{\sigma} \geq \beta - 1$.[67] At high level of debt, the credibility required to enforce a stationary allocation is decreasing in the equilibrium money printing rate. This builds upon the intuition that at higher equilibrium money printing rate $\sigma > \beta - 1$, the relative gains for the monetary authority to renouncing to the rule are lower. Accordingly, the credibility of the rule is higher.[68]

[66] In particular, $W^s(z_{-1}, \beta - 1) = W^{ta}(z_{-1})$.

[67] Appendix B.6 provides elements related to the numerical characterization of $\bar{\kappa}_2(z_{-1}, \tilde{\sigma})$

[68] On the contrary, at lower levels of debt, deviating from the Friedman money growth rate increases required credibility, since the relative gains to renege the promise are low and the permanent welfare cost on equilibrium higher.

Figure 9: Credibility of Strategic Rules: Equilibrium Money Printing Rates



This figure compares credibility cut-offs $\bar{\kappa}_2(z_{-1}, \tilde{\sigma})$ for strategic rules under different levels of equilibrium money growth rates $\tilde{\sigma} \geq \beta - 1$. At higher levels of debt, an increase in the equilibrium money growth rate reduces the relative gains to renouncing the rule, which lowers the commitment intensity required to sustain a stationary allocation. $\hat{z}_k$ for $k \in \{m, h\}$ correspond to cut-off levels of outstanding debt z_{-1} such that the dashed line are actually an upper bound on $\bar{\kappa}_2(z_{-1}, \tilde{\sigma}_k)$ if $z_{-1} < \hat{z}_k$.

Equilibrium path under lack of credibility. Finally, what is the equilibrium path of the economy when the central bank commitment technology cannot sustain the strategic rule given z_{-1}? Consider z_{-1} such that $\kappa < \bar{\kappa}_2(z_{-1})$, smooth deleraging up to z such that $\kappa = \bar{\kappa}(z)$. The initial spells of central bank renouncing its promise weigh on welfare, there might be another strategic rule which given κ improves welfare when z_{-1} is such that $\kappa = \bar{\kappa}(z)$. The analysis of this constrained problem is beyond the scope of our analysis.

3 Conclusions

Can a central bank design monetary interventions to share its commitment technology with a treasury? Does it endanger the credibility of monetary promises?

In this paper, we study how the presence of asymmetric and limited commitment influences the conduct of public policy in environments with fiscal and monetary time consistency problems. The analysis recommends that the central bank designs strategic rules. These are interventions conditional on fiscal decisions to correct the incentives of the treasury and eliminate the temptation to challenge the policy rule of the monetary authority. The benefits of these interventions are twofold. First, the induced equilibrium policy plan is better from a welfare perspective. Second, it relaxes the commitment intensity required for the central bank to reach a monetary objective. This is particularly true in circumstances where the commitment problem of the central bank is the most intense.

We apply our concept of monetary interventions in a standard cash-credit economy, and show how the nature and level of public debt influences the design of monetary strategies and the associated credibility. We unveil an interesting insight. When debt is nominal, unconditional money growth rate interventions curb the time inconsistency problem of fiscal policy plans, but they require significant commitment intensity to be supported in equilibrium. In contrast, when debt is real, monetary interventions need to rely on off equilibrium threats to eliminate fiscal deviations from the dynamic policy plan, but the associated commitment intensity is lower.

Our intention is to further investigate these type of monetary fiscal interactions with political economy frictions, such as situations where the fiscal authority is biased toward short run payoffs.

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A Linear Quadratic Framework

A.1 Policy choices under cooperation - section 1.1.2

Under cooperation, a single government authority decides on policy instruments. Under commitments, the government sets (τ, π) before private agents form expectations $e = \tau^e + \alpha\pi^e$. The objective is to minimize the welfare loss (1), subject to the Phillips curve (2) and private agents expectations (3). Substituting the constraints into the loss function yields:

$$\min_{\tau,\pi} \frac{1}{2}\Big[(\tau - \tau^*)^2 + \lambda(\pi - \pi^*)^2 + \gamma(0 - y^*)^2\Big] \tag{62}$$

The first order conditions naturally leads to $\tau^c = \tau^*$ and $\pi^c = \pi^*$.

When the government does not have a commitment technology, it takes policy decisions after private agents formed expectations $e = \tau^e + \alpha\pi^e$. The objective is to minimize the welfare loss (1), subject to the Philips curve (2), given e:

$$\min_{\tau,\pi} \frac{1}{2}\Big[(\tau - \tau^*)^2 + \lambda(\pi - \pi^*)^2 + \gamma\big(\tau - \tau^e + \alpha(\pi - \pi^e) - y^*\big)^2\Big] \tag{63}$$

The first order conditions give the policy reaction functions to private agents expectations:

$$\tau - \tau^* + \gamma\big(\tau - \tau^e + \alpha(\pi - \pi^e) - y^*\big) = 0 \tag{64}$$

$$\lambda(\pi - \pi^*) + \gamma\alpha\big(\tau - \tau^e + \alpha(\pi - \pi^e) - y^*\big) = 0 \tag{65}$$

In equilibrium, private agents hold rational expectations (3), which yields $\tau^d = \tau^* + \gamma y^*$ and $\pi^d = \pi^* + \frac{\gamma\alpha}{\lambda} y^*$.

A.2 Monetary Regime 1 - unconditional inflation target - section 1.3.1

Equilibrium outcome

Consider the case where the central bank commitment technology is unrestricted: $\pi_1 = \pi^e = \pi^*$. In this case, the fiscal authority solves (7):

$$\tilde{\tau} = \text{argmin}_\tau\ \mathcal{L}(e, \tau, \pi^*), \tag{66}$$

subject to (2). The first order condition yields:

$$\tau - \tau^* + \gamma(\tau - \tau^e - y^*) = 0 \tag{67}$$

In equilibrium, $\tau^e = \tau$, which gives $\tau_1 = \tau^* + \gamma y^*$ and $e_1 = e^* + \gamma y^*$. The loss in equilibrium is then:

$$\mathcal{L}(e_1, \tau^* + \gamma y^*, \pi^*) = \frac{\gamma(1+\gamma)}{2}(y^*)^2. \tag{68}$$

Proposition 1

We want to characterize the credibility cut-off required to deter a scenario where, given a fiscal choice $\tau \neq \tau_1$, the central bank renounces π^* and implements $\pi^d(\mathcal{S})$. Given $e_1 = e^* + \gamma y^*$, we characterize the monetary choice $\pi^d(\cdot)$ conditional on *renege*, the set of possible fiscal deviations and then identify the credibility intensity required for the central bank not to renounce π^* within this set.

