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Efficient Taxation of Fuel and Road Use
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# Efficient taxation of fuel and road use 

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#### Abstract

:

This study calculates efficient taxes on gasoline and road use designed to combat driving related externalities when motorists avoid taxes due to an excessive economic driving-style. The efficient tax on gasoline is reduced below the Pigouvian rate due to such avoidance. The current US tax rate on gasoline is below the efficient tax rate while the current UK rate is slightly above the efficient rate in this case. A GPS-based road user charge prevents such avoidance. The efficient GPS-based road user charge should be combined with a tax on gasoline which promotes an economic driving-style that lowers accidents.


Keywords: Transportation, optimal taxation, environmental taxation, global warming.

JEL classification: H2, H21, H23, Q58, R48.
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## 1. Introduction

Road transport is essential to maintain an efficient flow of goods, services and people, but generates costly negative externalities in the form of $\mathrm{CO}_{2}$-emissions, local air pollution, accidents, congestion and noise. Many countries have implemented taxes on fuels to curb externalities linked to both fuel and mileage. However, the tax-induced gain in terms of reduced externalities is diminished as less than half of the reduction in fuel use is due to reduced driving, see Parry and Small (2005) which argue that the optimal tax rates on gasoline are reduced accordingly. The optimal tax rate on gasoline in the United States is more than twice the current rate, while that for the United Kingdom is about half the current rate according to Parry and Small (2005). The remaining reduction in fuel use is due to improved fleet fuel efficiency.

Fleet fuel efficiency improves as households purchase more fuel-efficient vehicles. Bjertnæs (2019) shows that such avoidance should be prevented by imposing heavier taxation of fuel-efficient vehicles, and hence, that it is sub-optimal to lower the tax rate on gasoline. The efficient tax rate on gasoline is above current tax rates in both the US and the UK in this case. However, fleet fuel efficiency also improves as households choose a more fuel-efficient driving-style, see e.g. Barkenbus (2010). A tax on fuel may create an incentive to save fuel rather than lowering mileage-related damage in this case. A fuel-saving driving-style may on the other hand reduce driving-related externalities connected with accidents, see Aarts and van Schagen (2006). A tax on fuel may harvest gains due to a reduction in accidents in this case. A tax on fuel combined with GPS-based road user charges may on the other hand prevent avoidance due to an economic driving-style. These unexplored objections related to the optimal tax rate on gasoline calls for further investigations.

The present study contributes to this literature by exploring how taxes on gasoline and road use should be designed to combat driving related externalities. A new model framework is developed where agents choose the amount of driving and driving-style. The model framework generates tax formulas that are comparable with current tax rates on gasoline and road use. The study shows that the efficient road user charge on gasoline is below the marginal mileage-related damage. The explanation is that the tax-induced reduction in driving, and hence gain in terms of reduced externalities, is diminished as households respond by choosing a more fuel-efficient driving-style. The more fuel-efficient drivingstyle also contributes to reduce accidents, and hence, increase the tax-induced gain in terms of reduced externalities. This latter effect is however more modest. By use of a numerical model it is found that the current US tax rate on gasoline is way below the efficient rate while the current UK rate is slightly above the efficient rate in this case.

The study also calculates optimal combination of taxes on gasoline and road use. The efficient tax on gasoline promotes an economic driving-style which lowers accidents when the tax is combined with a GPS-based tax on road use. The efficient GPS-based tax rate on road use is reduced below the marginal damage of mileage-related externalities in this case to prevent excessive taxation of driving.

The rest of the paper is divided into six sections; Section 2 provides a literature review, Section 3 presents the model and Section 4 presents optimal taxes on gasoline and road use. Parameter values are presented in Section 5. Current taxes in selected countries are compared with optimal taxes in section 6. Section 7 concludes.

## 2. Literature review

Parry and Small (2005) show that the optimal uniform tax rate on gasoline in the United States is more than twice the current rate, while that for the United Kingdom is about half the current rate. Their optimal tax rate on gasoline consists of an adjusted Pigouvian tax component which includes damage from carbon emissions and other driving-related externalities, a Ramsey tax component designed to raise tax revenue, and a congestion feedback component which captures welfare gains as labor supply increases as congestion decreases. Driving-related externalities due to congestion and accidents as well as the Ramsey tax component are dominant, while global warming and congestion feedback are modest. Anderson and Auffhammer (2014) estimate higher accident-related externalities, which suggests that the UK gasoline tax is closer to the optimal level than the US tax. Several objections can be made to the methodology in Parry and Small (2005), however. First, a general set of assumptions excludes the Ramsey tax component from a welfare-maximizing tax system, see Atkinson and Stiglitz $(1976)^{2}$. Indeed, Jacobs and de Mooij (2015) show that a Pigouvian tax on polluting goods is part of a welfare-maximizing tax system within a Mirrlees-economy framework ${ }^{3}$. Second, the gain in terms of reduced externalities per liter of fuel is diminished by the fact that households avoid the mileagerelated tax component on fuel by purchasing more fuel-efficient vehicles according to Parry and Small $(2005)^{4}$. Bjertnæs (2019) on the other hand shows that it is more efficient to prevent such avoidance by imposing heavier taxation of fuel-efficient vehicles compared to fuel-intensive vehicles. He shows that the optimal tax rate on gasoline is above current tax rates in both the US and the UK. Bjertnæs (2019)

[^1]however did not consider avoidance of road user charges on fuel due to an excessive economic driving-style.

Innes (1996), Fullerton and West (2002) and Montag (2015) show different cases where vehicle specific taxes on fuel consumption or mileage perfectly reflect driving related externalities, and hence, implement first-best allocations. Fullerton and West (2002) explore tax combinations that implement the social planner choices of mileage, engine size, pollution control equipment, and fuel type. Montag (2015) shows that efficient vehicle specific taxes on fuel or mileage implements first-best choices of vehicles, driving distance, and fuel economy due to driving-style. His mileage tax depends on actual fuel consumption, and hence, is equivalent to a differentiated tax on fuel ${ }^{5}$. A vehicle specific tax on fuel may lead to costly monitoring to prevent high-tax fuel vehicles from using low-tax fuel. A uniform tax on fuel is not hampered by this shortcoming. A uniform tax on fuel may however create an incentive to save fuel rather than lowering mileage-related damage. A tax on fuel is also likely to reduce accidents, see Grabowski and Morrissey, 2004 and 2006. Motorists can certainly improve their fuel economy by choosing a more economic driving-style, see Barkenbus 2010, Carrico et al., 2009; Onoda, 2009; Tong et al., 2000; Van Mierlo et al., 2004; Vandenbergh et al., 2008. The impact on mileage-related damage is however more uncertain. Economic driving is not likely to reduce traffic congestion problems. A larger spread in speeds may boost accidents and traffic congestion according to Lave (1985), Aarts and van Schagen (2006) and Elvik (2014). Less aggressive driving at lower speeds however contribute to lower accidents according to Aarts and van Schagen, 2006, van Benthem, 2015, Rodriguez (1990) and Montag, 2014. Lower speeds also generate less noise pollution, see Bendtsen (2004). Hence, further investigations are required to determine the efficient tax rate on fuel when motorists choose driving-style.

