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Optimal Exit from QE

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Abstract

The paper analyses optimal asset-purchase policies in a macroeconomic model with banks, which face occasionally-binding balance-sheet constraints. It finds that asset-purchase policies should be used to offset large financial disturbances even if interest rates stay unconstrained by an effective lower bound. Optimal exit from a large central bank balance sheet is gradual: the balance sheet should remain positive even after financial tranquillity returns and the banks' balance sheet constraints stop binding. Otherwise, an abrupt exit could reintroduce financial turbulence by pushing banks back to their balance sheet constraints.

Keywords: Large-scale Asset Purchases, Balance-Sheet-Constrained Financial Intermediaries

JEL codes: E32, E44, E52

1 Introduction

Many of the world's leading reserve banks, including the US Federal Reserve and the European Central Bank, have built up large balance sheets to mitigate the macroeconomic fallout of recent financial disturbances, such as the default of Lehman Brothers or the European sovereign debt crisis. A notable feature of these balance sheet policies is their persistence: even though financial conditions and the macroeconomic environment have since turned benign, the central bank balance sheets are still large and are being reduced only gradually, if at all.

Figure 1 illustrates this point. It shows the evolution of the central bank balance sheets and the parallel evolution of two relevant measures of credit conditions: a high-yield corporate bond spread and the excess bond premium in the US and euro area. The excess persistence

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of the balance sheets are apparent. At face value, their disconnect might raise doubts about the realism and optimality of simple QE rules that link central bank balance sheet size to the excess interest rate premium (Gertler and Karadi, 2011).

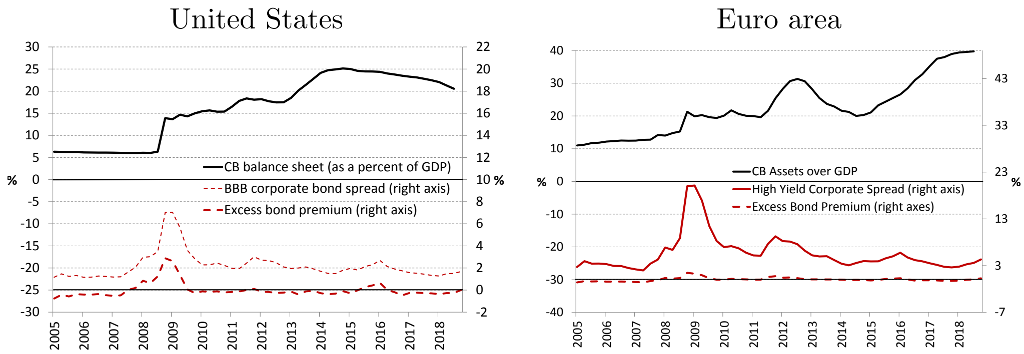

Figure 1: Central bank balance sheets corporate credit spreads in the US and euro area
Note: The figures plot the evolution of the balance sheet of the US Federal Reserve and the European Central Bank as a fraction of US and euro area annual GDP, the Bank of America - Merrill Lynch high-yield option-adjusted corporate bond spreads, and measures of the excess bond premia in the US (Gilchrist and Zakrajsek, 2012) and in the euro area (De Santis, 2018).

Is such a gradual exiting strategy optimal? Should the central bank keep its balance sheet size elevated to avoid returning to an adverse financial environment, or should it quickly reduce the size of its balance sheets to minimize market distortions? Should balance sheet policies become a part of the standard central banking toolkit in the future? Can QE rules, which respond systematically to excess premium, approximate the optimal policy? We address these questions through the lens of a stylized macroeconomic model.

In our model, the central bank purchases long-term government bonds (QE) optimally under commitment. QE can be effective, because financial intermediaries (or, simply, banks) face balance sheet constraints (Gertler and Karadi, 2011), which occasionally bind. When the banks' constraints bind, QE eases credit conditions through freeing up banks' balance sheet capacity and allowing them to extend extra credit to the private sector. The improved credit conditions mitigate the downturn, raise asset prices, improve banks' equity position and further ease banks' credit supply constraints in a positive feedback loop. When banks' constraints are loose, however, additional QE is ineffective because credit is ample and QE fully crowds out private lending. In our calibration, we assume that the latter benign conditions describe the long-term steady state; so, absent disturbances, the optimal central bank portfolio of long-term government bonds is zero.[1] Adverse financial disturbances, however, can push banks to their constraints and lead to impaired credit conditions and a macroeconomic downturn. These financial disturbances are the focus of our analysis, and we model them as exogenous shocks to banks' equity capital.

[1]Admittedly, this is a conservative assumption that pits our calibration against asset purchase policies. If one assumes that financial constraints are binding also in the steady state, this would imply the optimal use of QE also in the long term.

We find that the central bank optimally responds to adverse financial disturbances with asset purchase policies only and keeps interest rates unchanged, provided that QE policies are costless and the central bank itself faces no balance sheet constraints. The reason is that QE policies are better suited to mitigate the root cause of the disturbance, which is the credit crunch caused by the tight balance sheet constraints of banks. In fact, the optimal QE policy fully offsets the macroeconomic impacts of the shock: financial and economic conditions stay as benign and efficient as in the steady state. To achieve this, the central bank follows a linear targeting rule that mirrors the capital shortfall of the banking sector. The central bank immediately increases the size of its balance sheet and commits to exit very gradually, only as fast as the banking sector recapitalizes. The policy eases banks' balance sheet constraints, and allows them to fully satisfy all credit demand while they rebuild their equity capital. Remarkably, optimal QE policy slows down bank recapitalization: benign credit conditions and competition between banks reduces banks' profitability, and, therefore, curtails their means and incentives to rebuild their equity capital quickly. But the slow bank recapitalization is not detrimental for welfare, because the central bank can costlessly offset its impact on credit supply.[2] The results imply that the central bank optimally maintains a positive balance sheet long after the banks' balance sheet constraint stops binding. This may sound surprising: when banks' balance sheet constraints are loose, the reduction of the CB's balance sheet does not have any negative impact on the margin. But financial conditions are benign in the first place precisely because of the large central bank balance sheet: without it, credit supply would be limited. Therefore, a quick reduction of the central bank's balance sheet would be suboptimal, as it would bring back the impaired credit conditions.

We assess the robustness of our conclusions to the case of positive costs and constraints on QE. In particular, we introduce a small quadratic efficiency cost to QE as a reduced-form substitute for unmodelled distortions and political costs of maintaining a positive central bank balance sheet. The optimal exit from QE remains very gradual; the costs, however, change the optimal path of phasing in of QE. The optimal entry is delayed, because staying idle in the presence of deteriorating credit conditions - while detrimental to general welfare and worsening the impact of the shock on banks' balance sheet initially - speeds up the recapitalization of the banking sector by providing banks with excess premium on their lending activity. The quicker recapitalization reduces the necessary size of future central bank balance sheets. Therefore, the central bank is willing to tolerate the short term welfare losses in return for the future savings on the efficiency costs of QE.

We also consider the role of an upper bound on the central bank balance sheet, such as the issue and issuer limits currently honoured by the ECB. We find that our conclusions stay mostly unchanged: the optimal exit policy is still very gradual. The QE upper bound modifies the optimal commitment policy by delaying the date of the first balance-sheet reduction. Such a delay can be implemented as a commitment to reinvesting the principal payments of the

[2] This fact does not generalize to the case when the costs of QE are positive.

maturing assets for a period of time. The delay helps the central bank to partly offset the impact of the upper bound constraint. Optimal exit remains gradual also if the interest rate policy has a sizable room for manoeuvre to complement QE policy. We find that optimal interest rate eases substantially only for a short period of time to counterbalance the negative impact of the credit crunch on the economy, but QE remains the main credit easing instrument in later periods.

How should the central bank use QE policies in the future? Our non-linear model allows us to assess the welfare gains from QE interventions. A key implication of our model is that the welfare benefits of QE policies depend on the size of the financial disturbances: welfare gains are small for small shocks and they increase with the shock size. If QE were costless, then financial shocks should be optimally offset by QE policies alone even if interest rate policies are unconstrained by the lower bound. If, instead, the central bank faces a fixed cost for initiating a QE programme – a reasonable assumption if the central bank risks its reputation with an active programme – then small future disturbances might not justify embarking on QE. For such small shocks, the central bank can rely solely on standard interest rate policies to stabilize the economy. Depending on the size of the variable efficiency costs of QE, initiating QE policies can become optimal again, when the financial shocks are sufficiently large and/or when the policy interest rate becomes constrained by its effective lower bound.

Related literature Our paper is related to an active ongoing research area that assesses optimal asset-purchase policies in dynamic stochastic general equilibrium models. There are two key differences in our framework relative to the literature. First, we analyse an environment, where banks' balance sheet constraints bind only occasionally, so asset purchases are not always effective, while most of the related literature assumes that QE is always effective (Gertler and Karadi, 2011; Carlstrom et al., 2017; Harrison, 2017; Darracq-Paries and Kuehl, 2017).[3] This allows us to analyse the optimal conduct of policy after banks' financial constraints stop binding, a relevant question that previous research has neglected.