Discretionary monetary intervention. Given (e, τ) and conditional on *renege*, the central bank implements $\pi^d(e, \tau) = \text{argmin}_\tau \ \mathcal{L}(e, \tau, \pi^d)$. Simple computations lead to:

$$\pi^d(\mathcal{S}) = \frac{\lambda\pi^* + \gamma\alpha(y^* + e - \tau)}{\lambda + \gamma\alpha^2}. \tag{69}$$

Set of profitable fiscal deviations. Using (8) evaluated at (10):

$$\mathrm{T}(e_1) = \big\{\tau \ \text{ s.t. } \ \mathcal{L}\big(e_1, \tau, \pi^d(e_1, \tau)\big) \leq \mathcal{L}(e_1, \tau_1, \pi^*)\big\}, \tag{70}$$

where $\pi^d(\cdot)$ is given by (69) evaluated at $e_1 = e^* + \gamma y^*$:

$$\pi^d(e_1, \tau) = \pi^* + \frac{\gamma\alpha}{\lambda + \gamma\alpha^2}\big((1+\gamma)y^* + \tau^* - \tau\big). \tag{71}$$

Given the quadractic form of the loss function $\mathcal{L}(\cdot)$, we get $\mathrm{T}(e_1) = [\tau_l, \tau_h]$, where τ_x are the solutions to:

$$\mathcal{L}\big(e_1, \tau, \pi^d(e_1, \tau)\big) = \frac{\gamma(1+\gamma)}{2}(y^*)^2. \tag{72}$$

Rewrite this equation as:

$$(\tau - \tau^*)^2 + \frac{\gamma\lambda}{\lambda + \gamma\alpha^2}\big(\tau - \tau^* - (1+\gamma)y^*\big)^2 = \gamma(1+\gamma)(y^*)^2, \tag{73}$$

and derive

$$\tau_x - \tau^* = y^* \frac{\eta(1+\gamma) \pm \sqrt{(1+\gamma)(\gamma - \eta)}}{1+\eta}, \tag{74}$$

where $\eta = \frac{\gamma\lambda}{\lambda + \gamma\alpha^2}$.

Credibility cut-off. Given $e_1 = e^* + \gamma y^*$ and applying Definition 2, Monetary Regime 1 is credible if and only if:

$$\kappa \geq \max_{\tau \in \mathrm{T}(e_1)} \ \bar{\kappa}(\tau) = \mathcal{L}^k(e_1, \tau, \pi^*) - \mathcal{L}^r\big(e_1, \tau, \pi^d(e_1, \tau)\big), \tag{75}$$

Note that $\bar{\kappa}(\tau)$ is a second order positive polynomial, so its highest value is reached for either τ_l or τ_h. As $\bar{\kappa}(\tau)$ is minimum for $\tau = \tau^* + (1+\gamma)y^*$ and that $[\tau_l, \tau_h]$ is centered around $\tau = \tau^* + (1+\gamma)y^* \frac{\gamma\lambda}{\lambda + \gamma\alpha^2 + \gamma\lambda}$, with $\frac{\gamma\lambda}{\lambda+\gamma\alpha^2+\gamma\lambda} < 1$, we get $\bar{\kappa}(\tau_l) > \bar{\kappa}(\tau_h)$. Accordingly,

$$\bar{\kappa}_1 = \bar{\kappa}(\tau_l) = \mathcal{L}^k(e_1, \tau_l, \pi^*) - \mathcal{L}^r\big(e_1, \tau_l, \pi^d(e_1, \tau_l)\big). \tag{76}$$

Computations lead to:

$$\bar{\kappa}_1 = \frac{(y^*)^2}{2}(\gamma - \eta)(1+\gamma)\Big(\frac{\sqrt{1+\gamma}+\sqrt{\gamma-\eta}}{1+\eta}\Big)^2. \tag{77}$$

Comparative statics. From (77), derive:

$$\frac{d\bar{\kappa}_1}{d\eta} = -\frac{(y^*)^2}{2}(1+\gamma)\Big[\frac{\sqrt{1+\gamma}+\sqrt{\gamma-\eta}}{1+\eta}\Big]\Big[\frac{\sqrt{1+\gamma}+\sqrt{\gamma-\eta}}{1+\eta} + 2\frac{\gamma-\eta}{(1+\eta)^2}\Big(\frac{1}{2}\frac{1+\eta}{\sqrt{\gamma-\eta}} + \sqrt{1+\gamma}+\sqrt{\gamma-\eta}\Big)\Big], \tag{78}$$

which is unambiguously negative. Together with

$$\frac{d\eta}{d\lambda} = \frac{(\gamma\alpha)^2}{(\lambda+\gamma\alpha^2)^2} > 0 \qquad\qquad \frac{d\eta}{d\alpha} = -\frac{2\alpha\gamma^2\lambda}{(\lambda+\gamma\alpha^2)^2} < 0, \tag{79}$$

we get

$$\frac{d\bar{\kappa}_1}{d\lambda} < 0 \qquad\qquad \frac{d\bar{\kappa}_1}{d\alpha} > 0. \tag{80}$$

A.3 Monetary Regime 2 - Strategic Rule - section 1.3.2

Credibility cut-off. Start from (18):

$$\bar{\kappa}_2 = \mathcal{L}^k(e_2, \tau^*, \pi^*) - \min_{\tau,\pi} \mathcal{L}^r(e_2, \tau, \pi), \tag{81}$$

with $e_2 = e^* = \tau^* + \alpha\pi^*$. We get $\mathcal{L}^k(e_2, \tau^*, \pi^*) = \frac{\gamma(y^*)^2}{2}$.

Consider the optimization program $\min_{\tau,\pi} \mathcal{L}^r(e_2, \tau, \pi)$. The first order conditions w.r.t. each policy choice are:

$$(\tau - \tau^*) + \gamma\big(\tau - \tau^* + \alpha(\pi - \pi^*) - y^*\big) = 0, \tag{82}$$

$$\lambda(\pi - \pi^*) + \gamma\alpha\big(\tau - \tau^* + \alpha(\pi - \pi^*) - y^*\big) = 0. \tag{83}$$

Solving this system with unknown variables $\tau - \tau^*$ and $\pi - \pi^*$ leads to the following solution:

$$\tau - \tau^* = \frac{\gamma\lambda y^*}{\lambda + \gamma\alpha^2 + \lambda\gamma} \qquad \text{and} \qquad \pi - \pi^* = \frac{\gamma\alpha y^*}{\lambda + \gamma\alpha^2 + \lambda\gamma}. \tag{84}$$

Evaluating the loss function at this policy outcome:

$$\mathcal{L}^r(\cdot) = \frac{1}{2}\frac{\lambda\gamma(y^*)^2}{\lambda + \gamma\alpha^2 + \lambda\gamma}. \tag{85}$$

Altogether, we get:

$$\bar{\kappa}_2 = \frac{\gamma(y^*)^2}{2}\frac{\gamma(\lambda+\alpha^2)}{\lambda+\gamma\alpha^2+\lambda\gamma}. \tag{86}$$

Comparative statics. From (86), derive:

$$\frac{d\bar{\kappa}_2}{d\lambda} = -\frac{\gamma(y^*)^2}{2}\frac{\gamma\alpha^2}{(\lambda+\gamma\alpha^2+\lambda\gamma)^2} < 0 \qquad ; \qquad \frac{d\bar{\kappa}_2}{d\alpha} = \frac{\gamma(y^*)^2}{2}\frac{2\alpha\lambda\gamma}{(\lambda+\gamma\alpha^2+\lambda\gamma)^2} > 0. \tag{87}$$

Proposition 3. We want to derive conditions on parameters (α, λ), such that $\frac{\bar{\kappa}_1}{\bar{\kappa}_2} > 1$, where $\bar{\kappa}_1$ is given by (77) and $\bar{\kappa}_2$ in (86). Note $f(\alpha,\lambda) = \frac{\bar{\kappa}_1}{\bar{\kappa}_2}$ and derive:

$$f(\alpha,\lambda) = \frac{\alpha^2(1+\gamma)(\lambda+\gamma\alpha^2)}{(\lambda+\lambda\gamma+\gamma\alpha^2)(\lambda+\alpha^2)}\left(\sqrt{1+\gamma}+\sqrt{\gamma-\eta}\right)^2, \tag{88}$$

with $\eta = \frac{\gamma\lambda}{\lambda+\gamma\alpha^2}$. Expanding the quadratic factor:

$$\left(\sqrt{1+\gamma}+\sqrt{\gamma-\eta}\right)^2 = \frac{\lambda+\gamma\lambda+\gamma\alpha^2}{\lambda+\gamma\alpha^2} + \frac{2(\gamma\alpha)^2}{\lambda+\gamma\alpha^2} + 2\gamma\alpha\sqrt{\frac{1+\gamma}{\lambda+\gamma\alpha^2}}, \tag{89}$$

and get:

$$f(\alpha,\lambda) = \frac{\alpha^2(1+\gamma)}{\lambda+\alpha^2} + \frac{2\gamma^2(1+\gamma)\alpha^4}{(\lambda+\lambda\gamma+\gamma\alpha^2)(\lambda+\alpha^2)} + \frac{2\gamma(1+\gamma)^{\frac{3}{2}}\alpha^3\sqrt{\lambda+\gamma\alpha^2}}{(\lambda+\lambda\gamma+\gamma\alpha^2)(\lambda+\alpha^2)}. \tag{90}$$