There are several other shortcomings connected with a uniform road user charge on fuel, see e.g. Parry et al. (2007) and Ashley et al. (2017). First, a road user charge on fuel does not differentiate between geographic locations or peak and off-peak periods. Second, electric vehicles are exempt from a road user charge levied on fuel. Such problems are avoided if the road user charge on fuel is replaced with GPS-based road user charges. It is however challenging to implement a GPS-based system which rewards reductions in damage due to an economic driving-style. A tax on fuel might be a useful complement. Further investigations are required to determine the efficient combination of taxes on fuel and mileage in this case.

The main aim of the present study is to compliment results in Bjertnæs (2019) with more fine-tuned tax formulas for fuel and road use when choice of economic driving-style is considered. Taxation of

[^2]vehicles is excluded from the present study. The aim is consequently not to improve efficient tax formulas for vehicles in Bjertnæs (2019). Plausible assumptions about driving related behavior imply that the optimal road user charge on fuel is below the average marginal damage of road use ${ }^{6}$. The optimal $\mathrm{CO}_{2}$ tax on fuel however equals the marginal damage of $\mathrm{CO}_{2}$-emissions. The current US tax on gasoline is below the optimal tax rate in this case. The current UK tax rate is slightly above the optimal tax rate. The study also calculates optimal combination of taxes on fuel and road use. The optimal tax on fuel exceeds the marginal damage of $\mathrm{CO}_{2}$-emissions when the tax is combined with a GPS-based tax on road use. The marginal tax burden of driving however equals the total marginal damage of driving. Hence, the optimal GPS-based tax rate on road use is reduced below the marginal damage of mileage-related externalities in this case.

## 3. The model

An economical driving style can result in a significant reduction in fuel consumption according to studies mentioned above. A more economical driving-style entails, among other things, less aggressive acceleration, driving at lower and more stable speeds, driving in high gear, less running at idle, less use of air conditioning, more planning of road selection, choice of tires with low rolling resistance, engine maintenance, and avoiding excess weight and load that increases air resistance, see Carrico et al. 2009, Dietz et al. 2009, Gonder et al. 2012, Wang et al. 2008 and Ko et al. 2010. Studies also show that motorists are able to change behavior, see for example Barkenbus (2010), Onoda (2009) and Beusen et al. (2009). Most of the measures can be implemented without investment costs. However, economical driving requires taking time to run economically. Households therefore faces a trade-off. A more economical driving-style means longer travel time. The benefit is reduced fuel consumption, and hence, lower fuel costs. The tax rate on fuel will affect this trade-off, as well as the household's choice of mileage. This problem is analyzed in a partial model framework where a representative household chooses mileage and driving-style. The tax on fuel affects the choice of mileage as well as driving-style. The government chooses tax rates so that welfare is maximized. Welfare is defined as households' utility minus damage associated with negative external effects of vehicle use.

### 3.1 Households

Household utility, $u$, net of externalities is given by the quasilinear utility function

$$
\begin{equation*}
u=e(m)-w h_{d} m+c \tag{1}
\end{equation*}
$$

The utility is determined by the sub-utility of driving, $e(m)$, minus the cost of time spent driving, $w h_{d} m$, plus consumption of a non-polluting good, $c$. Driving distance measured in kilometer is denoted $m$. Time spent driving per kilometer is denoted $h_{d}$. The marginal utility of an extra

[^3]kilometer, $e^{\prime}(m)$ is positive but decreasing, $e^{\prime \prime}(m)<0$. Time spent on driving has an alternative value equal to the after-tax wage rate, $w$. The working time has a higher alternative cost due to taxes on the return on working. The alternative cost of time spent driving would consequently exceed the private alternative cost. This creates a welfare gain connected to lowering time spent driving which should be considered when tax formulas are constructed. Such alternative costs is however not included in the model framework. Consumption of other consumer goods, $c$, is given by total consumption opportunity, $y$, minus fuel consumption.
\[

$$
\begin{equation*}
c=y-p_{l} r\left(h_{d}\right) m \tag{2}
\end{equation*}
$$

\]

Fuel costs depend on the price of fuel, $p_{l}$, the number of kilometers driven, $m$, and consumption of fuel per kilometer, $r\left(h_{d}\right)$. Consumption per kilometer decreases with time spent driving per kilometer, $r^{\prime}\left(h_{d}\right)<0$, but with a decreasing rate, $r^{\prime \prime}\left(h_{d}\right)>0$, so that the second order condition for the household problem is satisfied.

### 3.2 Damage of driving

Damage from $\mathrm{CO}_{2}$-emissions is determined by the accumulated number of liters consumed, $r\left(h_{d}\right) m$, multiplied by the cost of $\mathrm{CO}_{2}$-emissions per liter consumed, $p_{\mathrm{CO}}$.

$$
\begin{equation*}
S_{\mathrm{CO} 2}=p_{\mathrm{CO} 2} r\left(h_{d}\right) m \tag{3}
\end{equation*}
$$

Mileage-related damage is determined by the number of kilometers driven and the price of damage per kilometer, $p_{d}\left(h_{d}\right)$. The cost of damage per kilometer depends on driving time per kilometer, $h_{d}$.

$$
\begin{equation*}
S_{d}=p_{d}\left(h_{d}\right) m \tag{4}
\end{equation*}
$$

This damage function is tailor maid for the present study. The function separates between mileage, $m$, and damage per kilometer, $p_{d}\left(h_{d}\right)$. Hence, the function is able to separate between policy which affects damage due to mileage, and policy which affects damage due to driving-style.

### 3.3 The social planner solution

The social planner solution maximizes the representative consumer's utility minus the damage caused by road transport. The problem is

$$
\begin{equation*}
\operatorname{Max}_{m, h_{d}} e(m)+y-p_{l} r\left(h_{d}\right) m-w h_{d} m-p_{C O 2} r\left(h_{d}\right) m-p_{d}\left(h_{d}\right) m . \tag{5}
\end{equation*}
$$

First order conditions imply that

$$
\begin{equation*}
e_{m}^{\prime}-p_{l} r\left(h_{d}\right)-w h_{d}-p_{C O 2} r\left(h_{d}\right)-p_{d}\left(h_{d}\right)=0 \tag{6}
\end{equation*}
$$

and

$$
\begin{equation*}
-p_{l} r^{\prime}\left(h_{d}\right) m-w m-p_{C O 2} r^{\prime}\left(h_{d}\right) m-p_{d}{ }^{\prime}\left(h_{d}\right) m=0 \tag{7}
\end{equation*}
$$

Hence,

$$
\begin{equation*}
e_{m}^{\prime}=\left(p_{l}+p_{\mathrm{CO} 2}\right) r\left(h_{d}\right)+w h_{d}+p_{d}\left(h_{d}\right) \tag{8}
\end{equation*}
$$

and

$$
\begin{equation*}
-\left(p_{l}+p_{\mathrm{CO} 2}\right) r^{\prime}\left(h_{d}\right)-p_{d}{ }^{\prime}\left(h_{d}\right)=w . \tag{9}
\end{equation*}
$$

Second order conditions are satisfied, see appendix A.