Second, we analyze the optimal asset-purchase policies under commitment. This is different from Gertler and Karadi (2011), who analyze optimal simple asset-purchase rules. We find that the simple rule analyzed in Gertler and Karadi (2011), where the central bank's balance sheet tracks the interest rate premium (which is a measure of the tightness of *current* balance sheet constraints), converges to the optimal QE policy as the policy coefficient increases towards infinity. We clarify that in the limiting case of such a policy, QE stays active even after the banks' balance sheet constraints stop binding and interest rate premia stabilizes at zero. Our exercise also complements the work of Harrison (2017), who solves for optimal discretionary policy. In contrast to him, we disregard uncertainty, but derive the Ramsey solution to the non-linear problem under perfect foresight. Similarly to him, we find that gradual exit is optimal, but the mechanism is different: in our case it is driven by the gradual recapitalization

[3] Bocola (2016) also analyzes quantitative easing policies in a related framework where banks face occasionally binding balance sheet constraints. Differently from us, however, the paper concentrates on the impact of sovereign risk on the transmission of QE and does not analyze optimal asset-purchase policies.

of banks, while in his case it is the consequence of households' aversion to quick portfolio-adjustment. Our work is also related to Woodford (2016), who uses a stylized model, Ellison and Tischbirek (2014), who analyze optimal combination of simple interest-rate and asset-price rules, and Darracq-Paries and Kuehl (2017), who analyze optimal commitment policies from a timeless perspective. Similarly to these papers, we find that optimal policy uses both interest-rate and quantitative-easing policies also in normal times, and not just in crisis scenarios when the interest rate lower bound is binding.

We structure our paper as follows. In Section 2, we present our model. Section 3 characterizes optimal policy under costless QE, and Section 4 shows the robustness of the results under costly QE. We conclude in Section 5.

2 Model

We assess optimal asset purchase and interest rate policies in a New Keynesian model (see e.g. Christiano, Eichenbaum and Evans, 2005; Smets and Wouters, 2007) with financial intermediaries (banks, for short, Gertler and Karadi (2011, 2013)) that face occasionally binding balance sheet constraints. In our exercises, the central bank implements these policies to mitigate the negative impact of a financial disturbance. The model also takes into account that the interest rate policy can be constrained by a lower bound and the asset purchase policy can be constrained by an upper bound (issue/issuer limit).

The model has seven agents: households, banks, intermediate-good producers, capital-good producers, retail firms, a fiscal authority and a central bank. Households consume, work, hold short-term deposits at banks and hold long-term government bonds, the latter subject to adjustment cost (Gertler and Karadi, 2013). Banks combine household deposits with equity capital and purchase long-term corporate bonds and long-term government bonds. They face an occasionally binding balance sheet constraint. Intermediate-good producers issue corporate bonds to finance their capital holdings, and use capital and labor to produce intermediate goods. Capital good producers create new capital subject to an investment adjustment cost (Christiano, Eichenbaum and Evans, 2005). Retail firms differentiate intermediate goods and set prices in a staggered fashion a la Calvo (1983). The fiscal authority holds a fixed supply of long-term government bonds, and the central bank conducts interest rate policy subject to an interest rate lower bound and purchases long-term government bonds subject to an upper bound on the total stock of bonds it holds.

2.1 Households

There is a continuum of identical households of measure unity. Households consume, work and invest their savings into bank deposits and short-term and long-term government bonds, the latter subject to adjustment costs.

Each household is comprised of a fraction $1-f$ of workers and a fraction f of bankers. Workers supply labor and return their earnings to the household. Each banker manages a financial intermediary and transfers any earnings similarly back to the household. The households save by holding long-term government bonds (see below) and by depositing funds to intermediaries they do not own. Within the family there is perfect consumption insurance.

The banker has a finite expected lifetime. Each banker stays a banker with probability σ or becomes a worker with probability $1-\sigma$, independently of history. The exiting bankers transfer their net worth to their families. We introduce finite horizon for bankers to insure that over time they do not accumulate so many assets that they never hit any financing constraints. The exiting bankers are replaced by a similar number of workers randomly becoming bankers, keeping the relative proportion of each type fixed. The household provides its new bankers with a small amount of start-up funds. These funds sum to a potentially time-varying amount ω_t across all households. Exogenous time-variation of start-up funds generates fluctuations in the equity position of the banking sector. Financial shocks generated by variation in ω_t will be the focus of our analysis.

Let C_t be consumption and L_t family labor supply. The discounted utility of the household is

$$E_t \sum_{i=0}^{\infty} \beta \left[\ln(C_{t+i} - hC_{t+i-1}) - \frac{\chi}{1+\varphi} L_{t+i}^{1+\varphi} \right] \tag{1}$$

with $0 < \beta < 1$, $0 < h < 1$ and $\chi, \varphi > 0$.

The household saves by holding deposits, short- and long-term government bonds. Both intermediary deposits and government debt are one period real bonds that pay the gross real return R_t from $t-1$ to t. In the equilibrium we consider, the instruments are both riskless and are thus perfect substitutes. Thus, we impose this equilibrium condition from the outset. We denote the total quantity of short-term debt the household acquires by D_{ht}.

We assume long-term government bonds are perpetuities with geometrically decaying coupons: they pay $\varrho^i \Xi$ dollars in periods $i = 0, 1, 2, ...$ We denote the bond holdings of the household by B_{ht}. Let q_t be the price of the bond, and Π_t be the gross inflation rate. Then the real rate of return on the bond R_{bt+1} is given by

$$R_{bt+1}\Pi_{t+1} = \frac{\Xi + \varrho q_{t+1}}{q_t} \tag{2}$$

The variables q_t and Π_t are determined in the general equilibrium of the model, as we show later.

While holding short-term assets is costless, we assume that households can hold long-term government bonds subject to transaction costs.[4] In particular, we suppose that for government

[4]For simplicity, we exclude households from holding long-term private assets. In the euro area, only around 10 percent of outstanding liabilities of non-financial corporations are held directly by non-leveraged institutions according to the sectoral accounts. Incorporating direct capital holdings into our model would only marginally change our quantitative conclusions.

bonds a household faces a holding cost equal to the percentage $\frac{1}{2}\kappa(B_{ht}-\overline{B}_h)^2/B_{ht}$ of the total value of government bonds held for $B_{ht} \geq \overline{B}_h$.

The household budget constraint is

$$C_t + D_{ht} + q_t[B_{ht} + \frac{1}{2}\kappa(B_{ht}-\overline{B}_h)^2] = \frac{W_t}{P_t}L_t + \Gamma_t + T_t + R_t D_{ht-1} + R_{bt}B_{ht-1}, \tag{3}$$

where Γ_t denotes the payouts to the household from ownership of both non-financial and financial firms and, T_t denotes lump sum taxes.

The household's objective is to choose C_t, D_{ht}, B_{ht} to maximize (1) subject to (3). Let $\mu_t = (C_t - hC_{t-1})^{-1} - \beta h(C_{t+1} - hC_t)^{-1}$ denote the marginal utility of consumption. Then the first order conditions for consumption/saving and labor supply are standard:

$$E_t \Lambda_{t,t+1} R_{t+1} = 1 \tag{4}$$

with

$$\Lambda_{t,t+1} \equiv \beta_{t+1}\frac{\mu_{t+1}}{\mu_t}.$$

$$w_t = \chi \mu_t^{-1} L_t^{\varphi} \tag{5}$$

The household's long-term asset demand is given by:

$$B_{ht} = \overline{B}_h + \frac{E_t\Lambda_{t,t+1}(R_{bt+1} - R_{t+1})}{\kappa} \tag{6}$$

Demand for long-term bonds above its frictionless capacity level is increasing in the excess return with an elasticity of the inverse of the curvature parameter κ.

2.2 Banks

Banks collect short-term liabilities from households and use them, together with their own equity capital, to purchase long-term corporate and government bonds.

Long-term corporate bonds provide funding for non-financial firms to finance capital. Let Z_t be the coupon payment from a security that is financing a unit of capital, Q_t, the market value of the security, and δ the depreciation rate of a unit of capital. Then the rate of return on the security, R_{kt+1}, is given by:

$$R_{kt+1} = \frac{Z_{t+1} + (1-\delta)Q_{t+1}}{Q_t} \tag{7}$$

The general equilibrium determines Z_t and Q_t.

2.2.1 The Bank's Problem

Let n_t be the amount of equity capital - or net worth - that a banker has at the end of period t; d_t the deposits the intermediary obtains from households, s_t the quantity of financial claims on non-financial firms that the intermediary holds and b_t the quantity of long-term government bonds. The intermediary balance sheet is then given by

$$Q_t s_t + q_t b_t = n_t + d_t \tag{8}$$

Net worth is accumulated through retained earnings. It is thus the difference between the gross return on assets and the cost of liabilities:

$$n_t = R_{kt} Q_{t-1} s_{t-1} + R_{bt} q_{t-1} b_{t-1} - R_t d_{t-1} \tag{9}$$

From (8) and (9), net worth evolves as

$$n_t = (R_{kt} - R_t) Q_{t-1} s_{t-1} + (R_{bt} - R_t) q_{t-1} b_{t-1} + R_t n_{t-1}. \tag{10}$$

The banker's objective is to maximize the discounted stream of payouts back to the household, where the relevant discount rate is the household's intertemporal marginal rate of substitution, $\Lambda_{t,t+i}$. To the extent the intermediary faces financial market frictions, it is optimal for the banker to retain earnings until exiting the industry. Accordingly, the banker's objective is to maximize expected terminal wealth.

$$V_t(s_t, b_t, n_t) = E_t \sum_{i=1}^{\infty} (1-\sigma)\sigma^{i-1} \Lambda_{t,t+i} n_{t+i} \tag{11}$$

To motivate a limit on the bank's ability to obtain deposits, banks face the following moral hazard problem: At the beginning of the period the banker can choose to divert funds from the assets it holds and transfer the proceeds to the household of which he or she is a member. The cost to the banker is that the depositors can force the intermediary into bankruptcy and recover the remaining fraction of assets.