We immediately get:

$$f(0,\lambda) = 0 \qquad \lim_{\alpha\to+\infty} f(\alpha,\lambda) > 1+\gamma \tag{91}$$

$$f(\alpha,0) > 1+\gamma \qquad \lim_{\lambda\to+\infty} f(\alpha,\lambda) = 0 \tag{92}$$

Note $g(\alpha,\lambda) = \frac{\alpha^2}{\lambda+\alpha^2}$. This function is increasing in α and decreasing in λ. Note $h(\alpha,\lambda) = \frac{\alpha^4}{(\lambda+\lambda\gamma+\gamma\alpha^2)(\lambda+\alpha^2)}$. This function is decreasing in λ. Further,

$$\frac{dh(\cdot)}{d\alpha} = \frac{\frac{dN}{d\alpha}D - \frac{dD}{d\alpha}N}{D^2} \tag{93}$$

with

$$N = \alpha^4 \qquad \frac{dN}{d\alpha} = 4\alpha^3 \tag{94}$$

$$D = (\lambda+\lambda\gamma+\gamma\alpha^2)(\lambda+\alpha^2) \qquad \frac{dD}{d\alpha} = 2\alpha(\lambda+2\gamma\alpha^2+2\lambda\gamma) \tag{95}$$

$\frac{dh(\cdot)}{d\alpha}$ has the sign of H:

$$
\begin{aligned}
H =& \frac{1}{2\alpha^3}\Big(\frac{dN}{d\alpha}D - \frac{dD}{d\alpha}N\Big) && (96)\\
=& 2(\lambda + \lambda\gamma + \gamma\alpha^2)(\lambda + \alpha^2) - \alpha^2(\lambda + 2\gamma\alpha^2 + 2\lambda\gamma) && (97)\\
=& 2\lambda^2 + \lambda\alpha^2 + \lambda^2\gamma + 2\lambda\gamma\alpha^2 > 0 && (98)
\end{aligned}
$$

Note $k(\alpha, \lambda) = \frac{\alpha^2\sqrt{\lambda+\gamma\alpha^2}}{(\lambda+\lambda\gamma+\gamma\alpha^2)(\lambda+\alpha^2)}$. Deriving the monotonicity properties:

$$
\frac{dk(\cdot)}{d\alpha} = \frac{\frac{dN}{d\alpha}D - \frac{dD}{d\alpha}N}{D^2} \tag{99}
$$

with

$$
\begin{aligned}
N &= \alpha^3(\lambda + \gamma\alpha^2)^{\frac{1}{2}} & \frac{dN}{d\alpha} &= \alpha^2(\lambda + \gamma\alpha^2)^{-\frac{1}{2}}(3\lambda + 3\gamma\alpha^2) && (100)\\
D &= (\lambda + \lambda\gamma + \gamma\alpha^2)(\lambda + \alpha^2) & \frac{dD}{d\alpha} &= 2\alpha(\lambda + 2\gamma\lambda + 2\gamma\alpha^2) && (101)
\end{aligned}
$$

$\frac{dk(\cdot)}{d\alpha}$ has the sign of K:

$$
\begin{aligned}
K =& \frac{(\lambda + \gamma\alpha^2)^{-\frac{1}{2}}}{\alpha^2}\Big(\frac{dN}{d\alpha}D - \frac{dD}{d\alpha}N\Big) && (102)\\
=& (3\lambda + 4\gamma\alpha^2)(\lambda + \lambda\gamma + \gamma\alpha^2)(\lambda + \alpha^2) - 2\alpha^2(2\gamma\lambda + 2\gamma\alpha^2 + \lambda)(\lambda + \gamma\alpha^2) && (103)\\
=& \lambda^2\alpha^2 + \gamma\lambda\alpha^{+}4\lambda\gamma^2\alpha^4 + \lambda^2(3\lambda + 4\gamma\alpha^2) + 3\lambda^3\gamma + 6\lambda^2\gamma\alpha^2 > 0 && (104)
\end{aligned}
$$

Overall:

$$
\frac{df(\cdot)}{d\alpha} = (1+\gamma)\frac{dg(\cdot)}{d\alpha} + 2\gamma^2(1+\gamma)\frac{dh(\cdot)}{d\alpha} + 2\gamma(1+\gamma)^{\frac{3}{2}}\frac{dk(\cdot)}{d\alpha} > 0, \tag{105}
$$

which together with (91) gives:

$$
\forall\lambda > 0\ \exists\bar{\alpha} > 0 \text{ s.t. } \forall\alpha > \bar{\alpha},\ \bar{\kappa}_1 > \bar{\kappa}_2. \tag{106}
$$

Similarly,

$$
\frac{dk(\cdot)}{d\lambda} = \frac{\frac{dN}{d\alpha}D - \frac{dD}{d\alpha}N}{D^2} \tag{107}
$$

with

$$
\begin{aligned}
N &= \alpha^3(\lambda + \gamma\alpha^2)^{\frac{1}{2}} & \frac{dN}{d\lambda} &= \frac{\alpha^3}{2}(\lambda + \gamma\alpha^2)^{-\frac{1}{2}} && (108)\\
D &= (\lambda + \lambda\gamma + \gamma\alpha^2)(\lambda + \alpha^2) & \frac{dD}{d\lambda} &= \alpha^2 + 2(\lambda + \lambda\gamma + \gamma\alpha^2). && (109)
\end{aligned}
$$

$\frac{dk(\cdot)}{d\lambda}$ has the sign of K:

$$K = \frac{2(\lambda+\gamma\alpha^2)^{-\frac{1}{2}}}{\alpha^3}\Big(\frac{dN}{d\alpha}D - \frac{dD}{d\alpha}N\Big) \tag{110}$$

$$= (\lambda+\lambda\gamma+\gamma\alpha^2)(\lambda+\alpha^2) - 2\big(\alpha^2 + 2(\lambda+\lambda\gamma+\gamma\alpha^2)\big)(\lambda+\gamma\alpha^2) \tag{111}$$

$$= -(3\lambda^2 + \lambda\alpha^2 + 3\lambda^2\gamma + 6\lambda\gamma\alpha^2 + \gamma\alpha^4 + 4\lambda\gamma^2\alpha^2 + 4\gamma^2\alpha^4) < 0. \tag{112}$$

Overall:

$$\frac{df(\cdot)}{d\lambda} = (1+\gamma)\frac{dg(\cdot)}{d\lambda} + 2\gamma^2(1+\gamma)\frac{dh(\cdot)}{d\lambda} + 2\gamma(1+\gamma)^{\frac{3}{2}}\frac{dk(\cdot)}{d\lambda} > 0, \tag{113}$$

which together with (92) gives:

$$\forall\alpha > 0\ \exists\bar{\lambda} > 0 \text{ s.t. } \forall\lambda < \bar{\lambda},\ \bar{\kappa}_1 > \bar{\kappa}_2. \tag{114}$$

A.4 Extension - Different monetary and fiscal preferences - Section 1.4.