## 4. Optimal tax rates

### 4.1 Pigouvian taxes

Pigouvian taxes is defined as tax rates on market activities which equals the marginal external damage generated by this market activity. There are two types of market activities within the present model framework, mileage, $m$, and time spent driving, $h_{d}$. Implementation of Pigouvian taxes therefore require tax rates on each of these activities which equals the marginal damage of these activities ${ }^{7}$. The marginal damage of these activities is found by taking the derivative of the damage functions, equation (3) and (4), with respect to $m$ and $h_{d}$. This gives

$$
\begin{equation*}
t_{m}=p_{\text {CO2 }} r\left(h_{d}\right)+p_{d}\left(h_{d}\right) \tag{10}
\end{equation*}
$$

and

$$
\begin{equation*}
t_{h_{d}}=p_{C O 2} m r^{\prime}\left(h_{d}\right)+p_{d}^{\prime}\left(h_{d}\right) m \tag{11}
\end{equation*}
$$

The Pigouvian tax on driven kilometers is positive because it represents a cost associated with $\mathrm{CO}_{2^{-}}$ emissions, as well as a cost associated with driving-related externalities. The Pigouvian tax on driving time per kilometer is negative because $\mathrm{CO}_{2}$-emissions are reduced and because mileage-related damage are dampened by a more economical driving style.

Assume that tax revenue from these taxes is transferred lump sum to the representative consumer.

$$
\begin{equation*}
k=t_{m} m+t_{h_{d}} h_{d} \tag{12}
\end{equation*}
$$

Hence, transfers, $k$, equals the tax rate on driving, $t_{m}$, multiplied with driving distance plus the tax rate on time spent driving per kilometer, $t_{h_{d}}$, multiplied with time spent driving. The household budget constraint is given by equation (13).

$$
\begin{equation*}
c=y+k-p_{l} r\left(h_{d}\right) m-t_{m} m-t_{h_{d}} h_{d} \tag{13}
\end{equation*}
$$

The household's problem then becomes

$$
\begin{equation*}
\operatorname{Max}_{m, h_{d}} e(m)-w h_{d} m+y+k-p_{l} r\left(h_{d}\right) m-t_{m} m-t_{h_{d}} h_{d} . \tag{14}
\end{equation*}
$$

First order conditions imply that

$$
\begin{equation*}
e_{m}^{\prime}-p_{l} r\left(h_{d}\right)-w h_{d}-t_{m}=0 \tag{15}
\end{equation*}
$$

and

$$
\begin{equation*}
-p_{l} r^{\prime}\left(h_{d}\right) m-w m-t_{h_{d}}=0 \tag{16}
\end{equation*}
$$

Implementing Pigouvian taxes, equation (10) and (11), into equation (15) and (16), and comparing with the social planner solution, equation (6) and (7), shows that these solutions are identical.

[^4]Implementation of such Pigouvian tax rates are however challenging as the government is unable to observe driving-style. Substantial monitoring is required to observe driving. Hence, the government is unable to implement such taxes.

### 4.2 An optimal tax on fuel

Many countries have chosen to implement a uniform tax on fuel only to alleviate external damage from road traffic. The sections analyze the case where a tax on fuel is implemented to correct for external effects of road transport. This represents a second-best strategy when Pigouvian taxes are unavailable. An optimal environmental tax on polluting goods equals the optimal difference between tax rates on polluting and non-polluting goods according to Fullerton (1997). Hence, the optimal tax rate on fuel in the present study should be interpreted as an optimal difference in tax rates between fuel and non-polluting goods.

The household chooses driving distance and driving time per kilometer to maximize utility. The consumer price on fuel equals the producer price plus the tax on fuel. The household's problem is

$$
\begin{equation*}
\operatorname{Max}_{m, h_{d}} e(m)+y+k-\left(p_{l}+t_{l}\right) r\left(h_{d}\right) m-w h_{d} m \tag{17}
\end{equation*}
$$

First order conditions imply that

$$
\begin{equation*}
e_{m}^{\prime}-\left(p_{l}+t_{l}\right) r\left(h_{d}\right)-w h_{d}=0 \tag{18}
\end{equation*}
$$

and

$$
\begin{equation*}
-\left(p_{l}+t_{l}\right) r^{\prime}\left(h_{d}\right) m-w m=0 \tag{19}
\end{equation*}
$$

Hence,

$$
\begin{equation*}
e_{m}^{\prime}=\left(p_{l}+t_{l}\right) r\left(h_{d}\right)+w h_{d} \tag{20}
\end{equation*}
$$

and

$$
\begin{equation*}
-\left(p_{l}+t_{l}\right) r^{\prime}\left(h_{d}\right)=w \tag{21}
\end{equation*}
$$

Equation (20) and (21) determines $m$ and $h_{d}$ as a function of $t_{l}$. Second order conditions are satisfied, see appendix B. A tax increase leads to lower mileage and more economic driving, see appendix E.

The government maximizes the indirect utility function minus external damage. Tax revenue are transferred to the representative agent. The government's problem is

$$
\begin{gather*}
\underset{t_{l}}{\operatorname{Max}} e\left(m\left(t_{l}\right)\right)+y-p_{l} r\left(h_{d}\left(t_{l}\right)\right) m\left(t_{l}\right)-w h_{d}\left(t_{l}\right) m\left(t_{l}\right)-p_{C O 2} r\left(h_{d}\left(t_{l}\right)\right) m\left(t_{l}\right)- \\
p_{d}\left(h_{d}\left(t_{l}\right)\right) m\left(t_{l}\right) . \tag{22}
\end{gather*}
$$

First order conditions imply that

$$
\left.\begin{array}{l}
t_{l}^{*}=p_{C O 2}+\frac{p_{d}\left(h_{d}\right)}{r\left(h_{d}\right)}\left[\frac{1}{1+\frac{E l_{h_{d}} r\left(h_{d}\right) E l_{t_{l}} h_{d}\left(t_{l}\right)}{E l_{t_{l}} m\left(t_{l}\right)}}\right]  \tag{23}\\
+\frac{p_{d}}{r\left(h_{d}\right)} \frac{E l_{h_{d}} p_{d}\left(h_{d}\right)}{E l_{h_{d}} r\left(h_{d}\right)}\left[\frac{1}{1+\frac{1}{E l_{h_{d}} r\left(h_{d}\right) E l_{t} h_{d}\left(t_{l}\right)}} E\right.
\end{array}\right],
$$

see Appendix C. Parameter values are restricted to those that satisfies the second order condition, see Appendix D. Equation (23) shows that the welfare maximizing tax rate, $t_{l}^{*}$, is determined by marginal damages connected to driving and traffic related elasticities. The optimization problem in expression (22) reveals that the environmental tax rate is optimized when the welfare cost of reduced driving due to a marginal tax increase equals the welfare gain of reduced damage due to the tax increase. The welfare cost equals the reduction in the marginal utility of driving due to the marginal tax increase minus the cost of time spent driving minus the cost of fuel. The welfare gain equals the environmental benefit connected to the marginal increase in the tax on fuel, represented by a reduction in greenhouse gas emissions and reduced mileage-related damage.