We assume that it is easier for the bank to divert funds from its holdings of private loans than from its holding of government bonds: In particular, it can divert the fraction θ of its private loan portfolio and the fraction $\Delta\theta$ with $0 \leq \Delta < 1$, from its government bond portfolio.

Accordingly, for depositors to be willing to supply funds to the banker, the following incentive constraint must be satisfied

$$V_t(s_t, b_t, n_t) \geq \theta Q_t s_t + \Delta\theta q_t b_t. \tag{12}$$

The left side is what the banker would lose by diverting a fraction of assets. The right side is the gain from doing so.

The bankers maximization problem is to choose s_t, b_t and to maximize $V_t(s_t, b_t, n_t)$ subject to (10) and (12).

In the resulting equilibrium banks face an occasionally binding constraint on their 'risk-adjusted' leverage:

$$\phi_t = \frac{Q_t s_t + \Delta q_t b_t}{n_t} \leq \overline{\phi}_t \tag{13}$$

where $\overline{\phi}_t$ is the maximum leverage ratio:

$$\overline{\phi}_t = \frac{E_t \Omega_{t,t+1} R_{t+1}}{\theta - E_t \Omega_{t,t+1}(R_{kt+1} - R_{t+1})} \tag{14}$$

with Ω_t being the banks' discount factor.[5] The maximum leverage ratio $\overline{\phi}_t$ ensures that the bank abstains from absconding with a fraction of its assets. Furthermore, the two assets do not enter the same way into the balance sheet constraint. Government bonds burden the banks' balance sheet capacity less than private assets. This is a direct consequence of the difference in the assets' absconding rates: households are willing to extend more funds to banks that hold more government bonds, because they could abscond with less of these. The relative absconding rate Δ determines the 'risk-weight' of the government bonds relative to private assets.

The equilibrium also requires that the banks are indifferent between investing into corporate and into government bonds. Their arbitrage condition is

$$\Delta E_t \Omega_{t+1}(R_{kt+1} - R_{t+1}) = E_t \Omega_{t+1}(R_{bt+1} - R_{t+1}). \tag{15}$$

The condition accounts for the lower relative absconding rate of government bonds, which allows banks to raise more outside funding (face lower margin requirements) for their government bond holdings.

If the leverage constraint is loose ($\phi_t < \overline{\phi}_t$) then excess returns $E_t \Omega_{t,t+1}(R_{kt+1} - R_{t+1}) = E_t \Omega_{t,t+1}(R_{bt+1} - R_{t+1}) = 0$ are zero.

2.2.2 Aggregation

Let S_t be the total quantity of corporate bonds that banks intermediate, B_{bt} the total number of government bonds they hold, and N_t their total net worth. We consider a symmetric equilibrium, where each bank maintains the same leverage (note that their net worth can differ). In this case, we can simply sum across all individual banks (13) to obtain

$$\phi_t = \frac{Q_t S_t + \Delta q_t B_{bt}}{N_t} \leq \overline{\phi}_t \tag{16}$$

[5] The banks' discount factor is $\Omega_t = \Lambda_{t-1,t}(1 - \sigma + \sigma \eta_t)$, where η_t is the shadow value of a unit of net worth at the beginning of the period. The banks' discount factor is different from the household discount factor whenever bank net worth exceeds unity as a result of a balance sheet constraint that binds or has the potential to bind in the future.

Equation (16) restricts the aggregate banking system leverage to be less than or equal to the maximum leverage. When the constraint is binding, variation in N_t will induce fluctuations in overall asset demand by intermediaries.

Total net worth evolves as the sum of retained earnings by the fraction σ of surviving bankers and the transfers that new bankers receive, ω_t, as follows.

$$N_t = \sigma[(R_{kt} - R_t)\frac{Q_{t-1}S_{t-1}}{N_{t-1}} + (R_{bt} - R_t)\frac{q_{t-1}B_{bt-1}}{N_{t-1}} + R_t]N_{t-1} + omega_t \tag{17}$$

Changes in the transfer of new bankers $\omega_t = \omega + e_{\omega,t}$, where ω is the steady state value of the transfers and $e_{\omega,t}$ is an iid innovation, causes exogenous variation in the banking system's aggregate net worth (N_t). The variation is endogenously amplified by the ex post return on loans R_{kt} and the ex post return on bonds R_{bt}. Further, the percentage impact of this return variation on N_t in each case, is increasing in the bank's degree of leverage, reflected by the respective ratios of assets to net worth, $Q_{t-1}S_{t-1}/N_{t-1}$ and $q_{t-1}B_{bt-1}/N_{t-1}$.

2.3 Central Bank Asset Purchases

If private intermediation is balance sheet constrained, excess returns on assets arise with negative consequences for the cost of capital and real activity. Within our model, large-scale asset purchases provide a way for the central bank to reduce excess returns and thus mitigate the consequences of a disruption of private intermediation.

In particular, we now allow the central bank to purchase long-term government bonds[6] in quantity B_{gt} for a market price q_t. Let B_t be the total supply of long term government bonds. The purchases will reduce the private holdings of these bonds

$$B_t = B_{bt} + B_{ht} + B_{gt} \tag{18}$$

where as before B_{bt} are the total amounts that are privately intermediated, and B_{ht} is the direct government bond holding of the households determined by equation (6).

When banks' balance sheet constraints are binding the central bank's acquisition of long-term government bonds will bid up the price of this asset. In turn, this will ease banks' constraints, and allow them to extend new lending to non-financial corporations. The easier credit conditions will stimulate demand, raise the value of corporate bonds and reduce their excess returns. To finance asset purchases, the central bank issues riskless short term debt D_{gt} that pays the safe market interest rate R_{t+1}. In particular, the central bank's balance sheet is given by

$$q_t B_{gt} = D_{gt} \tag{19}$$

[6]For simplicity, we disregard government purchases of corporate bonds. In the euro area, only around 20 percent of the Eurosystem asset holdings originates outside of the public sector (including corporate bonds, covered bonds and asset-backed securities). Taking their purchases explicitly into account, which can be done with our model, would not change our qualitative conclusions.

where we assume that the central bank turns over any profits to the Treasury and receives transfers to cover any losses. We suppose that the central bank issues the short term debt to households.[7]

These kinds of asset purchases essentially involve substituting central bank intermediation for private intermediation. What gives the central bank an advantage in this situation is that, unlike private intermediaries it is able to obtain funds elastically by issuing short-term liabilities. It is able to do so because within our framework the government can always commit credibly to honoring its debt. Accordingly, there is no agency conflict that inhibits the central bank from obtaining funds from the private sector. Put differently, in contrast to private financial intermediation, central bank intermediation is not balance sheet constrained.

At the same time, we allow the central bank to be less efficient than the private sector at making loans. In particular, we assume the central bank pays a quadratic efficiency cost of τ on the square of the government bonds it intermediates. Accordingly, for asset purchases to produce welfare gains, the central bank's advantage in obtaining funds cannot be offset by its disadvantage in making loans. Its advantage in obtaining funds is greatest when excess returns are large (i.e when limits to private arbitrage are tight).

When banks' balance sheet constraints are loose $\phi_t < \overline{\phi}_t$, positive purchases ($B_{gt}$) are ineffective on the margin. In this case, the amount of lending to non-financial corporations (S_t^*) is determined by a no-arbitrage condition $E_t\Omega_{t,t+1}(R_{kt+1} - R_{t+1}) = 0$, and stays unchanged if banks sell part of their government bond holdings to the government and reduce their leverage further. The situation, however, is not symmetric: it can turn effective when the central bank resells long-term bonds it holds. In this case, even if banks' balance sheet constraints are loose, the policy can have real effects if it raises banks' leverage so much that it makes the leverage constraints binding again.

2.4 Production, Fiscal Policy and Equilibrium

We now close the model by describing the non-financial production sector, and the general equilibrium.

There are three types of non-financial firms in the model: intermediate goods producers, capital producers, and monopolistically competitive retailers. The latter are in the model to introduce nominal price rigidities. We describe each in turn.

2.4.1 Intermediate Goods Producers

Intermediate goods producers produce output and sell to retailers. They are competitive and earn zero profits in equilibrium. Each operates a constant returns to scale technology with

[7] Alternatively, we could interpret D_{gt} as interest bearing reserves (essentially overnight government debt) held by banks on account at the central bank. It is equivalent to our baseline case with short-term debt to households, if we assume that banks cannot abscond with reserves.

capital and labor. Let Y_t be output, A_t total factor productivity, L_t labor, K_t capital, Then:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{20}$$

Let P_{mt} be the relative price of intermediate goods. Then the firm's demand for labor is given by

$$W_t = P_{mt}(1-\alpha)\frac{Y_t}{L_t} \tag{21}$$

It follows that we may express gross profits per unit of capital Z_t as follows:

$$Z_t = P_{mt}\alpha\frac{Y_t}{K_t}. \tag{22}$$

The acquisition of capital works as follows. At the end of any period t, the intermediate goods producer is left with a capital stock of $(1-\delta)K_t$. It then buys I_t units of new capital from capital producers. Its capital stock for $t+1$ is then given by

$$K_{t+1} = I_t + (1-\delta)K_t \tag{23}$$

To finance the new capital, the firm must obtain funding from a bank. For each new unit of capital it acquires it issues a state-contingent claim to the future stream of earnings from the unit. Banks are able to perfectly monitor firms and enforce contracts. As a result, through competition, the security the firm issues is perfectly state-contingent with producers earning zero profits state-by-state. In addition, the value of the security Q_t is equal to the market price of the capital underlying the security. Finally, the period $t+1$ payoff is $Z_{t+1}+(1-\delta)Q_{t+1}$: the sum of gross profits and the value of the leftover capital, which corresponds to the definition of the rate of return in equation (7).