A.4.1 Monetary Conservatism

This section details the derivation of the credibility cut-off $\bar{\kappa}_1$ under Regime 1, when the central bank is inflation conservative. The equilibrium outcome is (10). The set of profitable fiscal deviations reads:

$$\mathrm{T}(e_1) = \big\{\tau \text{ s.t. } \mathcal{L}\big(e_1, \tau, \pi^d(\cdot|\tilde{\lambda})|\lambda\big) \leq \mathcal{L}(e_1, \tau_1, \pi^*|\lambda)\big\}, \tag{115}$$

where λ is the preference parameter to the fiscal authority, $\tilde{\lambda}$ the preference parameter to the central bank. The credibility cut-off is then:

$$\bar{\kappa}_1 = \max_{\tau\in\mathrm{T}(e_1)} \mathcal{L}^k\big(e_1, \tau, \pi^*|\tilde{\lambda}\big) - \mathcal{L}^r\big(e_1, \tau, \pi^d(\cdot|\tilde{\lambda})|\tilde{\lambda}\big). \tag{116}$$

A.4.2 Short Sighted Treasury

Under Regime 1, the equilibrium outcome if the inflation target is credible is:

$$\tau_1 = \tau^* + \tilde{\gamma}y^* \qquad \pi_1 = \pi^* \qquad e_1 = \tau^* + \tilde{\gamma}y^* + \alpha\pi^* \qquad y_1 = 0, \tag{117}$$

where $\tilde{\gamma}$ is the preference parameter of the fiscal authority. The set of profitable fiscal deviations is:

$$\mathrm{T}(e_1) = \big\{\tau \text{ s.t. } \mathcal{L}\big(e_1, \tau, \pi^d(\cdot|\gamma)|\tilde{\gamma}\big) \leq \mathcal{L}(e_1, \tau_1, \pi^*|\tilde{\gamma})\big\}, \tag{118}$$

where γ is the preference parameter of the central bank and $\pi^d(\cdot|\gamma) = \text{argmin}\pi\mathcal{L}(e,\tau,\pi|\gamma)$. The credibility cut-off is then:

$$\bar{\kappa}_1 = \max_{\tau\in\mathrm{T}(e_1)} \mathcal{L}^k\big(e_1,\tau,\pi^*|\gamma\big) - \mathcal{L}^r\big(e_1,\tau,\pi^d(\cdot|\gamma)|\gamma\big). \tag{119}$$

A.5 Extension - Inflation target and strategic rule - Section 1.5.1

The formal derivation of the result follows Appendix A.3 with $e_2 = \tau^* + \alpha\pi^s$ and $\pi^s > \pi^*$:

$$\bar{\kappa}_2(\pi^s) = \mathcal{L}^k(e_2,\tau^*,\pi^s) - \min_{\tau,\pi} \mathcal{L}^r(e_2,\tau,\pi), \tag{120}$$

where

$$\mathcal{L}^k(e_2,\tau^*,\pi^s) = \frac{1}{2}\big[\lambda(\pi^s-\pi^*)^2 + \gamma(y^*)^2\big] \tag{121}$$

Consider the optimization program $\mathcal{L}^r(e_2,\tau,\pi)$. The first order conditions w.r.t. each policy choice:

$$\tau - \tau^* + \gamma(\tau + \alpha\pi - e_2 - y^*) = 0 \tag{122}$$

$$\lambda(\pi - \pi^*) + \gamma\alpha(\tau + \alpha\pi - e_2 - y^*) = 0 \tag{123}$$

Get $\tau - \tau^* = \frac{\lambda}{\alpha}(\pi - \pi^*)$ and then derive:

$$(\lambda + \lambda\gamma + \gamma\alpha^2)\pi = \lambda(1+\gamma)\pi^* + \gamma\alpha^2\pi^s + \gamma\alpha y^* \tag{124}$$

which rewrites:

$$\pi - \pi^* = \frac{\gamma\alpha\big(\alpha(\pi^s-\pi^*) + y^*\big)}{\lambda + \lambda\gamma + \gamma\alpha^2} \tag{125}$$

so that the loss function under *renege* is:

$$\mathcal{L}^r(\cdot) = \frac{\gamma\lambda}{2}\frac{\big(\alpha(\pi^s-\pi^*) + y^*\big)^2}{\lambda + \lambda\gamma + \gamma\alpha^2} \tag{126}$$

and

$$\bar{\kappa}_2(\pi^s) = \frac{1}{2}\Big[\lambda(\pi^s-\pi^*)^2 + \gamma(y^*)^2 - \frac{\gamma\lambda}{\lambda + \lambda\gamma + \gamma\alpha^2}\big(\alpha(\pi^s-\pi^*) + y^*\big)^2\Big] \tag{127}$$

Finally:

$$\frac{d\bar{\kappa}_2(\pi^s)}{d\pi^s}\Big|_{\pi^s=\pi^*} = -\frac{\gamma\lambda y^*}{\lambda + \lambda\gamma + \gamma\alpha^2} < 0. \tag{128}$$

A.6 Extension - Anchoring private agents' expectations

As mentionned in Section 1.2, our evaluation of the commitment intensity of a given strategy on fiscal-monetary interactions. In other terms, private agents' expectations are assumed "well-behaved". As such, variations in private agents' expectations cannot threaten the implementation of a strategy.

In this section, we highlight that a simple additional term in the central bank strategy function can overrule any concerns related to private agents' expectations. Consider a simplified game between a central bank and private agents. First, the central bank commits to π^*. Then private agents form expectations π^e and finally the central bank:

- either implements π^*, output is $y = \pi^* - \pi^e$ and the loss to the economy is

$$\mathcal{L}(\cdot) = \frac{1}{2}\big[(\pi - \pi^*)^2 + \gamma(y - y^*)^2\big], \tag{129}$$

- or renounces its promise, pays a welfare cost κ and implements

$$\pi^d = \text{argmin}_{\pi} \frac{1}{2}\big[(\pi - \pi^*)^2 + \gamma(\pi - \pi^e - y^*)^2\big]. \tag{130}$$

There are two possible equilibrium paths, related to the intensity of the commitment intensity κ:

- If private agents form $\pi^e = \pi^*$, then the central bank implements $\pi = \pi^*$ if and only if $\kappa \geq \bar{\kappa}$.
- If private agents form $\pi^e = \pi^d = \pi^* + \gamma y^*$, then the central bank renounces π^* and implements π^d if and only if $\kappa \leq \bar{\bar{\kappa}}$.

Importantly, $\bar{\kappa} \leq \bar{\bar{\kappa}}$, which means that for intermediate values of $\kappa \in [\bar{\kappa}, \bar{\bar{\kappa}}]$, the credibility of the monetary target π^* depends on self-fulfilling variations in private agents' expectations. To avoid this strategic uncertainty, the central needs to augment its strategy as follow:

$$\pi^s(\mathcal{S}) = \pi^* + \alpha(\pi^e - \pi^*) \qquad \text{with } \alpha \in (0, 1) \tag{131}$$

Under this strategy, if $\kappa \leq \bar{\kappa}$, then the only equilibrium path is one where $\pi^e = \pi^*$.