The Pigouvian solution is a special case where the government is able to tax emissions directly. The $\mathrm{CO}_{2}$-tax component on gasoline, equation (23), therefore equals the Pigouvian solution. A comparison of equation (23) with the Pigou tax on driving however shows that the Pigou tax represents a special case. The optimum fuel tax deviates from the Pigou tax when the tax affects choice of driving-style. There are two reasons for this. The optimal tax is reduced because a more economical driving-style leads to avoidance of the road user charge on fuel, see also Parry and Small (2005). The optimal tax is however increased when a more economical driving-style dampens mileage-related damage.

### 4.3 Optimal taxes on fuel and road use

GPS-based road user charges can be designed to matches mileage-related damage of driving. GPSpricing allows for geographic differentiation, differentiation between different vehicles, and differentiation based on traffic density throughout the day. It is however problematic to design GPSpricing which rewards an economic driving-style. It is therefore assumed that GPS-based road user charges are unable to reward an economic driving-style. Speed limits, traffic rules and taxes on liability insurance will to some extent regulate reckless behavior in road traffic. Such rules and regulations are however not likely to harvest all potential gains connected to an economic drivingstyle. This section calculates optimal combination of taxes on fuel and GPS-based road user charges within such scenarios.

The government chooses tax rates to maximize welfare when the representative household choose mileage and driving-style. A new tax is introduced on the number of kilometers driven, $t_{m}$.

Households' maximization problem is otherwise identical to the problem in section 4.2. The first-order conditions are

$$
\begin{equation*}
e_{m}^{\prime}=\left(p_{l}+t_{l}\right) r\left(h_{d}\right)+w h_{d}+t_{m} \tag{24}
\end{equation*}
$$

and

$$
\begin{equation*}
-\left(p_{l}+t_{l}\right) r^{\prime}\left(h_{d}\right)=w \tag{25}
\end{equation*}
$$

The government now has two instruments to meet 2 objectives. It is therefore possible to implement the social planner solution. This solution is the welfare maximizing solution, as tax rates are chosen to maximize the same expression as in the social planner solution. The optimum fuel tax is found by setting the tax on fuel in equation (25) so that driving time per kilometer is chosen as in equation (9) in the social planner solution. This gives

$$
\begin{equation*}
t_{l}^{* *}=p_{C O 2}+\frac{p_{d}\left(h_{d}\right) E l_{h_{d}} p_{d}\left(h_{d}\right)}{r\left(h_{d}\right) E l_{h_{d}} r\left(h_{d}\right)} \tag{26}
\end{equation*}
$$

The optimum tax on driven kilometers is found by inserting the tax on fuel in equation (26) into equation (24). The tax on mileage in equation (24) is chosen so that the number of kilometers driven is identical with the social planner solution, equation (8). Note that the tax on mileage represents a GPSbased tax on road use. This simple modelling approach is feasible and tractable because of the tailor maid mileage-related damage function.

$$
\begin{equation*}
\frac{t_{m}^{* *}}{r\left(h_{d}\right)}=\frac{p_{d}\left(h_{d}\right)}{r\left(h_{d}\right)}-\frac{p_{d}\left(h_{d}\right) E l_{h_{d}} p_{d}\left(h_{d}\right)}{r\left(h_{d}\right) E l_{h_{d}} r\left(h_{d}\right)} \tag{27}
\end{equation*}
$$

The government stimulates economic driving by setting the tax rate on fuel higher than the marginal damage of $\mathrm{CO}_{2}$-emissions. The tax on mileage is set below the mileage-related damage per kilometer. Summing equations (26) and (27) implies that

$$
\begin{equation*}
t_{l}^{* *}+\frac{t_{m}^{* *}}{r\left(h_{d}\right)}=p_{C O 2}+\frac{p_{d}\left(h_{d}\right)}{r\left(h_{d}\right)} . \tag{28}
\end{equation*}
$$

The tax on fuel plus the tax on mileage equals the marginal damage of $\mathrm{CO}_{2}$-emissions plus the mileage-related marginal damage measured per liter fuel. The sum of these taxes thus equals the sum of the marginal damage per liter fuel.

One may argue that a road user charge based on odometer readings or pay-as-you-drive insurance combined with congestion charges and toll roads resembles GPS-based road user charges. Optimal tax formulas presented in equation (26) and (27) should be employed in this case. However, such charges are costly to administer, susceptible to evasion, and lead to undesirable traffic planning designed to avoid toll stations, see Parry (2002).

## 5. Parameter values

Optimal tax rates are determined by the variables on the right side of equation (23), (26) and (27). This section presents empirical estimates of these variables.

### 5.1 Marginal damage

The marginal damage of $\mathrm{CO}_{2}$-emissions, or social cost of carbon, has been estimated by more than 100 peer-reviewed studies according to the report from the Intergovernmental Panel on Climate Change (IPCC) (2007). The average cost estimate is $\$ 43$ per ton of $\mathrm{CO}_{2}$. A cost estimate of $\$ 50$ is common, as some recent estimates are higher. Consumption of one liter of gasoline generates $2.32 \mathrm{~kg} \mathrm{CO}_{2}$, which amounts to approximately $\$ 0.44$ per gallon of gasoline; see table 1 . This estimate is relevant for countries facing a quota price which equals the social cost of carbon, and for countries concerned with the global damage of carbon emissions.

The average mileage-related marginal damage related to road transport for the US and the UK amounts to $\$ 1.92$ and $\$ 2.92$ per gallon of gasoline, respectively, according to Parry and Small (2005). Anderson and Auffhammer (2014) show that accident-related externalities are related to the weight of vehicles. Internalizing such externalities by a weight-varying mileage tax or a $\$ 0.97-\$ 2.17$ per gallon of gasoline tax is similar for most vehicles according to their estimates. The literature identifies substantial differences in mileage-related damage between rural and non-rural areas, peak and off-peak periods, heavy and light-duty vehicles, and between different speed levels. Such differentiation is to some extent feasible with toll roads and a GPS-based system. The average marginal damage is however relevant with a uniform tax on fuel, see Diamond (1973).

### 5.2 Traffic behavior

The assumption that motorists choose driving and time spent driving to maximize utility implies that utility maximizing fuel-saving measures are implemented. A change in fuel-saving behavior represents a change from a utility optimum. Elasticities are therefore chosen based on changes in optimal choices.

The elasticity of fuel consumption per kilometer with respect to time spent driving per kilometer, $E l_{h_{d}} r\left(h_{d}\right)$, equals the percentage change in liters of fuel consumption per kilometer when the driving time per kilometer increases by one per cent. Fuel economy testes uncover that this percentage reduction in fuel consumption is larger when the initial speed is higher. A highway speed reduction of 1 per cent results in a reduction in fuel consumption per kilometer of approximately 1 per cent, see e.g. Wang et al. (2008). The link between fuel consumption and time spent driving within city centers is more complex, see Tong et al. (2000). Time spent on economic driving however includes a range of techniques, see Barkenbus 2010, Carrico et al., 2009; Onoda, 2009; Tong et al., 2000; Van Mierlo et
al., 2004; Vandenbergh et al., 2008. An elasticity equal to minus one is chosen in a base line scenario based on these studies.

Studies show that increases in fuel prices result in relatively modest reductions in vehicle miles traveled, see e.g. Johansson and Schipper (1997). $E l_{t_{l}} m\left(t_{l}\right)$ is therefore set equal to minus 0.1 in the base line scenario. Note that the elasticity of driving distance with respect to the tax rate, $E l_{t_{l}} m\left(t_{l}\right)$, deviates from the elasticity with respect to the price of fuel.