2.4.2 Capital Goods Producers

Capital producers make new capital using as input final goods and subject to adjustment costs. They sell the new capital to firms at the price Q_t. Given that households own capital producers, the objective of a capital producer is to choose I_t to solve:

$$\max E_t \sum_{\tau=t}^{\infty} \Lambda_{t,\tau} \left\{ Q_\tau^i I_\tau - \left[1 + f\left(\frac{I_\tau}{I_{\tau-1}}\right)\right] I_\tau \right\} \tag{24}$$

From profit maximization, the price of capital goods is equal to the marginal cost of investment goods production as follows,

$$Q_t = 1 + f\left(\frac{I_t}{I_{t-1}}\right) + \frac{I_t}{I_{t-1}} f'(\frac{I_t}{I_{t-1}}) - E_t\Lambda_{t,t+1}(\frac{I_{t+1}}{I_t})^2 f'(\frac{I_{t+1}}{I_t}) \tag{25}$$

Profits (which arise only outside of steady state), are redistributed lump sum to households.

2.4.3 Retail Firms

Final output Y_t is a CES composite of a continuum of mass unity of differentiated retail firms, that use intermediate output as the sole input. The final output composite is given by

$$Y_t = \left[\int_0^1 Y_{ft}^{\frac{\varepsilon-1}{\varepsilon}} df \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{26}$$

where Y_{ft} is output by retailer f.

Retailers simply re-package intermediate output. It takes one unit of intermediate output to make a unit of retail output. The marginal cost is thus the relative intermediate output price P_{mt}. As in Rotemberg and Woodford (1997), we assume a constant steady state tax (ς) on the revenue of the retail firms. The subsidy is set to offset steady state distortions caused by the retail firms' market power ($1-\varsigma = (\varepsilon-1)/\varepsilon$). We introduce nominal rigidities following Calvo. In particular, each period a firm is able to freely adjust its price with probability $1-\gamma$. Accordingly, each firms chooses the reset price P_t^* to maximize expected discounted profits subject to the restriction on the adjustment frequency. Following standard arguments, the first order necessary condition for this problem is given by:

$$E_t \sum_{i=0}^{\infty} \gamma^i \Lambda_{t,t+i} \left[\frac{P_t^*}{P_{t+i}} - \mu P_{mt+i} \right] Y_{ft+i} = 0 \tag{27}$$

with $\mu = \frac{1}{1-\varsigma}\frac{\varepsilon}{\varepsilon-1}$. From the law of large numbers, the following relation for the evolution of the price level emerges:

$$P_t = \left[(1-\gamma)(P_t^*)^{1-\varepsilon} + \gamma (P_{t-1})^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \tag{28}$$

2.4.4 Fiscal Policy

Government expenditures are composed of: government consumption, which we hold fixed as a share of output at g and the net interest payments from an exogenously fixed stock of long-term government debt, which we set at $\overline{B}$. Revenues consist of lump sum taxes and the earnings from central bank intermediation net of transaction costs. Central bank asset purchases are financed by short-term government debt. Given the central bank balance sheet (19), we can express the consolidated government budget constraint as:

$$gY_t + (R_{bt} - 1)\overline{B} = T_t + (R_{kt} - R_t - \tau) Q_{t-1} S_{gt-1} + (R_{bt} - R_t - \tau) q_{t-1} B_{gt-1} \tag{29}$$

2.4.5 Resource Constraint and Equilibrium

Output is divided between consumption, investment, government consumption, and expenditures on central bank intermediation Φ_t. The economy-wide resource constraint is thus given by

$$Y_t = C_t + [1 + f\left(\frac{I_t}{I_{t-1}}\right)]I_t + G + \Phi_t \tag{30}$$

with $\Phi_t = \tau \left(q_{t-1}B_{gt-1}\right)^2$.

The link between nominal and real interest rates is given by the Fisher relation

$$1 + i_t = R_{t+1}E_t\frac{P_{t+1}}{P_t} \tag{31}$$

Finally, to close the model, we require market clearing in markets for private securities, long term government bonds and labor. The supply of private securities at the end of period t is given by the sum of newly acquired capital I_t and leftover capital $(1-\delta)\,K_t$:

$$S_t = I_t + (1-\delta)\,K_t \tag{32}$$

The supply of long term government bonds is fixed by the government

$$B_t = \overline{B} \tag{33}$$

and labor market clears.

We note that because of Walras' Law, once the markets for goods, labor, and long-term securities clear, the market for riskless short-term debt will be cleared automatically. This completes the description of the model.

2.5 Central Bank Policy

The central bank sets its policies to maximize the present value of household welfare, equation (1), subject to the agents' first order conditions, the market-clearing conditions and the aggregate resource constraints, the occasionally binding balance sheet constraints of banks ($\lambda_t \geq 0$) and any policy constraints: the interest rate lower bound ($i \geq 0$) and QE upper bound $\left(\Gamma \leq \overline{\Gamma}\right)$.

Unless explicitly stated otherwise, we assume that the interest rate follows a standard Taylor type rule.

$$\exp\{i_t\} = \exp\{i_{t-1}\}^{\rho_i}\left[R^*\left(\frac{\Pi_t}{\Pi^*}\right)^{\kappa_\pi}\left(\frac{Y_t}{Y^*}\right)^{\kappa_Y}\right]^{1-\rho_i} \tag{34}$$

We set the inflation target Π^* consistent with zero inflation and therefore zero price dispersion in the long-run. In the absence of an effective lower bound on the nominal interest rate, the latter is optimal in the Calvo model of price adjustment. This is because inefficient price dispersion generates a distorted allocation of resources across firms or sectors as relative prices vary in ways not justified by sectoral or firm-level shocks, leading to suboptimal quantities of different goods being produced and consumed (Galí, 2015).

The values of Y^* and R^* are set equal to steady-state output and nominal interest rate, respectively. The reason Y^* appears in our measure of output gap instead of the more usual time varying "natural" level of output is that – in the presence of real imperfections – the *welfare-relevant* output gap is the one between actual output and its *efficient* counterpart (Blanchard and Galí, 2007). And in our model the efficient level of output remains unaffected by the shock to net worth, which is the main focus of our analysis.

3 Optimal Costless QE

In this section, we characterize key features of the optimal response of the asset purchase policy to a financial shock. We assume that the policy is costless and it faces no constraints. We relax these assumptions in later sections. We describe the results in the form of a proposition and relegate proofs to the appendix.

Proposition 1. *Under costless ($\tau \downarrow 0$) and unconstrained ($\overline{\Gamma} \uparrow \infty$) asset purchase policy*[8] *and an adverse financial shock ($e_{\omega,t}$), the Ramsey-optimal asset-purchase policy under commitment and perfect foresight follows a path that is linear in the net worth of the banking sector (N_t). In particular, the optimal value of long-term government bonds in the central bank's balance sheet ($\Gamma_t qB$) evolves as*

$$\Gamma_t qB = -\frac{\phi}{\Delta}\left(N_t - N\right),$$

where the variables without subscript denote steady state values. The policy completely offsets the impact of the shock on the non-financial macroeconomy. Optimal interest rate stays constant at its steady state value.

Proof. In the Appendix. □

Under the optimal policy, the central bank commits to purchase long-term government bonds in an amount that guarantees that the banking sector can continue to satisfy all demand for private credit. To achieve this, the optimal size of the central bank balance sheet evolves inversely with the banks' net worth gap ($N_t - N$), a measure of the equity scarcity in the banking sector. The policy responds stronger to the net worth gap if the steady state leverage (ϕ) is higher, because then the same gap would generate a larger credit-supply shortfall. In contrast, the policy is less responsive to the gap if the market-risk weight of the long-term government bonds relative to that of private credit (Δ) is higher, because then government bond purchases can ease more the risk-weighted balance-sheet constraint of the banks. As the financial shock only hits the banking sector, and the optimal policy fully neutralizes its impact on the macroeconomy, the policy achieves first best.

[8] The assumption implicitly presupposes that the shock is not large enough such that QE cannot fully offset it even if *all* the outstanding long-term government bonds are purchased by the central bank.

Along the equilibrium path, excess returns are zero ($R_{kt} - R_t = R_{bt} - R_t = 0$), because all excess demand for private credit is satisfied. Therefore, the law of motion of the net worth of the banking sector simplifies to a first-order autoregressive process

$$N_t = \sigma R N_{t-1} + \omega_t.$$

The autoregressive term is the product of the rate of return on net worth (R) and the survival rate of banks (σ). The survival rate determines the retained net worth of the banking sector after a proportional dividend payout, which is a fixed share of the outstanding equity capital (with a proportionality factor $1 - \sigma$). Applying the analogous steady state relationship $N = \sigma R N + \omega$, we can derive the law of motion for the net worth gap along the optimal path: $N_t - N = \sigma R(N_{t-1} - N) + e_{\omega,t}$, where we assumed a one-off financial shock. The law of motion of the banking sector net worth gap implies a useful corollary about the persistence of optimal QE exit.

Corollary 1. *The optimal central bank balance sheet along the equilibrium path follows a first order autoregressive process*

$$\Gamma_t qB = \sigma R \left(\Gamma_{t-1} qB\right) - \frac{\phi}{\Delta} e_{\omega,t}, \tag{35}$$

with an autoregressive parameter σR.