B Cash-Credit Economy

B.1 Policy representation of a competitve equilibrium

Proof of Lemma 1. Start by proving necessity. First, combine the resource constraint (29), binding cash-in-advance constraint and market clearing conditions to rewrite the household's budget constraint (31) in real terms, after dividing it by M_{t-1}:

$$z_t(1 + \sigma_t)q_t + (1 + \sigma_t) - \Big(\frac{1 - \tau_t}{d_t}\Big)g + \tau_t\Big(1 + \frac{c_t}{d_t}\Big) - 1 = z_{t-1},$$

where $z_t \equiv B_t/M_t$. Second, using household's optimality conditions (36) to (38):

$$\beta\left[\frac{(1-\alpha)\beta}{1+\sigma_{t+1}}\right]z_t - \alpha(1-\tau_t) - (1-\alpha)\beta\frac{(1-\tau_t)}{(1+\sigma_t)} + \Phi = \left[\frac{(1-\alpha)\beta}{1+\sigma_t}\right]z_{t-1},$$

where $\Phi \equiv \big(\beta(1-\alpha) + \alpha - \gamma g\big)$.

To prove sufficiency, consider a sequence $\{\tau_t, \sigma_t\}_{t=0}^{\infty}$ of policy instruments that satisfies implementability constraints (40) and let $\{B_t, M_t\}_{t=0}^{\infty}$ be the associated paths of government nominal liabilities. We derive a sequence of quantities and prices that satisfy Definition (3). Let $B_t^h = B_t$ and $M_t^h = M_t$ for all $t \geq 0$, cash good consumption sequence $\{d_t\}_{t=0}^{\infty}$ satisfies

$$d_t = \frac{\beta(1-\alpha)}{\gamma}\frac{(1-\tau_t)}{(1+\sigma_t)}, \tag{132}$$

credit good consumption $\{c_t\}_{t=0}^{\infty}$ and leisure $\{l_t\}_{t=0}^{\infty}$ be given by (36) and (29). Also, let bond prices $\{q_t\}_{t=0}^{\infty}$ be given by $q_t = \beta/(1+\sigma_{t+1})$. With this construction, (37) and (38) are satisfied and the sequence of implementability constraints (40) implies that (31) and (34) are satisfied. Hence, all conditions of a competitive equilibrium are met by these sequences.

Similar elements allow to derive (41) for real debt.

Proof of Lemma 2. Use the resource constraint (29) to substitute leisure into the utility function:

$$U(c_t, d_t) = \alpha\log(c_t) - \gamma c_t + (1-\alpha)\log(d_t) - \gamma d_t,$$

where constant terms independent of policy are discarded, without loss of generality. Next, substitue c_t and d_t with policy instrument choices τ_t and σ_t with (36) and (132):

$$U(\tau_t, \sigma_t) = \alpha\big[\log(1-\tau_t) - (1-\tau_t)\big] + (1-\alpha)\left[\log\left(\beta\frac{(1-\tau_t)}{(1+\sigma_t)}\right) - \beta\frac{(1-\tau_t)}{(1+\sigma_t)}\right].$$

B.2 Ramsey equilibrium: proof of Proposition 4.

Nominal debt. The Ramsey policy problem determines a whole sequence of policy instruments to maximize households welfare subject to the implementability constraints, given $z_{-1} > 0$:

$$\max_{\{\tau_t,\sigma_t\}} \quad \sum_{t=0}^{\infty}\beta^t\left\{\alpha\big[\log(1-\tau_t) - (1-\tau_t)\big] + (1-\alpha)\left[\log\left(\beta\frac{(1-\tau_t)}{(1+\sigma_t)}\right) - \beta\frac{(1-\tau_t)}{(1+\sigma_t)}\right]\right\}$$

$$\text{subject to} \quad \sum_{t=0}^{\infty}\beta^t\left\{\Phi - \alpha(1-\tau_t) - (1-\alpha)\beta\frac{(1-\tau_t)}{(1+\sigma_t)}\right\} = \left[\frac{(1-\alpha)\beta}{(1+\sigma_0)}\right]z_{-1}, \tag{133}$$

where the intertemporal implementability constraint (133) is derived by forward substitution of (40) in the absence of Ponzi schemes. The first-order conditions read as follows:

$$1 = (1+\lambda)\left[\alpha(1-\tau_t) + (1-\alpha)\beta\frac{(1-\tau_t)}{(1+\sigma_t)}\right], \qquad \forall t \geq 0, \tag{134}$$

$$1 = (1+\lambda)\beta\frac{(1-\tau_t)}{(1+\sigma_t)}, \qquad \forall t \geq 1, \tag{135}$$

$$1 = (1+\lambda)\beta\frac{(1-\tau_0)}{(1+\sigma_0)} + \lambda\frac{\beta}{(1+\sigma_0)}z_{-1}, \tag{136}$$

where $\lambda \geq 0$ is the Lagrange multiplier attached to (133). (134) and (135) imply $\tau_t = \bar{\tau} \equiv \lambda/(1+\lambda)$ and $\sigma_t = \bar{\sigma} \equiv \beta - 1$ for all $t \geq 1$. Using (36), (132) and (29), it is straightforward to get that consumption of both goods, leisure and debt-to-money ratio are constant for all $t \geq 1$, with

$$z_t = \bar{z} \equiv \frac{1}{(1-\alpha)(1-\beta)}\left[\Phi - \frac{1}{1+\lambda}\right], \quad t \geq 0. \tag{137}$$

Contrasting optimality conditions for $t = 0$ and $t \geq 1$ yields $\tau_0 < \bar{\tau}$ and $\sigma_0 > \bar{\sigma}$. Finally, substituting (134) into (133) gives $\bar{z} = \frac{\beta}{1+\sigma_0}z_{-1}$, i.e. if $z_{-1} > 0$ then $\bar{z} < z_{-1}$.

Real debt. Under real debt, the right hand side of the intertemporal implementability condition (133) is $\frac{\gamma}{1-\tau_0}b_{-1}$. Let $\lambda > 0$ be the associated Lagrange multiplier. (134) holds for all $t \geq 1$, (135) holds for all $t \geq 0$. Replace (136) with:

$$1 = (1+\lambda)\left[\alpha(1-\tau_0) + (1-\alpha)\frac{\beta(1-\tau_0)}{1+\sigma_0}\right] - \lambda\frac{\gamma}{(1-\tau_0)}b_{-1}. \tag{138}$$

Stationary real debt level reads:

$$b_t = \bar{b} \equiv \frac{1}{\gamma(1-\beta)(1+\lambda)}\left[\Phi - \frac{1}{1+\lambda}\right], \quad t \geq 0. \tag{139}$$

Use (134) and (138) into the intertemporal implementability condition and get:

$$\bar{b} = \underbrace{\left[\frac{1+2\lambda}{1+\lambda}\right]}_{>1}\underbrace{\left[\frac{\beta}{1+\sigma_0}\right]}_{<1}b_{-1}. \tag{140}$$

To characterize the relation between $\bar{b}$ and b_{-1}, let's consider non negative level of debt such that $\bar{b} = b_{-1}$ from (140).[69] Obviously, $b_{-1} = 0$ is a fixed point, associated with Lagrange multiplier $\lambda = 1/\Phi - 1$. Next, we show there is unique $b^p > 0$ such that if $b_{-1} = b^p$, then $\bar{b} = b^p$. Equation (140) implies :

$$\left[\frac{1+2\lambda^p}{1+\lambda^p}\right]\left[\frac{\beta}{1+\sigma_0}\right] = 1. \tag{141}$$

[69] There is also a negative debt fixed point such that $\lambda = 0$ and the implied allocation coincides with the first-best.