The economic incentive to save fuel increases when the tax on fuel is increased. The price elasticity of gasoline differs between empirical studies. A bench-mark price elasticity of -0.55 is chosen in Parry and Small (2005). Approximately half of this reduction was attributed to changes in vehicles miles traveled. Hence, the remaining half is the result of improved fuel economy due to choic of vehicle and driving-style. Empirical studies show that choice of vehicle is influenced by the price of gasoline, see Sallee et al. (2016) and Busse et al. (2013). Empirical studies also show that economic driving improves fuel economy, see Barkenbus 2010, Carrico et al., 2009; Onoda, 2009; Tong et al., 2000; Van Mierlo et al., 2004; Vandenbergh et al., 2008. It is however challenging to pinpoint the exact contribution due to economic driving. Hence, three scenarios are presented where the elasticity of economic driving with respect to the tax rate on fuel, $E l_{t_{l}} h_{d}\left(t_{l}\right)$, is close to zero, equals 0.05 and $0.1^{8}$. Note that the elasticity is larger than zero within the model framework, see appendix E.

Less aggressive driving at lower speeds contribute to lower accidents according to Aarts and van Schagen, 2006, van Benthem, 2015, Rodriguez, 1990, Montag, 2014, and Elvik et al. 2019. Lower speeds also generate less noise pollution, see Bendtsen (2004). Lave (1985) however finds no connection between average speed and fatal accidents. He finds a clear connection between fatalities and spreading in speed, see also Aarts and van Schagen (2006) and Elvik (2014). An economic driving-style, and a larger spread in speeds, may however boost traffic congestion. The impact of economic driving on mileage-related damage is uncertain. Three scenarios are presented where the elasticity of mileage-related damage with respect to time spent driving, $E l_{h_{d}} p_{d}\left(h_{d}\right)$, equals $0,-0,3$ and $-0,5$.

[^5]
## 6. Optimal versus current tax rates

Optimal tax rates based on empirical estimates are compared with real world taxes in the US and the UK. Tax theory is however unable to produce a unique optimal tax rate on polluting goods due to choice of normalization, see Fullerton (1997). The explanation is that the allocation of resources is unchanged when a uniform tax increase on consumer goods is combined with a proportional, revenueneutral reduction in taxation of income. The welfare is unchanged by this reform even though the tax rate on polluting goods is increased. Hence, optimal tax rates derived above are compared to current differences in tax rates between fuel and non-polluting goods. Toll on toll roads represents a tax on road transport that comes in addition to today's fuel taxes. The current difference in tax rates between fuel and non-polluting goods therefore includes fees on toll roads. Marginal damage estimates and current tax rates on gasoline and road use are presented in table 1 . Current tax rates in table 1 are compared with optimal tax rates presented in table 2 and 3. Road user charges based on GPS-tracking is not implemented in full scale in the US and the UK. Hence, a comparison with optimal GPS-based taxes is omitted.

The tax rates on fuels differ substantially across countries. The average US tax on gasoline amounted to 47 cents per gallon in 2018 according to the US Energy Information Administration. The average combined sales tax ( 8.4 percent according to Thomson Reuters, 2015) of spending the cost of one gallon of gasoline on non-polluting goods amounts to approximately 17.8 cents. Thus, the current tax difference between gasoline and non-polluting goods amounts to approximately 29 cents per gallon of gasoline in the US. The average toll per gallon of gasoline amounts to approximately 9 cents. The current US tax difference between gasoline and non-polluting goods, including fees on toll roads, equals 0.38 dollar per gallon of gasoline. The tax difference between gasoline and non-polluting consumer goods in the UK amounts to the gasoline tax of $£ 0.5795$ per liter of gasoline, or $\$ 2.69$ per gallon, see UK (2019). The additional value-added tax is levied on most goods including gasoline, and thus does not influence the tax difference. Toll per gallon of gasoline on roads and bridges in the UK is marginal.

Table 1: Tax rates and costs: USD per gallon of gasoline.

|  | The cost of $\mathrm{CO}_{2}-$ <br> emissions | Average mileage- <br> related costs | Current tax difference, <br> gasoline vs. goods | Current fees, <br> toll road | Current tax <br> plus fees |
| :--- | :--- | :--- | :--- | :--- | :--- |
| USA | 0.44 | 1.92 | 0.29 | 0.09 | 0.38 |
| UK | 0.44 | 2.92 | 2.69 | 0 | 2.69 |

The current US tax in table 1 is compared with optimal tax rates presented in table 2 . Scenario A1 assumes that the choice of driving style does not affect the extent of mileage-related damage of road transport. The empirical support for this assumption is weak. The scenario is however constructed to
illustrate the impact of avoidance. The optimal combination of taxes on gasoline and mileage equals the marginal damage of gasoline and kilometers driven, respectively. A tax on gasoline only should be set equal to the marginal damage of $\mathrm{CO}_{2}$-emissions plus half of the mileage-related damage caused by road transport. The explanation is that the road user charge on gasoline is avoided by choosing a more economical driving-style. The percentage reduction in gasoline consumption per kilometer due to economic driving equals the percentage reduction in miles driven. Hence, gains due to lower mileagerelated damage generated by the tax on gasoline is halved as motorists responds by choosing a more economic driving-style. The optimal tax is reduced accordingly.

Table 2, Elasticities, marginal damage and optimal tax rates for the US, USD per gallon of gasoline

| Scenario | $E l_{h_{d}} r\left(h_{d}\right)$ | $E l_{t_{l}} m\left(t_{l}\right)$ | $E l_{t_{l}} h_{d}\left(t_{l}\right)$ | $E l_{h_{d}} p_{d}\left(h_{d}\right)$ | $p_{C O 2}$ | $\frac{p_{d}\left(h_{d}\right)}{r\left(h_{d}\right)}$ | $t_{l}^{*}$ | $t_{l}^{* *}$ | $\frac{t_{m}^{* *}}{r\left(h_{d}\right)}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A 1 | -1 | -0.1 | 0.1 | 0 | 0.44 | 1.92 | 1.4 | 0.44 | 1.92 |
| A 2 | -1 | -0.1 | 0.1 | -0.5 | 0.44 | 1.92 | 1.88 | 1.4 | 0.96 |
| A 3 | -1 | -0.1 | 0.1 | -0.3 | 0.44 | 1.92 | 1.69 | 1.02 | 1.34 |
| A 4 | -1 | -0.1 | 0.05 | -0.3 | 0.44 | 1.92 | 1.91 | 1.02 | 1.34 |
| A 5 | -1 | -0.1 | 0.02 | -0.3 | 0.44 | 1.92 | 2.14 | 1.02 | 1.34 |
| A 6 | -1 | -0.1 | 0 | -0.3 | 0.44 | 1.92 | 2.36 | 1.02 | 1.34 |

In scenarios A2 and A3, it is assumed that a more economical driving style generates a significant reduction in the mileage-related damage from road transport. The optimal combination of taxes on gasoline and driving changes drastically compared to scenario A1. The tax on gasoline is increased to 1.4 in scenario A2 and 1.02 in scenario A3 to reap the rewards of reduced mileage-related damage. The tax rate on mileage is reduced accordingly to maintain an optimal tax burden on driving. The total tax on gasoline and driving equals the marginal damage of driving. The optimal tax on gasoline only is significantly lower than the sum of the marginal damage from greenhouse gases and mileage-related damage. A more economic driving-style leads to avoidance of road user charges on gasoline, and hence contributes to lower the optimal tax rate. The economic driving-style also contributes to lower mileage-related damage. Such gains contribute to increase the optimal tax rate on gasoline. The first effect is stronger however. This explains why the optimal tax rate is below the total marginal damage of driving.