Proof. In the Appendix. □

Under optimal policy, banks' balance sheet constraints stay loose, all credit demand is satisfied and interest rate premium stays constant at zero. This is achieved through a QE policy that is large enough to guarantee sufficient credit supply despite the scarcity of banks' equity capital. Exit from QE is very gradual and mirrors the slow recapitalization of the banking sector. This recapitalization is gradual partly because optimal central bank size eliminates excess interest rate premia, which could speed up the recapitalization of the banking sector, but only at the cost of welfare detrimental aggregate credit scarcity. It should be emphasized that the lack of financial frictions along the equilibrium path does not mean that QE is ineffective. On the contrary, it is the optimal policy that guarantees that the credit supply satisfies credit demand. Furthermore, in the zero limit of positive QE costs ($\tau \downarrow 0$), the central bank purchases just enough government bonds to make credit supply satisfy credit demand. Any additional purchase would just increase the efficiency costs without increasing the already fully satisfied credit demand. Therefore, any exit path that is quicker than optimal would necessarily reintroduce credit frictions to the economy and would lead to an economic contraction.

Optimal policy can be implemented in multiple ways. One possibility is to follow the linear rule that responds to the net worth gap of the banking sector as specified in Proposition 1. The rule provides an intuitive target criterion, and prescribes a straightforward relationship between the evolution of the central bank balance sheet and the recapitalization of the banking sector. Alternatively, the same equilibrium QE path can be achieved, if the QE policy follows a rule that responds to the excess interest rate premium ($\Gamma_t = \kappa_R (R_{k,t} - R_t)$) with a response coefficient that increases without limit $\kappa_R \uparrow \infty$ (see Figure 3 later). Intuitively, such a policy, if it converges to a finite equilibrium QE path also needs to fully stabilize the premium. In line with Proposition 1, full premium stabilization requires the same optimal central bank balance sheet path as described in Proposition 1, which lifts banks' balance sheet constraint. The relationship between the finite optimal central bank balance sheet path and the zero equilibrium path of the interest rate premium is not as transparent as with the net worth gap rule, however.

4 Optimal Costly QE

In this section, we analyze the robustness of the results described in Section **??**, when the central bank asset purchases are costly and face upper bounds. We first describe the calibration of the model and the solution method, before presenting our results.

4.1 Calibration and Solution

We calibrate the bulk of the parameters based on Coenen et al. (2018). The parameters are partly calibrated and partly estimated on euro area data using standard Bayesian methods. Table 1 lists the parameter values. Most of the parameters are standard. The discount rate β is calibrated to be 0.995, which implies a steady state real interest rate around 2 percent. As in Coenen et al. (2018), the consumption habit parameter is around 0.6, and the Frisch elasticity of labor supply is around 0.5. The capital share α is 0.36, and the capital depreciation rate δ is 0.025. The capital adjustment cost parameter is $\eta_i = 5.17$. The Calvo parameter of price rigidity is $\gamma = 0.92$, with a backward indexation parameter $\gamma_{P,-1} = 0.23$.[9] The duration of the long-term government bond is 7 years ($\varrho = 0.97$), and its overall supply is 70 percent of GDP. We assume that households hold three quarters of these bonds in the steady state, and banks hold the remaining one quarter.

The calibration of the financial sector follows Coenen et al. (2018) with one important difference. We assume that banks' balance sheet constraints are not binding in the non-stochastic steady state. This implies that in the steady state interest rate premium is zero. In contrast, Coenen et al. (2018) assumes that the constraints are binding in the steady state, which leads to an interest rate premium of 217 basis points. Our choice simplifies the analysis and guarantees that the optimal central bank balance sheet size is zero in the steady state: a positive central bank balance sheet would just cause distortions without easing the credit conditions. Our choice, however, does not influence the qualitative conclusions of our analysis

[9]The last three parameters are obtained from a previous version of the model (Christoffel et al., 2008), which estimated the model (i) without variable capital utilization, and (ii) without assuming kinked demand curves, which are both missing from our model.

of optimal asset-purchase policy conditional on a financial disturbance. As Coenen et al. (2018), banks' expected planning horizon is 10 years ($\sigma = 0.97$), and we calibrate the fraction of capital that can be diverted θ and the transfer to entering bankers ω such that interest rate premium is 0, and banks' leverage is just equal to the maximum leverage ($\overline{\phi}$) of 6. The relative absconding rate of government debt relative to private assets (Δ) is 0.83. The household portfolio adjustment cost is $\kappa = 0.009\%$, implying a moderate level of financial frictions.

We calibrate the cost of QE τ to be tiny, 0.01 basis point of the assets held by the central bank. For large QE interventions to stay beneficial in our calibration, we need such low values. This is partly the consequence of our choice to assume away distortions in the steady state to make some of the results more transparent. [10] The finding of slow optimal exit from large central bank balance sheets is not sensitive to this choice.

Table 1: Parameter values

	Households	
β	0.995	Discount rate
h	0.62	Habit parameter
χ	35	Relative utility weight of labor
B/Y	0.700	Steady state Treasury supply
ϱ	0.97	Geometric decay of government bond
$\bar{B}^h/B$	0.75	Proportion of long term Treasury holdings of the HHs
κ	0.009	Portfolio adjustment cost
φ	2	Inverse Frisch elasticity of labor supply
	Financial Intermediaries	
θ	0.167	Fraction of capital that can be diverted
Δ	0.83	Proportional advantage in absconding rate of government debt
ω	0.067	Transfer to the entering bankers
σ	0.972	Survival rate of the bankers
	Intermediate good firms	
α	0.36	Capital share
δ	0.025	Depreciation rate
	Capital Producing Firms	
η_i	5.17	Inverse elasticity of investment to the price of capital
	Retail Firms	
ϵ	3.86	Elasticity of substitution
γ_P	0.92	Probability of keeping the price constant
$\gamma_{P,-1}$	0.23	Price indexation parameter
	Government	
$\frac{G}{Y}$	0.200	Steady state proportion of government expenditures
τ	0.01 basis point	Cost of QE

[10] This is different from Gertler and Karadi (2011), for example, who consider a distorted steady state and find that large QE interventions are still welfare improving under higher (linear) costs (10 basis points).

We analyze optimal interest-rate and asset-purchase policies under commitment and perfect foresight conditional on a bank net worth shock. We solve for the Ramsey-optimal asset-purchase policy subject to three occasionally binding constraints: the banks' balance sheet constraint, the interest rate lower bound and the asset purchase upper bound. We solve the model using the non-linear equation solver for Ramsey-problems with occasionally binding constrains in the `dynare` package.

4.2 Results

To illustrate how QE policy works, in Figure 2 we first plot the impulse responses to an exogenous hump-shaped shock to asset purchases. This policy shock hits in our baseline scenario of a deep financial recession, in which banks' balance sheet constraints are binding as a result of a negative transitory shock to banks' equity. Without policy intervention, the shock would reduce output by around 0.8 percent and inflation by around 0.4 percent. The policy shock is calibrated such that the purchases reach 12 percent of GDP by the 8th quarter, before slowly returning towards zero. The solid black lines show the baseline case when the effective lower bound on the nominal interest rate is not binding, while red dashed lines are for the case of an interest rate floor that is binding for 4 quarters. In both cases asset purchase policies are effective in a deep financial recession, because banks' balance sheet constraints are binding and the policy frees up banks' balance sheet capacity. This leads to lower lending spreads and more credit to the private sector. The improved credit conditions raise asset prices and improve banks' equity positions, which further eases banks' credit supply constraints in a positive feedback loop. Overall, output increases by about 0.45 percent and inflation by 0.06 percent in the case without the ELB. When the ELB is binding, output increases by 0.8 percent and inflation rises by 0.14 percent. These effects are comparable to those found in Coenen et al. (2018) in a linearized model.

We then turn to our central question concerning optimal policy. The baseline is a deep financial recession as in the previous exercise. As a result of the negative shock, banks' balance sheets would deteriorate and their leverage constraints would become binding for years to come. The binding constraints would lead to stressed financial conditions reflected in inefficiently elevated lending spreads. Figure 3 depicts in black solid lines the unconstrained optimal policy assuming zero QE cost. In line with Proposition 1, the policy that maximizes household welfare in response to a financial disturbance is one which stabilizes completely the macroeconomy. Namely, inflation and output remain constant, as do asset prices, lending spreads and banks' leverage. Net worth under the optimal policy falls persistently below trend, while the central bank balance sheet increases in a mirror image. Indeed, as shown in Section 3, the optimal targeting rule in the case with zero QE cost relates asset purchases negatively to bank's net worth gap. Figure 3 also demonstrates that a policy which targets the lending spread as in Gertler and Karadi (2011) (instead of banks' net worth) converges towards the fully optimal policy as the coefficient on the spread rises towards infinity. Note, however, that unless the