Using (141) and (135), rewrite (138) to characterize b^p using the associated Lagrange multiplier λ^p:

$$b^p = \frac{\alpha}{\gamma}\frac{(1+2\lambda^p)}{(1+\lambda^p)^3} > 0, \tag{142}$$

which together with the definition of $\bar{b}$ (139) implies that the Lagrange multiplier λ^p is a solution to

$$(1+\lambda)\big[\Phi(1+\lambda)-1\big] - \alpha(1-\beta)(1+2\lambda) = 0. \tag{143}$$

This expression is quadratic in λ with one negative and one positive root. The positive root is an admissible solution, which implies existence of a unique positive debt fixed point $b^p > 0$. Further, $\lambda^p > 1/\Phi - 1$ from (139).

Finally, we need to show that $\bar{b} > b_{-1} > 0$ if and only if $0 < b_{-1} < b^p$. Following the steps (140) to (142), one gets that $\bar{b} > b_{-1}$ for all $b_{-1} > 0$ if and only if $b_{-1} < \frac{\alpha}{\gamma}\frac{(1+2\lambda)}{(1+\lambda)^3}$. As λ is increasing in b_{-1} and $\frac{(1+2\lambda)}{(1+\lambda)^3}$ is decreasing in λ, we get that $\bar{b} > b_{-1}$ for lower values of b_{-1}, i.e. for $b_{-1} \in (0, b^p)$.

B.3 Joint government, without commitment

Definition. Consider the economy with nominal debt. A Markov Perfect equilibrium is a triplet of functions $\{\tau(z_{-1}), \sigma(z_{-1}), z(z_{-1}))\}$ and value function $V(z_{-1})$ that solves:

$$V(z_{t-1}) = \max_{\tau_t,\sigma_t,z_t} \left\{\alpha\big[\log(1-\tau_t)-(1-\tau_t)\big] + (1-\alpha)\left[\log\left(\beta\frac{(1-\tau_t)}{(1+\sigma_t)}\right) - \beta\frac{(1-\tau_t)}{(1+\sigma_t)}\right] + \beta V(z_t)\right\},$$

subject to

$$\beta\left[\frac{(1-\alpha)\beta}{1+\sigma(z_t)}\right]z_t - \alpha(1-\tau_t) - (1-\alpha)\beta\frac{(1-\tau_t)}{(1+\sigma_t)} + \Phi = \left[\frac{(1-\alpha)\beta}{1+\sigma_t}\right]z_{t-1}. \tag{144}$$

A differentiable Markov-Perfect equilibrium satisfies (144) and the following optimality conditions:

$$1 = (1+\theta_t)\left[\alpha(1-\tau_t) + (1-\alpha)\beta\frac{(1-\tau_t)}{1+\sigma_t}\right], \tag{145}$$

$$1 = (1+\theta_t)\beta\frac{(1-\tau_t)}{(1+\sigma_t)} + \theta_t\frac{\beta}{1+\sigma_t}z_{t-1}, \tag{146}$$

$$0 = \theta_{t+1} - \theta_t\left[1 - \frac{z_t}{(1+\sigma_{t+1})}\frac{d\sigma(z_t)}{dz_t}\right], \tag{147}$$

where $\theta_t \geq 0$ is the Lagrange multiplier attached to (144).

Long run convergence. In steady state, equation (147):

$$0 = \bar{\theta}\frac{\bar{z}}{(1+\bar{\sigma})}\frac{d\sigma(\bar{z})}{dz}, \tag{148}$$

which indicates the possible existence of different steady states. Steady state properties and transition dynamics depend on the derivative $d\sigma(z_t)/dz_t$.[70]. Our numerical simulations display an equilibrium which

[70]This feature echoes the analysis of fiscal policy under discretion with real debt in Debortoli and Nunes (2013).

converges to a zero debt steady state, $\bar{z} = 0$.

Real debt. Constraint (144) is replaced by (41), state variable is b_{t-1} instead of z_{t-1}, and the optimality conditions (145)–(147) for a differentiable equilibrium:

$$1 = (1+\theta_t)\left[\alpha(1-\tau_t) + (1-\alpha)\beta\frac{(1-\tau_t)}{1+\sigma_t}\right] - \theta_t\frac{\gamma}{(1-\tau_t)}b_{t-1}, \tag{149}$$

$$1 = (1+\theta_t)\beta\frac{(1-\tau_t)}{(1+\sigma_t)}, \tag{150}$$

$$0 = \theta_{t+1} - \theta_t\left[1 + \frac{b_t}{(1-\tau_{t+1})}\frac{d\tau(b_t)}{db_t}\right]. \tag{151}$$

In steady state, equation (151):

$$0 = \bar{\theta}\frac{\bar{b}}{(1-\bar{\tau})}\frac{d\tau(\bar{b})}{db}.$$

As with nominal debt, our numerical simulations feature the steady state with zero debt, $\bar{b} = 0$.

Numerical solution. The model is solved using value function iteration to search for a fixed point of the value function and associated decision rules.

1. Define a grid over s and guess equilibrium functions V^0, τ^0, σ^0.

2. For every grid point s_{t-1}, solve the optimization problem of the government

$$\max_{\tau_t,\sigma_t,s_t}\left\{\alpha\big[\log(1-\tau_t) - (1-\tau_t)\big] + (1-\alpha)\left[\log\left(\beta\frac{(1-\tau_t)}{(1+\sigma_t)}\right) - \beta\frac{(1-\tau_t)}{(1+\sigma_t)}\right] + \beta V^0(s_t)\right\},$$

   subject to

$$0 = f\big(s_t, s_{t-1}, \sigma^0(s_t), \sigma_t, \tau^0(s_t), \tau_t\big).$$

   Call the decision rules that solve this problem (τ^1, σ^1, s^1) and let

$$\begin{aligned} V^1(s_{t-1}) = {} & \alpha\big[\log(1-\tau^1(s_{t-1})) - (1-\tau^1(s_{t-1}))\big] \\ & + (1-\alpha)\left[\log\left(\beta\frac{(1-\tau^1(s_{t-1}))}{(1+\sigma^1(s_{t-1}))}\right) - \beta\frac{(1-\tau^1(s_{t-1}))}{(1+\sigma^1(s_{t-1}))}\right] + \beta V^0(s^1(s_{t-1})). \end{aligned}$$

3. Check convergence of the decision rules and the value function. If convergence is above a desired threshold, set $V^0 = V^1, \tau^0 = \tau^1, \sigma^0 = \sigma^1$ and return to step 2.

The algorithm relies on interpolating the value function and the decision rules off the grid points. The results presented in the paper are based on the implementation of the algorithm in Matlab. Equilibrium functions are interpolated using the cubic splines routine from the CompEcon toolbox; see Miranda and Fackler (2002). The optimization problem of the government is solved using IPOPT, an open source nonlinear optimization solver, in the OPTI toolbox; see Currie and Wilson (2012). Computation speed is improved by parallelizing the step of solving the optimization problem on the grid.