In scenario A4 and A5, it is assumed that the choice of economic driving style is less sensitive to changes in the tax on gasoline compared to scenario A3. The scenario is otherwise identical to scenario A3. The change does not affect the optimal combination of taxes on gasoline and kilometers driven. However, the optimal tax rate on gasoline only increases from \$ 1.69 to $\$ 1.91$ in scenario A4, and to $\$ 2,14$ in scenario A5. The explanation is that the tax on gasoline leads to less tax avoidance
due to economic driving. The tax becomes a more efficient tool designed to lowering mileage-related damage in this scenario. Hence, it is optimal to set a higher tax rate on gasoline.

Scenario A6 illustrates the case where taxes on gasoline do not affect the choice of driving style. The empirical support for this assumption is weak. The scenario is however constructed to illustrate the solution when choice of driving-style is not considered. The scenario is otherwise identical to scenarios A3, A4 and A5. The optimal combinations of taxes on gasoline and driving is identical with combinations in scenarios A3, A4 and A5. The optimal tax rate on gasoline only equals the marginal damage of $\mathrm{CO}_{2}$-emissions plus the marginal damage of mileage-related externalities. The explanation is that the tax does not lead to efficiency losses due to avoidance. Hence, it is optimal to set the tax equals to the total marginal damage of driving.

Optimal UK tax rates are presented in table 3. A comparison with current tax rates in table 1 shows that the optimal tax rate on gasoline only is below the current tax rate on gasoline in scenario B1-B4. The optimal tax rate is higher than the current tax rate in scenario B5 and B6, i.e. where the impact of taxation on economic driving is marginal or absent. A transition to GPS-based road user charges leads to different results. The optimal tax rate on gasoline exceeds the marginal damage of $\mathrm{CO}_{2}$-emissions by a substantial margin in scenarios where economic driving lowers mileage-related damage. The explanation is that the tax on gasoline is able to harvest such gains, while the GPS-based tax is unable to harvest these gains.

Table 3, Elasticities, marginal damage and optimal tax rates for the UK, USD per gallon of gasoline

| Scenario | $E l_{h_{d}} r\left(h_{d}\right)$ | $E l_{t_{l}} m\left(t_{l}\right)$ | $E l_{t_{l}} h_{d}\left(t_{l}\right)$ | $E l_{h_{d}} p_{d}\left(h_{d}\right)$ | $p_{C O 2}$ | $\frac{p_{d}\left(h_{d}\right)}{r\left(h_{d}\right)}$ | $t_{l}^{*}$ | $t_{l}^{* *}$ | $\frac{t_{m}^{* *}}{r\left(h_{d}\right)}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| B1 | -1 | -0.1 | 0.1 | 0 | 0.44 | 2.92 | 1.9 | 0.44 | 2.92 |
| B2 | -1 | -0.1 | 0.1 | -0.5 | 0.44 | 2.92 | 2.63 | 1.9 | 1.46 |
| B3 | -1 | -0.1 | 0.1 | -0.3 | 0.44 | 2.92 | 2.34 | 1.32 | 2.04 |
| B4 | -1 | -0.1 | 0.05 | -0.3 | 0.44 | 2.92 | 2.39 | 1.32 | 2.04 |
| B5 | -1 | -0.1 | 0.02 | -0.3 | 0.44 | 2.92 | 3,02 | 1.32 | 2.04 |
| B6 | -1 | -0.1 | 0 | -0.3 | 0.44 | 2.92 | 3.36 | 1.32 | 2.04 |

## 7. Conclusion

Many countries have implemented taxes on fuel to curb externalities linked to both fuel and mileage. However, the tax-induced gain in terms of reduced externalities is diminished as less than half of the
reduction in fuel use is due to reduced driving, see Parry and Small (2005) which argue that the optimal tax rates on gasoline are reduced accordingly. The remaining reduction is due to improved fleet fuel efficiency. Fleet fuel efficiency improves as households avoid the mileage-related tax component on fuel by purchasing more fuel-efficient vehicles. Bjertnæs (2019) however shows that such avoidance should be prevented by imposing heavier taxation of fuel-efficient vehicles compared to fuel-intensive vehicles, and hence, that it is sub-optimal to lower the tax rate on gasoline. However, fleet fuel efficiency also improves as households choose a more fuel-efficient driving-style. The tax on fuel creates an incentive to save fuel rather than lowering mileage-related damage. The present study contributes to the literature by exploring how taxes on fuel and road use should be designed to combat driving related externalities when agents avoid the tax on fuel by choosing a fuel-efficient drivingstyle.

The study shows that the efficient road user charge on fuel is below the marginal mileage-related damage to prevent tax avoidance due to an excessive economic driving-style, even though accidents are reduced. The current US tax rate on gasoline is way below the efficient rate while the current UK rate is slightly above the efficient rate in this case. The study also calculates optimal combination of taxes on fuel and road use. An efficient tax on fuel exceeds the marginal damage of $\mathrm{CO}_{2}$-emissions to promote an economic driving-style when the tax is combined with a GPS-based tax on road use. The efficient GPS-based tax rate is reduced below the marginal damage of mileage-related externalities in this case to prevent excessive taxation of driving. Electric vehicle owners avoid all taxes levied on fuel. A GPS-based road user charge may of course be levied on electric vehicles.

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## Appendix

A. Second order conditions for the social planner problem:

$$
\begin{align*}
& e^{\prime}{ }_{m}{ }^{\prime}{ }_{m}<0  \tag{A1}\\
& -p_{l} r^{\prime}\left(h_{d}\right)-w-p_{C O 2} r^{\prime}\left(h_{d}\right)-p_{d}{ }^{\prime}\left(h_{d}\right)=0  \tag{A2}\\
& -p_{l} r^{\prime}{ }_{h_{d}}{ }^{\prime} h_{d} m-p_{C O 2} r^{\prime}{ }_{h_{d}}{ }^{\prime} h_{d} m-p_{d}{ }^{\prime} h_{d}{ }^{\prime} h_{d} m<0 \tag{A3}
\end{align*}
$$

The condition for a local optimum is satisfied. The condition for a global optimum is assumed to be satisfied.
B. Second order conditions for the household problem:

$$
\begin{align*}
& e_{m}^{\prime}{ }_{m}^{\prime}<0  \tag{A4}\\
& -\left(p_{l}+t_{l}\right) r^{\prime}\left(h_{d}\right)-w=0  \tag{A5}\\
& -\left(p_{l}+t_{l}\right) r^{\prime}{ }_{h_{d}{ }^{\prime} h_{d} m<0}{ }^{\prime} m<0 \tag{A6}
\end{align*}
$$