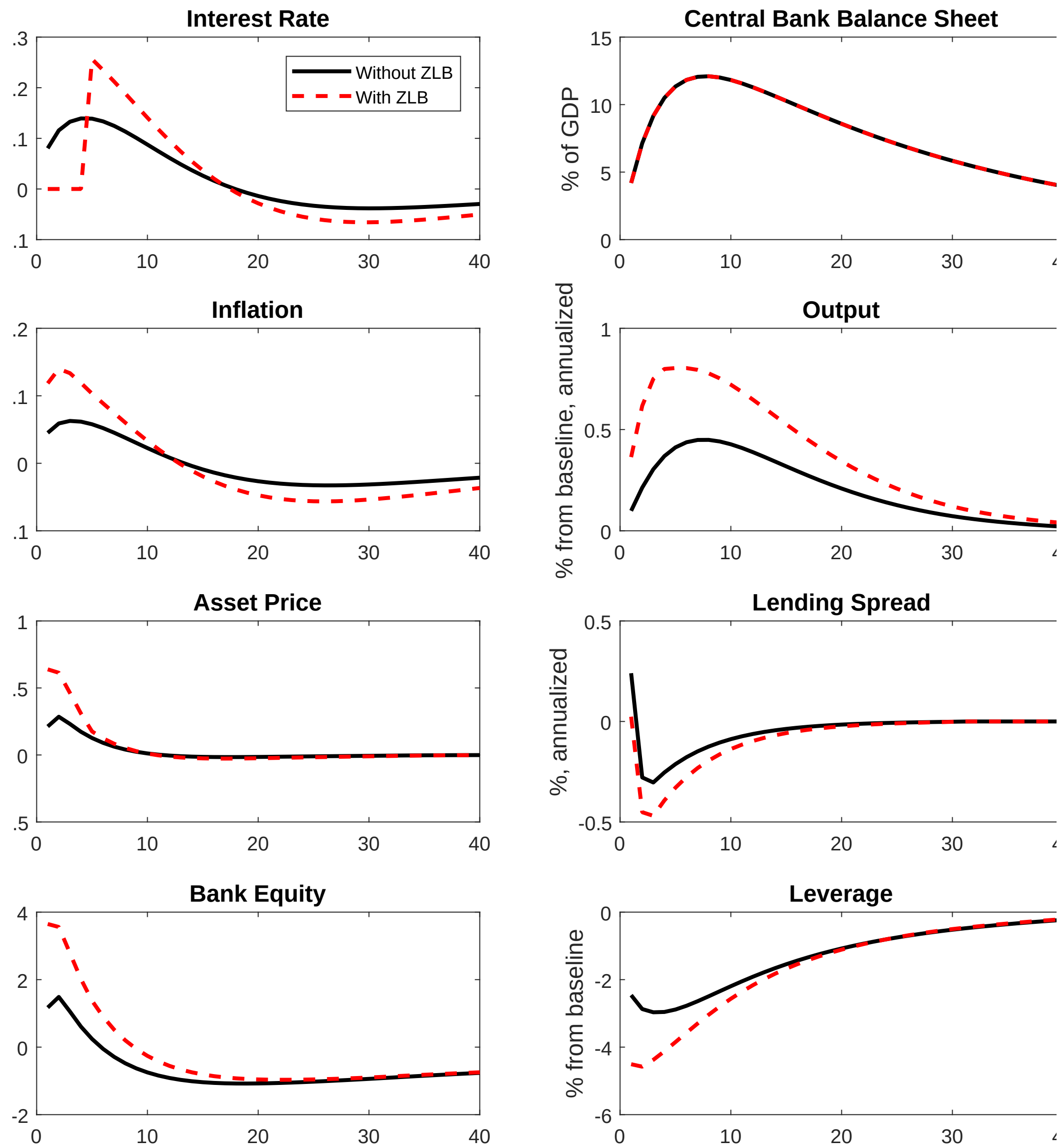

Figure 2: Responses to an asset purchase shock at the ELB

response coefficient is very large, the financial shock would produce non-trivial fluctuations in output, inflation, and financial variables, as shown by the red dashed lines.

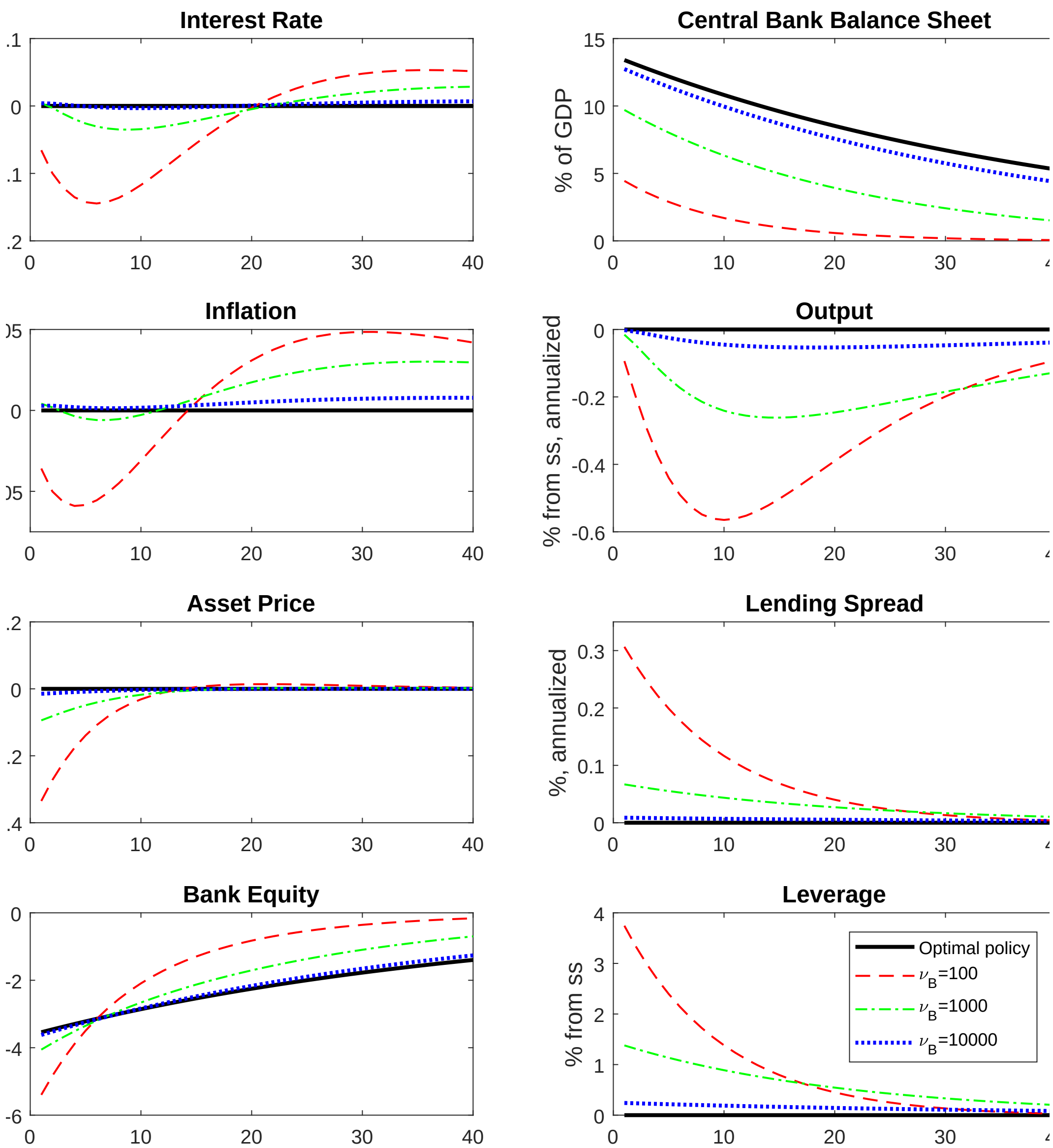

Figure 3: Responses to a deep financial crisis under optimal policy (black solid line) and simple rules (dashed line)

Note: In response to a negative shock to banks' equity, the optimal policy (black solid line) raises the central bank balance sheet on impact and commits to undo it very gradually as banks recapitalize. Interest rates stay constant. The figure contrasts the optimal policy to equilibrium paths with simple QE rules (dashed lines) that respond to excess premium with increasing aggressiveness.

Next, we turn to the characterization of optimal policy in the more realistic case with a positive QE cost and an upper bound on the central bank's balance sheet. Figure 4 illustrates that the unconstrained optimal policy (red dashed lines) with a positive QE cost is similar to the one without a cost. Importantly, the exit from asset purchases remains very gradual. A notable difference, however, is that the phase-in of the QE policy becomes delayed. Delayed QE entry contributes to a positive lending spread, which leads to a faster bank recapitalization. This allows the central bank to save on QE-related costs in the future at a cost of worse financial conditions in the short term. Output, inflation and interest rates are all nearly constant, while financial variables exhibit some short-lived volatility, except bank equity which is persistently below trend again. To see how an upper bound on QE affects the results, the black solid lines show the responses when QE is capped at 15% of outstanding assets, and the blue dash-dotted line show when the cap is 14%. As a result, the output drop is somewhat worse and likewise inflation is slightly below target. QE remains at its upper bound longer than it would absent the constraint, but then drops off a bit faster than in the case without a constraint. During the period of a binding QE upper bound, the central bank reinvests the proceeds of maturing assets in its portfolio. In the appendix, we show that our key result – the optimally gradual exit from QE – remains robust also if we relax the assumption that the central bank follows the Taylor rule, and assume instead that it sets policy rates optimally jointly with the QE policy.

How should the central bank respond to financial shocks in the future, when the interest rate is not constrained by its lower bound? Our model can shed light on some of the issues. As Proposition 1 has shown, if QE were costless and unconstrained, the optimal response to financial shocks is to use asset purchase policies without any interest rate easing even if interest rate policy is available. The reason is that these policies address the root cause of the downturn: the credit crunch. With positive QE costs, however, the results are more subtle, and it is far from obvious that embarking on QE is welfare improving even if the downturn is caused by a financial disturbance. We draw two conclusions based on our model.