B.4 Nominal debt, standard rule

Proof of Proposition 5. The monetary authority with commitment follows a constant money growth rate rule: $\sigma_t = \sigma \geq \beta - 1$. The Markov-Perfect equilibrium of the game is associated with a solution of the following dynamic programming problem of the fiscal authority:

$$V^F(z_{t-1}) = \max_{\tau_t, z_t} \left\{ \alpha\big[\log(1-\tau_t) - (1-\tau_t)\big] + (1-\alpha)\left[\log\left(\beta\frac{(1-\tau_t)}{(1+\sigma)}\right) - \beta\frac{(1-\tau_t)}{(1+\sigma)}\right] + \beta V^F(z_t) \right\},$$

subject to

$$\beta\left[\frac{(1-\alpha)\beta}{1+\sigma}\right] z_t - \alpha(1-\tau_t) - (1-\alpha)\beta\frac{(1-\tau_t)}{(1+\sigma)} + \Phi = \left[\frac{(1-\alpha)\beta}{1+\sigma}\right] z_{t-1}. \tag{152}$$

The first-order envelope conditions result in the following expressions for all $t \geq 0$:

$$1 = (1+\theta_t)\left[\alpha(1-\tau_t) + (1-\alpha)\beta\frac{(1-\tau_t)}{(1+\sigma)}\right], \tag{153}$$

$$0 = \theta_{t+1} - \theta_t, \tag{154}$$

where $\theta_t \geq 0$ is the Lagrange multiplier associated with (152). (154) implies $\theta_t = \bar{\theta}$ for all $t \geq 0$. Hence, starting from $t = 0$, $\tau_t = \bar{\tau}$ as can be seen from (153) and the equilibrium allocation is constant as per characterization from Lemma 1. Importantly, using the implementability constraint (152) we get that the debt-to-money ratio is equal to its initial outstanding value, i.e., $\bar{z} = z_{-1}$.

Proof of Corollary 1. When $\sigma = \beta - 1$, conditions (153) and (154) imply $\tau_t = \bar{\tau} \equiv \bar{\theta}/(1+\bar{\theta})$. The Lagrange multiplier $\bar{\theta} \geq 0$, in turn, is implicitly determined by the following equation derived from (152):

$$z_{-1} = \frac{1}{(1-\alpha)(1-\beta)}\left[\Phi - \frac{1}{1+\bar{\theta}}\right]. \tag{155}$$

This characterization coincides with the "timeless allocation" induced by the choice of a joint government under commitment for $t \geq 1$, as described in Proposition 5. In particular, comparing (155) and (137) shows that $\bar{\theta} = \lambda$ when z_{-1} is equal to the long-run debt-to-money ratio in a Ramsey equilibrium. The corresponding tax rate and money growth rates also match. Therefore, the equilibrium of the game described in Definition (4) coincides with the stationary "timeless" allocation of a Ramsey plan with stationary level of debt $\bar{z} = z_{-1}$.

Real debt. Constraint (152) is replaced by (41), the state variable is b_{t-1} instead of z_{t-1}, and the optimality conditions (153)–(154):

$$1 = (1+\theta_t)\left[\alpha(1-\tau_t) + (1-\alpha)\beta\frac{(1-\tau_t)}{1+\sigma_t}\right] - \theta_t\frac{\gamma}{(1-\tau_t)} b_{t-1}, \tag{156}$$

$$0 = \theta_{t+1} - \theta_t\left[1 + \frac{b_t}{(1-\tau_{t+1})}\frac{d\tau(b_t)}{db_t}\right]. \tag{157}$$

In contrast to the nominal debt case, the implied equilibrium allocation is not stationary whenever $b_{-1} > 0$.[71]

B.5 Strategic rule - Proposition 6

Monetary rule ϱ^{Mk}. For all $s_{t-1} \geq 0$,

- if $\tau_t = \tau^*(s_{t-1})$, then $\varrho^{Mk}(s_{t-1}, \tau_t) = \beta - 1$,
- if $\tau_t \in \mathrm{T}(s_{t-1})$ and $\tau_t \neq \tau^*(s_{t-1})$, then $\varrho^{Mk}(s_{t-1}, \tau_t) = \sigma$ such that:

$$V^{Fk}(s_{t-1}, \tau_t) = W^{ta}(s_{t-1}), \tag{158}$$

where

$$\mathrm{T}(s_{t-1}) = \big\{\tau \; | V^{Fr}(s_{t-1}, \tau) \geq V^{Fk}\big(s_{t-1}, \tau^*(s_{t-1})\big) = W^{ta}(s_{t-1})\big\}. \tag{159}$$

Equilibrium path if credible. Given $s_{-1} > 0$, if credible, the strategic monetary rule is designed to implement on equilibrium the timeless allocation, i.e. the solution to the game induces the following dynamic path of policy choices:

$$\tau_t = \tau^*(s_{-1}) \qquad \sigma_t = \beta - 1 \qquad s_t = s_{-1}, \quad \forall t \geq 0, \tag{160}$$

where $\tau^*(s_{-1})$ solves a stationary (40), or (41) if debt is real. In that case, the welfare to both monetary and fiscal authorities is $W^{ta}(s_{-1})$, given by (44).

Existence of monetary threats. To prove existence, consider the case of nominal debt and the following sequential policy decision: the fiscal authority chooses $\tau \in \mathrm{T}(s_{-1})$, then the central bank can:

- implement the equilibrium money growth rate $\sigma^* = \beta - 1$, in which case the welfare to the fiscal authority is lower than at the desired equilibrium path by virtue of Proposition 5:

$$\forall \tau, V^F(\tau, \sigma^*, z_{-1}) \leq W^{ta}(\tau^*, \sigma^*, z_{-1}),$$

- or it can optimizes and implements $\sigma^r(\tau, z_{-1}) \geq \sigma^*$, in which case by definition of $\mathrm{T}(z_{-1})$:[72]

$$\forall \tau \in \mathrm{T}(z_{-1}), V^F(\tau, \sigma^r(\cdot), z_{-1}) \geq W^{ta}(\tau^*, \sigma^*, z_{-1}).$$

By continuity, there is $\sigma \in [\sigma^*, \sigma^d]$ s.t.:

$$\forall \tau \in \mathrm{T}(z_{-1}), V^{Fk}(\tau, \sigma, z_{-1}) = W^{ta}(\tau^*, \sigma^*, z_{-1})$$

[71] While this characterization is similar to that of the jointly optimal policy under discretion, the lack of adjustment in money growth rate changes the derivative $d\tau(b_t)/db_t$.

[72] Formally, $\sigma^r(\cdot) = \mathrm{argmax}_\sigma V^M(\cdot)$.

This expression implicitly defines $\varrho^{Mk}(z_{-1}, \tau)$ for $\tau \in \mathrm{T}(z_{-1})$.

Characterization of credibility. This section follows Section 2.3.1 to characterize the commitment intensity requried to implement the strategic monetary rule given s_{-1}. By construction, the welfare to the monetary authority (on and off equilbrium) conditional on the central bank *keeps* its rule

$$V^{Mk}(s_{t-1}, \tau_t) = W^{ta}(s_{t-1}), \tag{161}$$

and the welfare (off equilibrium) conditional on central bank *reneges* its rule

$$V^{Mr}(s_{t-1}, \tau_t) = \max_{\sigma, s} U(\tau, \sigma) + \beta V^M\big(s, \varrho^F(\cdot)\big). \tag{162}$$

The rule is credible at (s_{t-1}, τ_t) if and only if

$$\kappa \geq \Delta(s_{t-1}, \tau_t) = V^{Mr}(s_{t-1}, \tau_t) - V^{Mk}(s_{t-1}, \tau_t) = V^{Mr}(s_{t-1}, \tau_t) - W^{ta}(s_{t-1}), \tag{163}$$

and at s_{t-1} if and only if

$$\kappa \geq \max_{\tau_t \in \mathrm{T}(s_{t-1})} \Delta(s_{t-1}, \tau_t) \tag{164}$$

In particular

$$\max_{\tau_t} V^{Mr}(s_{t-1}, \tau_t) = \max_{\tau_t} \max_{\sigma_t, s_t} U(\tau_t, \sigma_t) + \beta V^M\big(s_t, \varrho^F(\cdot)\big) \tag{165}$$

$$\leq W^{ra}(s_t) = \max_{\tau_t, \sigma_t, s_t} U(\tau_t, \sigma_t) + \beta W^{ta}(s_t) \tag{166}$$

where the inequality comes from the fact that the highest welfare that can be reached given s_{t-1} is induced by a Ramsey plan, see Proposition 4.