The condition for a local optimum is satisfied. The condition for a global optimum is assumed to be satisfied.
C. First order conditions for the government optimization problem.

$$
\begin{align*}
& e_{m}^{\prime} m_{t_{l}}^{\prime}-p_{l} r\left(h_{d}\left(t_{l}\right)\right) m^{\prime}{ }_{t}-p_{l} r^{\prime}\left(h_{d}\right) h_{d}{ }^{\prime}{ }_{l} m\left(t_{l}\right)-w h_{d}{ }^{\prime}{ }_{l} m\left(t_{l}\right)-w h_{d}\left(t_{l}\right) m_{t_{l}}^{\prime}  \tag{A7}\\
& -p_{C O 2} r\left(h_{d}\left(t_{l}\right)\right) m^{\prime}{ }_{t}-p_{C_{l}} r^{\prime}\left(h_{d}\right) h_{d}{ }^{\prime}{ }_{t}{ }_{l} m\left(t_{l}\right)-p_{d}{ }^{\prime}\left(h_{d}\right) h_{d}{ }^{\prime} t_{l} m\left(t_{l}\right)-p_{d}\left(h_{d}\left(t_{l}\right)\right) m^{\prime}{ }_{t l}=0
\end{align*}
$$

so that

$$
\begin{align*}
& e^{\prime}{ }_{m} m^{\prime}{ }_{t_{l}}=p_{l} r\left(h_{d}\left(t_{l}\right)\right) m_{t_{l}}^{\prime}+w h_{d}\left(t_{l}\right) m_{t_{l}}^{\prime}+p_{C O 2} r\left(h_{d}\left(t_{l}\right)\right) m_{t_{l}}^{\prime}+p_{d}\left(h_{d}\right) m_{t_{l}}^{\prime}  \tag{A8}\\
& \quad+p_{l} r^{\prime}\left(h_{d}\right) h_{d}{ }^{\prime} t_{l} m\left(t_{l}\right)+w h_{d}{ }^{\prime} t_{l} m\left(t_{l}\right)+p_{C O 2} r^{\prime}\left(h_{d}\right) h_{d}{ }^{\prime} t_{l} m\left(t_{l}\right)+p_{d}{ }^{\prime}\left(h_{d}\right) h_{d}{ }^{\prime} t_{l} m\left(t_{l}\right)
\end{align*}
$$

Multiplying first order conditions for the household (18) with $m^{\prime}{ }_{t_{l}}$ and (19) with $h_{d}{ }^{\prime} t_{l}$ gives

$$
\begin{equation*}
e_{m}^{\prime} m_{t_{l}}^{\prime}-p_{l} r\left(h_{d}\right) m_{t_{l}}^{\prime}-w h_{d} m_{t_{l}}^{\prime}=t_{l} r\left(h_{d}\right) m_{t_{l}}^{\prime} \tag{A9}
\end{equation*}
$$

and

$$
\begin{equation*}
p_{l} r^{\prime}\left(h_{d}\right) h_{d}{ }^{\prime}{ }_{l} m\left(t_{l}\right)+w h_{d}{ }^{\prime}{ }_{l} m\left(t_{l}\right)=-t_{l} r^{\prime}\left(h_{d}\right) h_{d}{ }^{\prime}{ }_{l} m\left(t_{l}\right) \tag{A10}
\end{equation*}
$$

Implementing equation (A9) and (A10) into equation (A8) gives

$$
\begin{align*}
t_{l} r\left(h_{d}\right) m^{\prime}{ }_{t_{l}}=p_{\mathrm{CO} 2} r & \left(h_{d}\left(t_{l}\right)\right) m^{\prime}{ }_{t_{l}}+p_{d}\left(h_{d}\right) m^{\prime}{ }_{t}-t_{l} r^{\prime}\left(h_{d}\right) h_{d}{ }^{\prime}{ }_{t} m\left(t_{l}\right)  \tag{A11}\\
& +p_{\mathrm{CO} 2} r^{\prime}\left(h_{d}\right) h_{d}{ }^{\prime}{ }_{t_{l}} m\left(t_{l}\right)+p_{d}{ }^{\prime}\left(h_{d}\right) h_{d}{ }^{\prime}{ }_{t} m\left(t_{l}\right)
\end{align*}
$$

Hence,

$$
\begin{align*}
& t_{l}=p_{C O 2}+\frac{p_{d}\left(h_{d}\right)}{r\left(h_{d}\right)}\left[\frac{1}{1+\frac{r^{\prime}\left(h_{d}\right) h_{d^{\prime}} t_{l} m\left(t_{l}\right)}{r\left(h_{d}\right) m^{\prime} t_{l}}}\right]  \tag{A12}\\
& +p_{d}{ }^{\prime}\left(h_{d}\right) \frac{h_{d} t_{l} m\left(t_{l}\right)}{r\left(h_{d}\right) m^{\prime} t_{l}}\left[\frac{1}{1+\frac{r^{\prime}\left(h_{d}\right) h_{d^{\prime} t_{t} m\left(t_{l}\right)}^{r\left(h_{d}\right) m^{\prime} t_{l}}}{1}}\right]
\end{align*}
$$

Hence,

$$
\left.\begin{array}{l}
t_{l}=p_{C O 2}+\frac{p_{d}\left(h_{d}\right)}{r\left(h_{d}\right)}\left[\frac{1}{1+\frac{E l_{h_{d}} r\left(h_{d}\right) E l_{t_{l} h_{d}\left(t_{l}\right)}^{E t_{t}} m\left(t_{l}\right)}{E l}}\right]  \tag{A13}\\
+\frac{p_{d}^{\prime}\left(h_{d}\right)}{r^{\prime}\left(h_{d}\right)}\left[\frac{1}{1+\frac{1}{E l_{h_{d}} r\left(h_{d}\right) E l_{t} h_{d}\left(t_{l}\right)}} \overline{E l_{t_{l}} m\left(t_{l}\right)}\right.
\end{array}\right]
$$

Hence,

$$
\left.\begin{array}{l}
t_{l}^{*}=p_{C O 2}+\frac{p_{d}\left(h_{d}\right)}{r\left(h_{d}\right)}\left[\frac{1}{1+\frac{E l_{h_{d}} r\left(h_{d}\right) E l_{t_{l}} h_{d}\left(t_{l}\right)}{E l_{t} m\left(t_{l}\right)}}\right]  \tag{A14}\\
+\frac{p_{d}}{r\left(h_{d}\right)} \frac{E l_{h_{d}} p_{d}\left(h_{d}\right)}{E l_{h_{d}} r\left(h_{d}\right)}\left[\frac{1}{1+\frac{1}{E l_{h_{d}} r\left(h_{d}\right) E l_{t_{l}} h_{d}\left(t_{l}\right)}} E E l_{t_{l} m\left(t_{l}\right)}\right.
\end{array}\right]
$$