First, if the central bank faces a fixed cost of embarking on a new asset purchase programme, because an active program might lead to reputational costs, for example, then it should activate the program only if financial shocks are sufficiently large. The reason is that the welfare gains from a QE policy intervention decline as the shocks becomes smaller, as illustrated by the first panel in the first row of Figure 5. Two factors drive this result. First, as emphasized also in standard quadratic-linear welfare analyses, the value of a marginal intervention declines as the shock becomes smaller and stops pushing the representative household far away from its optimum. This mitigates the increase in the household's marginal utility, which determines its valuation for an additional intervention. Second, and this is revealed only by our non-linear analysis and illustrated by the second and third panels in the first row of Figure 5: the effectiveness of a large QE intervention declines fast as the shock size becomes small. The effectiveness of QE decreases, because without it the counterfactual downturn would also become smaller. We find that for larger shocks, which are not fully offset by a given size QE intervention, the effectiveness of the intervention decreases somewhat. The reason for this is that without an offsetting QE intervention, the financial shock limits the outstanding credit

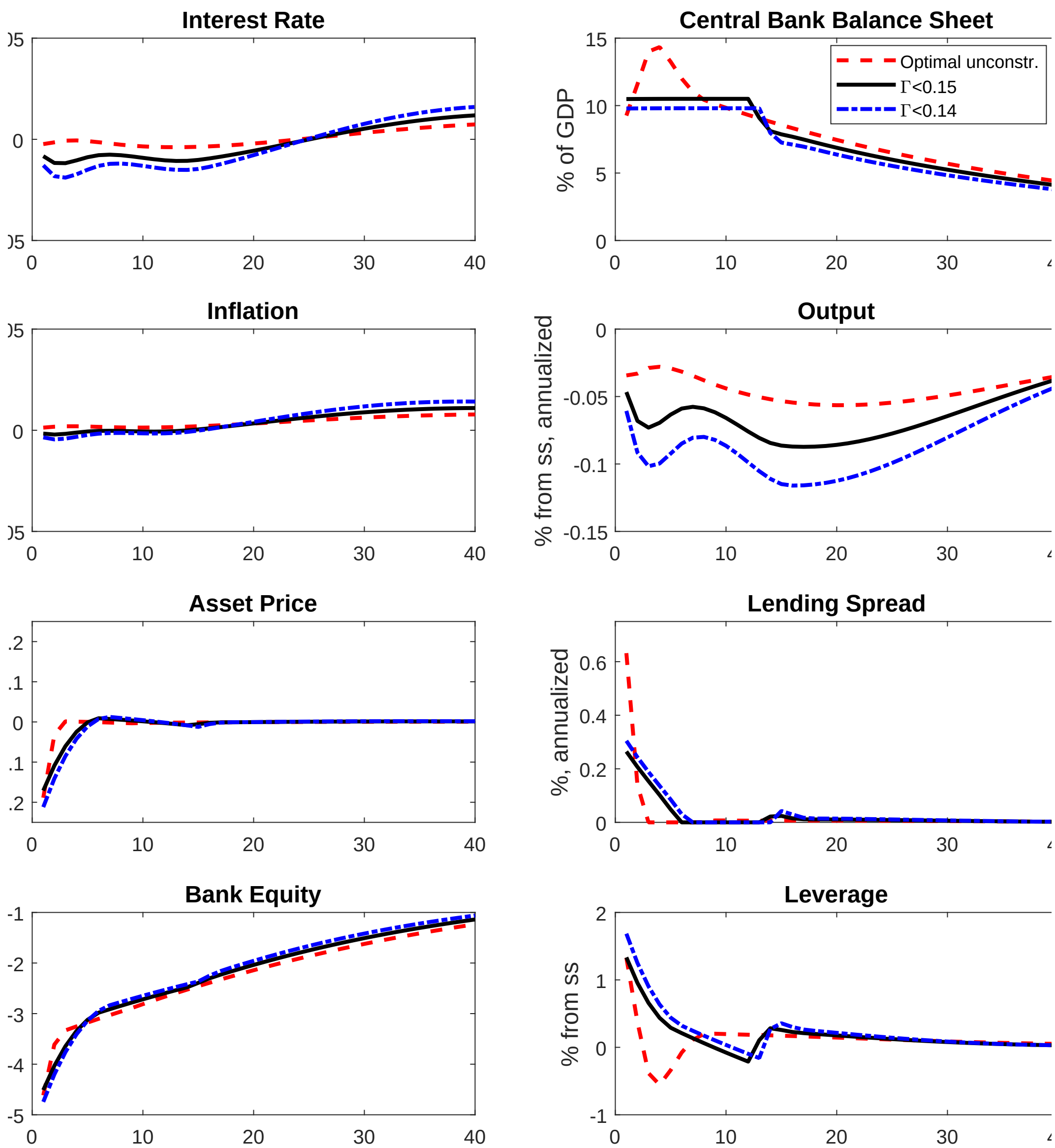

Figure 4: Responses to a deep financial crisis under positive QE costs (red dashed line) and upper bound constraints (black solid line and blue dash-dotted line)

Note: Under positive costs and upper bound constraint, the exit from QE policy stays very gradual. The phase-in of the QE becomes delayed in the presence of positive costs. The optimal policy reacts to the upper bound constraint by delaying the first reduction of the central bank balance sheet.

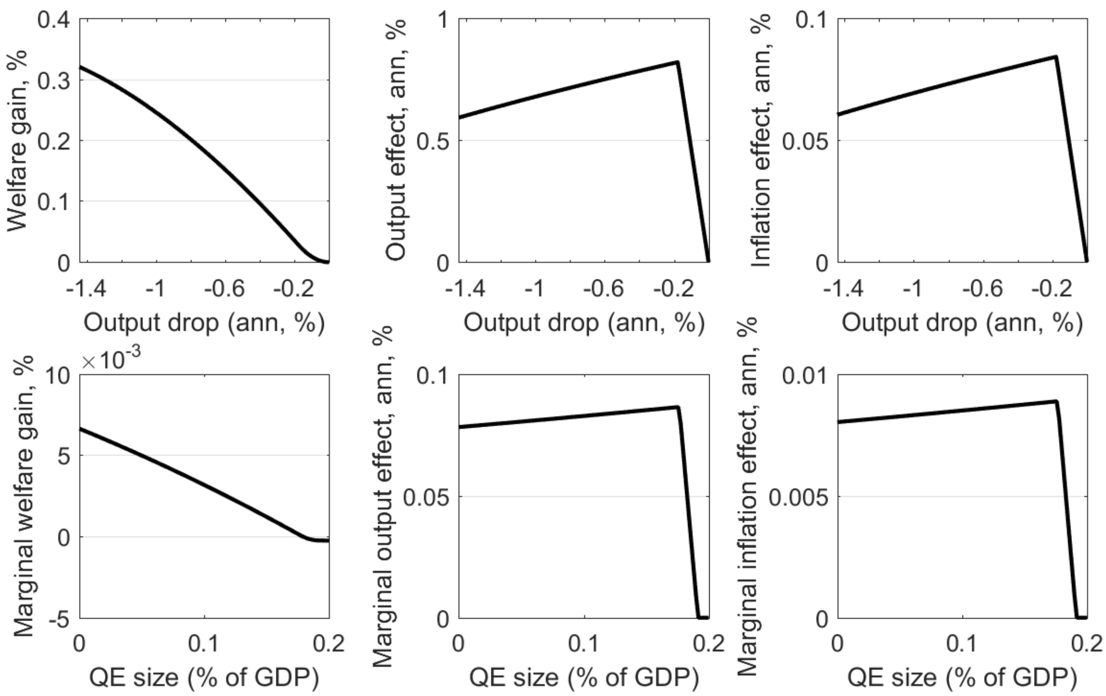

Figure 5: Welfare gains and effectiveness of QE as a function of financial shock size and QE size

Note: The first row of the figure shows the welfare gains, and the output and inflation effects of a QE as a function of different size of financial shocks. The policy follows an AR(1) process with initial size of 10 percent of GDP with persistence σR. The size of the financial shock is expressed in terms of the peak output drop. The welfare gains from an infinitesimal shock is zero, and the gains monotonically increase with shock size. The effectiveness of the policy shock increases steeply from zero as the shock size increases, it peaks when financial shock reaches a size when the QE policy cannot completely offset it any more, and falls back somewhat afterwards. The second row shows the marginal impact of an additional QE purchase in the amount of 1 percent of GDP under a baseline financial shock as a function of different QE interventions. The welfare gains from marginal QE decreases with QE size, and stabilizes at a negative value when the QE size completely offsets the financial shock. The effectiveness of a marginal QE initially increases with QE size, but the effectiveness abruptly disappears, after the QE has already fully offset the impact of the financial shock.

in the economy. This reduces the strength of the general equilibrium feedback mechanisms triggered by the QE policy intervention, because the improvement in outlook and asset valuations brought about the intervention improves banks' balance sheet conditions to a lesser degree if their outstanding credit is lower. The decline in effectiveness, however, does not offset the increase in the value of intervention for a realistic range of financial shocks in our parametrizaton.