We form and then verify the following conjecture: if ϱ^{Mk} is credible at $s_{-1} \geq 0$, then it is credible at $0 \leq s'_{-1} \leq s_{-1}$. Under the conjecture, and if the continuation debt level s_t of a Ramsey plan given s_{t-1} satisfies $s_t \leq s_{t-1}$,

$$\max_{\tau_t} V^{Mr}(s_{t-1}, \tau_t) = W^{ra}(s_{t-1}). \tag{167}$$

Indeed, (i) $\tau^{ra}(s_{t-1})$ the "initial" Ramsey deviation belongs to $\mathrm{T}(s_{t-1})$. (ii) given $\tau^{ra}(s_{t-1})$, the maximum welfare to the monetary authority is reached if the central bank reoptimizes, implements $\sigma^{ra}(s_{t-1})$ and induces $s_t = s^{ra}(s_{t-1})$ with continuation welfare $W^{ta}(s_t)$. (iii) if the rule is credible today, then the timeless allocation is credible at s_t if $s_t < s_{t-1}$, in which case the continuation welfare in (165) is $V^M(s_t, \varrho^F(s_t)) = W^{ta}(s_t)$ since under these conditions $\varrho^F(s_t) = \tau^{ta}(s_t)$. Overall, the strategic rule is credible at s_{t-1} if and only if:

$$\kappa \geq \tilde{\kappa}(s_{-1}) = W^{ra}(s_{t-1}) - W^{ta}(s_{t-1}), \tag{168}$$

and the continuation level of debt under the Ramsey plan is $s_t \leq s_{t-1}$. The credibility cut-off writes then:

$$\bar{\kappa}_2(s_{t-1}) = \tilde{\kappa}(s_{-1}) = W^{ra}(s_{t-1}) - W^{ta}(s_{t-1}). \tag{169}$$

If the continuation level of debt under a Ramsey plan is $s_t > s_{t-1}$, as in (58), then the inequality (166) is strict since the timeless allocation (off equilibrium tomorrow) under s_t might not be induced by the central bank given its commitment intensity κ. In that case:

$$\bar{\kappa}_2(s_{t-1}) < \tilde{\kappa}(s_{-1}) = W^{ra}(s_{t-1}) - W^{ta}(s_{t-1}). \tag{170}$$

Verify the conjecture. Consider the following program:

$$W(z_{-1}) = \max_{\tau,\sigma,z} U(\tau,\sigma) + \beta W^{ta}(z) \tag{171}$$

subject to the implementability constraint (40) and possibly an additional constraint $z = z_{-1}$. Note $\lambda > 0$ and $\mu > 0$ the respective Lagrange multipliers. With both constraints, $W(z_{-1}) = W^{ta}(z_{-1})$ and with the implementability condition only, $W(z_{-1}) = W^{ra}(z_{-1})$. Then:

$$\frac{d\bar{\kappa}_2(z_{-1})}{dz_{-1}} = \frac{dW^{ra}(z_{-1})}{dz_{-1}} - \frac{dW^{ta}(z_{-1})}{dz_{-1}}, \tag{172}$$

$$= -\lambda^{ra}\frac{(1-\alpha)\beta}{1+\sigma^{ra}} - \big(-\lambda^{ta}(1-\alpha) - \mu^{ta}\big), \tag{173}$$

where the second equality comes from the enveloppe conditions of each program. Reorganizing,

$$\frac{d\bar{\kappa}_2(z_{-1})}{dz_{-1}} = (1-\alpha)\Big[\lambda^{ta} - \lambda^{ra}\frac{\beta}{1+\sigma^{ra}}\Big] + \mu^{ta} \tag{174}$$

Since $\lambda^{ta} \geq \lambda^{ra}$ and $\sigma^{ra} > \beta - 1$, one gets $\frac{d\bar{\kappa}_2(z_{-1})}{dz_{-1}} \geq 0$.

When debt is real, a similar argument holds for the upper bound $\tilde{\kappa}(b_{-1})$:

$$\frac{d\tilde{\kappa}(b_{-1})}{db_{-1}} = \gamma\Big[\frac{\lambda^{ta}}{1-\tau^{ta}} - \frac{\lambda^{ra}}{1-\tau^{ra}}\Big] + \mu^{ta} \geq 0, \tag{175}$$

since $\tau^{ta} \leq \tau^{ra}$ and $\lambda^{ta} \geq \lambda^{ra}$.

B.6 Annex on Numerical Computations

This section provides elements regarding numerical simulations reported in Section 2.4.3, and in particular (51), i.e.

$$\bar{\kappa}_i(s_{-1}) = \max_{\tau_0 \in \mathrm{T}(s_{-1})} V^{Mr}(s_{-1}, \tau^0) - V^{Mk}(s_{-1}, \tau^0) \tag{176}$$

Standard (unconditional rule) with nominal debt. The central bank commits unconditionally to $\sigma = \beta - 1$, what is the credibility cut-off given z_{-1}? The numerical solution reported in Figure 7 panel (a) derives $\bar{\kappa}_1(z_{-1})$ as follows. Given κ, solve the game using the value function iteration algorythm described previously. Then compute $\Delta(z_{-1}) = \max_{\tau_0 \in \mathrm{T}} V^{Mr}(\tau_0, z_{-1}) - V^{Mk}(\tau_0, z_{-1})$. Then infer $\bar{z}_{-1}$ s.t. $\kappa = \Delta(\bar{z}_{-1})$, and get $\bar{\kappa}_1(\bar{z}_{-1}) = \kappa$.

Strategic monetary rule with nominal debt. The central band commits to a strategic monetary rule, but target an equilibrium money printing rate $\tilde{\sigma} \geq \beta - 1$. The derivation of associated credibility cut-of $\bar{\kappa}_2(z_{-1}, \tilde{\sigma})$ goes as follows. By proposition 5, if the rule is credible, then the implemented allocation is stationary $(\tilde{\tau}, \tilde{\sigma}, z_{-1})$ and the associated welfare reads

$$W^{st}(z_{-1}, \tilde{\sigma}) = \frac{\beta}{1-\beta} U(\tilde{\tau}, \tilde{\sigma}), \tag{177}$$

where the superscript *st* stands for stationary. In particluar, $W^{st}(z_{-1}, \beta - 1) = W^{ta}(z_{-1})$. Let $\tilde{\Delta}(z_{-1}, \tau_0, \tilde{\sigma})$ be the monetary temptation wedge at (z_{-1}, τ_0). It satisfies:

$$\tilde{\Delta}(z_{-1}, \tau_0, \tilde{\sigma}) \leq \tilde{\kappa}(z_{-1}, \tilde{\sigma}) = \max_{\tau_0, \sigma, z} U(\tau_0, \sigma) + \beta W^{st}(z, \tilde{\sigma}) - W^{st}(z_{-1}, \tilde{\sigma}) \tag{178}$$

As in (B.5), if $z \leq z_{-1}$, then $\bar{\kappa}_2(z_{-1}, \tilde{\sigma}) = \tilde{\kappa}(z_{-1}, \tilde{\sigma})$. If $z \geq z_{-1}$, then $\bar{\kappa}_2(z_{-1}, \tilde{\sigma}) \leq \tilde{\kappa}(z_{-1}, \tilde{\sigma})$, since

$$\max_{\tau_0, \sigma, z} U(\tau_0, \sigma) + \beta V^M(z, \tilde{\sigma}) \leq \max_{\tau_0, \sigma, z} U(\tau_0, \sigma) + \beta W^s(z, \tilde{\sigma}). \tag{179}$$