D. Second order conditions for the government problem

$$
\begin{aligned}
& e^{\prime}{ }_{m}{ }^{\prime}{ }_{m} m^{\prime}{ }_{t_{l}} m^{\prime}{ }_{t_{l}}+e^{\prime}{ }_{m} m^{\prime}{ }_{t_{l}}{ }^{\prime} t_{l}-p_{l} r^{\prime}\left(h_{d}\right) h_{d}{ }^{\prime} t_{l} m^{\prime}{ }_{t_{l}}-p_{l} r\left(h_{d}\left(t_{l}\right)\right) m^{\prime}{ }_{t_{l}}{ }^{\prime} t_{l} \\
& -p_{l} r^{\prime}{ }_{h_{d}}{ }^{\prime} h_{d} h_{d}{ }^{\prime}{ }_{t} h_{d}{ }^{\prime}{ }_{t} m\left(t_{l}\right)-p_{l} r^{\prime}{ }_{h_{d}} h_{d}{ }^{\prime} t_{l}{ }_{l}{ }_{l_{l}} m\left(t_{l}\right)-p_{l} r^{\prime}{ }_{h_{d}} h_{d}{ }^{\prime}{ }_{t} m^{\prime}{ }^{\prime}{ }_{t_{l}} \\
& -w h_{d}{ }^{\prime} t_{l}{ }_{l}{ }_{l}{ }_{l} m\left(t_{l}\right)-w h_{d}{ }^{\prime} t_{l} m^{\prime}{ }_{t_{l}}-w h_{d}{ }^{\prime}{ }_{t_{l}} m^{\prime}{ }_{t_{l}}-w h_{d} m^{\prime}{ }_{t_{l}}{ }^{\prime} t_{l}-p_{C O 2} r^{\prime}{ }_{h_{d}} h_{d}{ }^{\prime} t_{l} m^{\prime}{ }_{t_{l}} \\
& -p_{\mathrm{CO} 2} r\left(h_{d}\left(t_{l}\right)\right) m^{\prime}{ }_{t_{l}{ }^{\prime} t_{l}}-p_{\mathrm{CO} 2} r^{\prime}{ }_{h_{d}}{ }^{\prime}{ }_{h_{d}} h_{d}{ }^{\prime}{ }_{t_{l}} h_{d}{ }^{\prime}{ }_{t}{ }_{l} m\left(t_{l}\right)-p_{\mathrm{CO} 2} r^{\prime}{ }_{h_{d}} h_{d}{ }^{\prime}{ }_{t}{ }_{l}{ }^{\prime}{ }_{l} m\left(t_{l}\right) \\
& -p_{\mathrm{CO} 2} r^{\prime}{ }_{h_{d}} h_{d}{ }^{\prime}{ }_{t} m^{\prime}{ }_{t_{l}}-p_{d}{ }^{\prime} h_{d}{ }^{\prime} h_{d} h_{d}{ }^{\prime}{ }_{t} h_{d}{ }^{\prime}{ }_{t} m\left(t_{l}\right)-p_{d}{ }^{\prime} h_{d} h_{d}{ }^{\prime}{ }_{t}{ } m^{\prime}{ }_{t_{l}} \\
& -p_{d}{ }^{\prime}{h_{d}} h_{d}{ }^{\prime}{ }^{\prime}{ }_{l}{ }^{\prime}{ }_{l} m\left(t_{l}\right)-p_{d}{ }^{\prime}{ }_{d} h_{d}{ }^{\prime} t_{l} m^{\prime}{ }_{t_{l}}-p_{d}\left(h_{d}\left(t_{l}\right)\right) m^{\prime}{ }_{t_{l}}{ }^{\prime} t_{l}<0
\end{aligned}
$$

Parameter values are restricted to those that satisfies this condition.
E.

Taking the derivative of equation (19) implies that

$$
\begin{align*}
& -{r^{\prime}}_{h_{d}}-\left(p_{l}+t_{l}\right) r_{h_{d}}^{\prime}{ }^{\prime}{h_{d}}_{d} h_{d}{ }^{\prime} t_{l}=0  \tag{A16}\\
& h_{d}^{\prime} t_{l}=\frac{r_{h_{d}}^{\prime}}{-\left(p_{l}+t_{l}\right) r^{\prime} h_{d} h_{d}}>0 \tag{A17}
\end{align*}
$$

Taking the derivative of equation (18) implies that
$e^{\prime}{ }_{m}{ }^{\prime}{ }_{m} m^{\prime}{ }_{t_{l}}=r\left(h_{d}\left(t_{l}\right)\right)+\left(p_{l}+t_{l}\right) r^{\prime}{ }_{h_{d}} h_{d}{ }^{\prime} t_{l}+w h_{d}{ }^{\prime} t_{l}$
Hence,

$$
\begin{equation*}
m_{t_{l}}^{\prime}=\frac{r\left(h_{d}\left(t_{l}\right)\right)+\left(p_{l}+t_{l}\right) r^{\prime}{ }_{h_{d}} h_{d^{\prime} t_{l}}+w h_{d^{\prime} t_{l}}}{e^{\prime} m^{\prime} m} \tag{A18}
\end{equation*}
$$

Inserting for $h_{d}{ }^{\prime}{ }_{l}$ gives

$$
\begin{equation*}
m_{t_{l}}^{\prime}=\frac{r\left(h_{d}\left(t_{l}\right)\right)}{e^{\prime} m^{\prime} m}<0 \tag{A20}
\end{equation*}
$$


[^0]:    ${ }^{1}$ I am highly grateful for valuable comments from Vidar Christiansen, Bjart Holtsmark, Paal Brevik Wangsness and attendees at the OFS workshop on taxes 2018.

[^1]:    ${ }^{2}$ Taxation of consumer goods designed to redistribute income is not part of an optimal tax system.
    ${ }^{3}$ Results in the literature differ on the issue of whether environmental taxes should deviate from the Pigouvian rate due to tax revenue requirements. The optimal tax rate in Parry and Small (2005) is lower due to tax revenue requirements. Jaeger (2011), however, finds that the need for tax revenue contributes to increasing the optimal environmental tax wedge to higher than the Pigouvian tax rate. The optimal $\mathrm{CO}_{2}$-tax also exceeds the quota price when the government purchase quotas and the marginal cost of public funds exceed one, according to Bjertnæs et. al. (2013).
    ${ }^{4}$ A range of other studies have adopted their method to calculate optimal tax rates on gasoline in other countries; see e.g. Anton-Sarabia and Hernandez-Trillo (2014), Lin and Zeng (2014).

[^2]:    ${ }^{5}$ A tax based on odometer readings is also hampered by avoidance if drivers can roll back their odometers.

[^3]:    ${ }^{6}$ The average damage is relevant when a uniform tax is employed to correct for different externalities, see Diamond (1973).

[^4]:    ${ }^{7}$ Road transport in the production sector is ignored. Tax formulas within this study are however relevant if Pigouvian taxes are desirable in the production sector.

[^5]:    ${ }^{8}$ Example: Assume that the long-run price elasticity of gasoline consumption in the US equals minus one. This implies a gasoline tax rate elasticity of approximately -0.2 as the tax rate amounts to approximately 20 percent of the price. Assume that 50 percent of the price response is due to reductions in vehicle miles traveled. This implies that the vehicle miles traveled elasticity with respect to the gasoline tax rate equals -0.1 . The remaining 50 percent of the price response is due to lower fuel consumption per miles traveled. Assuming that choice of vehicle and choice of driving style is equally important implies a fuel consumption per mile traveled elasticity with respect to choice of driving style of -0.05 .