Second, the size of the optimal intervention should balance the marginal gains from QE, which decline with the size of the intervention, with its variable costs. The welfare gains are plotted on the first panel in the second row of Figure 5. The main driving force of the decline, as before, is the declining value of the marginal intervention as the large QE offsets a higher share of the financial shock and brings the households closer to their optimum. The second and third panel in the second row of Figure 5 shows that marginal effectiveness of QE actually increases initially as the QE intervention becomes larger. This happens because, as before, larger aggregate credit supply raises the financial amplification of a marginal intervention. Additional QE becomes ineffective abruptly, however, when the QE intervention fully offsets the financial shock. As before, the initial increase in the effectiveness of the marginal policy intervention is not sufficiently strong to counteract the decline in the value of the intervention.

5 Conclusion

We have analyzed the optimal asset-purchase policy after financial disturbances in a model with banks, which face occasionally binding financial constraints. We have found that optimal exit from balance sheet policies is very gradual and the central bank optimally maintains a large balance sheet long after banks' balance sheet constraints stopped binding. We have also argued that if quantitative easing is costless, than QE constitutes the optimal response to financial disturbances even if the interest rate is unconstrained by the effective lower bound. Any fixed costs of embarking on a quantitative easing policy, would, however, justify QE interventions against only large financial shocks or if interest rates are constrained.

The analysis has disregarded the role of macro-prudential policies, like capital requirements, which can potentially alter some of the conclusions. In so far as these policies can force the recapitalization of banks after a crisis, they might permit a quicker optimal exit. Such macroprudential policies, however, are not without costs, because they can limit the credit supply and can delay the recovery. The analysis of the optimal interaction of asset-purchase and macroprudential policies, therefore, is an interesting avenue for future research.

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A Proofs

We repeat our main proposition here, before sketching its proof.

Proposition 1. *Under costless ($\tau \downarrow 0$) and unconstrained ($\overline{\Gamma} \uparrow \infty$) asset purchase policy and an adverse financial shock ($e_{\omega,t}$), the Ramsey-optimal asset-purchase policy under commitment and perfect foresight follows a path that is linear in the net worth of the banking sector (N_t). In particular, the optimal value of long-term government bonds in the central bank's balance sheet ($\Gamma_t qB$) evolves as*

$$\Gamma_t qB = -\frac{\phi}{\Delta}\left(N_t - N\right),$$

where the variables without subscript denote steady state values. The policy completely offsets the impact of the shock on the non-financial macroeconomy. Optimal interest rate stays constant at its steady state value.

Proof. The financial shock $e_{\omega,t}$ reduces the net worth of the banking sector. A QE policy, which completely offsets the impact of the shock, and keeps the non-financial sector isolated from the financial turbulence and stable at its nonstochastic steady state achieves the first best. The reason is that the steady state is efficient: all credit demand is satisfied at the slack balance sheet constraint of the financial intermediaries, and any distortions coming from the market power of the retail firms are offset by steady state subsidies. We guess that such QE policy is feasible.

Under such policy, the values of the corporate and the government bonds (Q, q), as well as the banking sector corporate bond demand (S_p) stay constant at the steady state. We postulate that the risk-adjusted leverage of the banking system also stays constant at its steady state ϕ. The constant leverage implicitly defines the necessary evolution of the QE policy

$$\phi = \frac{QS_p + \Delta q B_{pt}}{N_t} = \frac{QS_p + \Delta q \left(B - B_h - \Gamma_t B\right)}{N_t},$$

where the second equality has taken into account that the household's demand for government bonds are also constant at the steady state.

The equation implies that $\phi N_t = QS_p + \Delta q B(1 - \Gamma_t) - \Delta q B_h$. We also know that in the steady state $\phi N = QS_p + \Delta q B - \Delta q B_h$. These two equations generate the optimal rule in the proposition. Provided the policy is unconstrained, and the shock is not too large such that the steady state supply of corporate credit S_p can be maintained without purchasing all outstanding long-term government bonds ($\Gamma_t \leq 1$), the policy keeps the private lending of the banking system at its steady state level. The leverage stays constant and the economy stays stable at its efficient steady state. Any interest rate policy is unnecessary. □

Corollary 1. *The optimal central bank balance sheet along the equilibrium path follows a first order autoregressive process*

$$\Gamma_t q B = \sigma R \left(\Gamma_{t-1} q B\right) - \frac{\phi}{\Delta} e_{\omega,t}, \tag{36}$$

with an autoregressive parameter σR.

Proof. Under optimal policy, credit premia $R_{kt} - R_t = R_{bt} - R_t$ are stabilized at 0. This implies that the banking system's net worth evolves as $N_t = \sigma R N_{t-1} + \omega_t$. Using the equation for steady state net worth N, we get that

$$N_t - N = \sigma R(N_{t-1} - N) + e_{\omega,t},$$

where $e_{\omega,t} = \omega_t - \omega$. As the optimal policy follows $\Gamma_t = -\phi(N_t - N)/\Delta q B$, $\Gamma_{t-1} = -\phi(N_{t-1} - N)/\Delta q B$. Substituting these two equations into the law of motion of the net worth gap $N_t - N$ implies the corollary. □

B Optimal joint interest rate and asset purchase policy

Figure 6 demonstrates the robustness of our main results if we allow both the interest rate and the asset purchase policy to respond optimally to a financial shock. The red dashed lines plot the responses with unconstrained QE, while the solid black lines correspond to the case with an upper bound on asset purchases. In both cases, our main finding – the persistence of optimal QE – remains valid. The dramatic initial interest rate drop results in inflation and output temporarily above their respective trends, while, as before, bank equity falls below steady state and the lending spread rises. The initial QE response in this case is not only delayed,

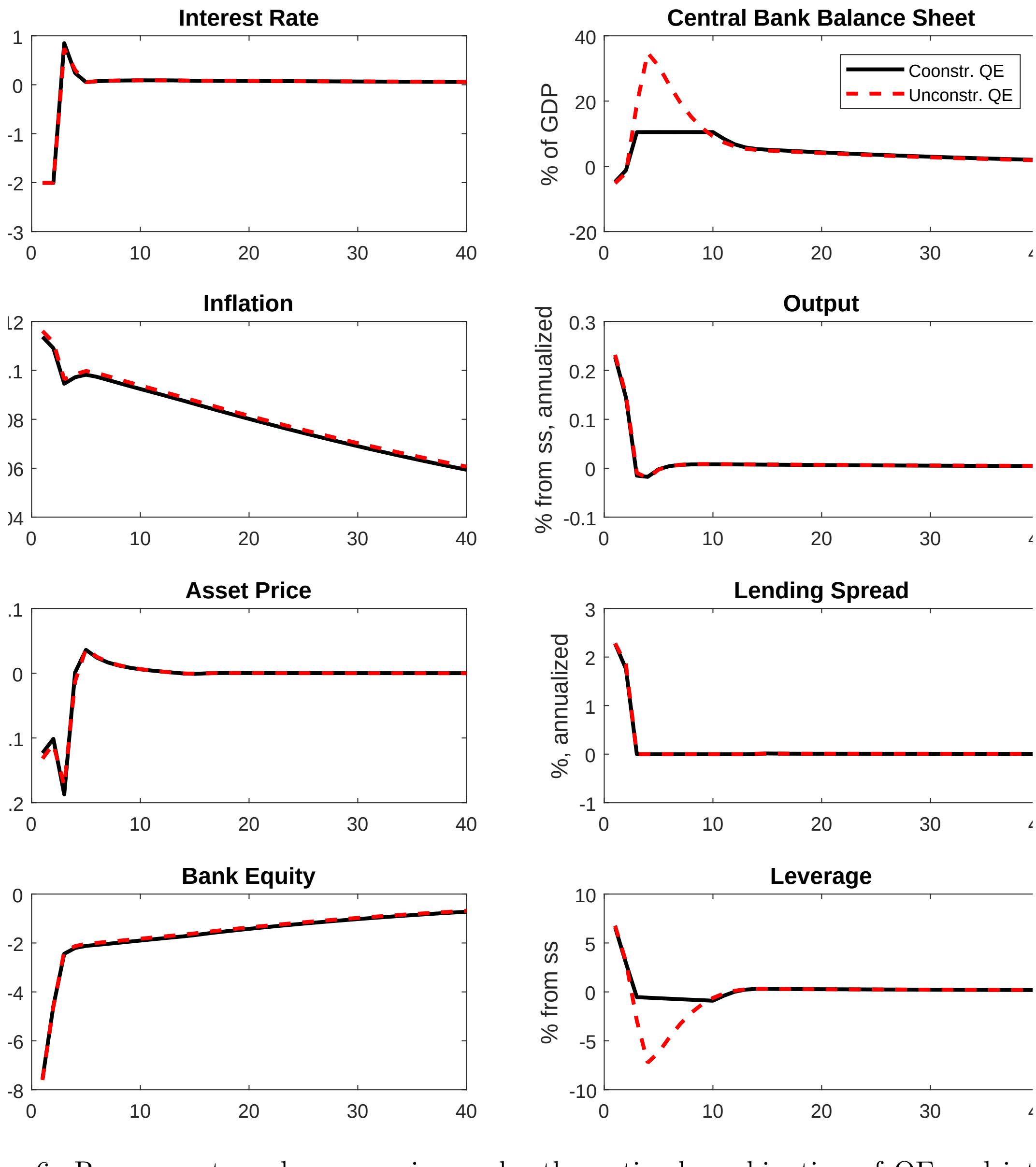

Figure 6: Responses to a deep recession under the optimal combination of QE and interest rate policy

it is actually negative. The central bank achieves multiple goals with this. First, as before, it amplifies the initial impact of the credit crunch, which raises the interest rate premium and helps banks to recapitalize faster. Second, with amplifying the initial credit crunch, the central bank mitigates the stimulative impact of the large interest rate cut.