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### Tax Debt Collection Enforcement: When Does Suspension of a Driver's License Help?

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## Abstract

This paper examines the enforcement of tax debt collection and explains when it is optimal to impose on delinquent taxpayers a collateral tax sanction such as the suspension of a driver's license or the revocation of a passport. I develop a dynamic model, where individuals are heterogeneous in income and in ability to escape tax debt payment: a debtor may not pay the tax debt either because of being income constrained or because of having a higher chance to escape from the collection process. To discourage tax debt, the tax authority can impose a monetary fine or a collateral tax sanction. I show that, when debtors differ in their ability to escape tax debt, the timing when a penalty affects the tax debtor is critical, therefore it may be optimal to use the collateral tax sanction in addition to the monetary fine. In contrast to the monetary fine that can be delayed and paid only when the tax debt is collected, the collateral tax sanction applies and influences immediately. In the case, when the utility is CRRA and the distributions of income and ability are uniform, it is optimal to use the collateral tax sanction if the upper bound of income distribution is sufficiently large.

**Keywords:** tax debt, enforcement, collateral tax sanctions, dynamic

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# 1 Introduction

Collection of tax debt is a critical and integral part of the tax enforcement process. It is the next step after tax evasion has been discovered and the due amount of taxes is assessed but failed to be paid. However, while tax evasion has received a lot of attention in the existing tax enforcement literature (see the surveys in Cowell (1990), Andreoni, Erard and Feinstein (1998), Slemrod and Yitzhaki (2002)), enforcement of tax debt has not been well studied. The nearly sole study of tax debt enforcement, by Perez-Truglia and Troiano (2015), presents a theory and experimental evidence supporting that it may be optimal to use shaming penalty that involves publishing the names of tax delinquents online in addition to monetary fine. In this paper, I continue to explore what instruments are optimal to use for tax debt enforcement. Specifically, I examine when it is optimal to use collateral tax sanctions for the enforcement of tax delinquencies. An example of a collateral tax sanction that has been used for this purpose by several states in the US is the suspension of a driver's license. Recently, such a collateral tax sanction as the denial of a new (or renewed) passport has been used in the US nationwide.

In the United States, the size of tax debt and especially uncollected tax debt is significant and surprising. In 2013, tax debt, that consists of past-due tax liabilities, penalties, and interest, was roughly 60 billions of dollars, which is 5 percent of individual, estate, and trust net income tax collections. According to Burman (2003), in a statement before the US House of Representatives Committee on the Budget, “the IRS assesses almost \$30 billion of taxes that it will never collect. This is not theoretical tax evasion. The \$30 billion represents underpayments of tax that the IRS has identified but cannot collect because its staff is spread so thin. [ . . . ] According to IRS estimates, 60 percent of identified tax debts are never collected. These unclosed cases include: 75 % of identified nonfilers; 79 % of taxpayers who use ‘known abusive devices’ to avoid taxes; 78 % of taxpayers identified through document matching programs. It is possible that some of these people simply cannot afford to pay their tax debts, but more than half—56 %—of noncompliant taxpayers with incomes over \$100 000 get off scot-free.”

The IRS has multiple instruments to enforce the collection of past-due tax liabilities. The common instrument is a monetary fine (i.e., an above-market interest rate on the debt amount). The tax authority has also collection tools that include liens (taking ownership of the taxpayer's property until a tax debt is paid) and levies (garnishing wages, seizing money from bank accounts or outright seizures of property). The tax authority starts the collection process by notifying a delinquent taxpayer about the tax amount owed and demanding the payment. Then, if a taxpayer does not take actions to pay the tax debt (like making a

payment or entering an installment agreement), the IRS places a lien on the taxpayer's assets in order to secure payment of taxes. Lastly, if still no actions are taken, the IRS proceeds by implementing a tax levy, which, depending on the taxpayer's situation, could be wage garnishment, bank levy, or asset seizure.

However, even with these instruments the tax authority has limited ability to collect tax debt. One reason for this is explained by Galmarini et al. (2014) who focus on post-audit collection efforts in Italy. In the case of Italian tax system, a critical step of the collection of past-due taxes is to notify the taxpayers with due amounts. Some taxpayers are, however, trying to escape the notice by "changing address". If the collection agency is not able to discover where the taxpayer hides, then the notice will not take place within the legally set time limit and the taxpayer will be cleared from the due tax amount.<sup>1</sup> Second, when a lien or levy is put in action, the collection process may be substantially delayed or impeded by lengthy court battles.

Recently, the IRS began to implement a new instrument for tax debt enforcement, namely, it began to deny new or renewed passports for taxpayers who have more than \$51,000 of overdue tax debt.<sup>2</sup> This measure is an example of a collateral tax sanction, which is a revocation of a privilege provided by the government, imposed for a failure to comply with tax obligations. Interestingly, this instrument's effects are immediately noticeable, according to an IRS spokesman, 220 people had handed over \$11.5 million to repay their full debts as of late June, 2018, while 1,400 others had set up payment plans to reduce their debts. Moreover, one debtor in particular had paid \$1 million in tax debts specifically to avoid passport denial.<sup>3</sup>

Another collateral tax sanction that has been used for tax debt enforcement is the suspension of a driver's license. For now, it has been implemented by three states – Louisiana, California, and New York State. After a notice to a tax delinquent about tax debt with a request to pay it, the tax authority can proceed by instructing Department of Motor Vehicles to suspend a driver's license. The suspension remains in effect until the taxpayer has satisfied her past-due tax liabilities. The process of a driver's license suspension is easy and almost

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<sup>1</sup>By using Italian data, Galmarini et al. (2014) found that the presence of a prior notice is associated with a reduced probability of changing address afterward.

<sup>2</sup>See news-releases "IRS Urges Travelers Requiring Passports to Pay Their Back Taxes or Enter into Payment Agreements; People Owing \$51,000 or More Covered", January 16, 2018, available at <https://www.irs.gov/newsroom/irs-urges-travelers-requiring-passports-to-pay-their-back-taxes-or-enter-into-payment-agreements-people-owing-51000-or-more-covered>. See also <https://www.irs.gov/businesses/small-businesses-self-employed/revocation-or-denial-of-passport-in-case-of-certain-unpaid-taxes>

<sup>3</sup>See press-release "Thousands of Americans stand to be denied passports due to unpaid taxes", July 6, 2018, available at <http://thehill.com/policy/finance/395869-hundreds-of-thousands-stand-to-be-denied-passports-due-to-unpaid-taxes>

costless for the tax authority.<sup>4</sup> According to New York State Governor Andrew M. Cuomo (March 17, 2014), initial results of tax scofflaw driver license suspension initiative show that as a result of the program tax collections increased nearly \$56.4 million on a state and local basis, which is 5 percent of total delinquency collections in 2013.<sup>5</sup> Furthermore, 37 percent of tax debtors who were contacted beginning in August, 2013 have either paid in full or have been making payments on their debt.

One important difference between a collateral tax sanction and a monetary fine lies in the timing of their effects on tax debtors. Collateral tax sanctions can be imposed in a short time and start to influence tax debtors right away.<sup>6</sup> In contrast, monetary fines affect taxpayers only at the moment when they are collected, which occurs together with the tax debt collection. Because in reality a number of tax delinquents delay or escape paying their taxes and fines, monetary fines imposed on paper are delayed and often not collected at all. As a result, the actual time until the monetary fine is collected and, thus, influences the taxpayer may be longer than the time needed for the revocation of driver's license to affect the taxpayer.

This prompt influence of the collateral tax sanction is critical when debtors are heterogeneous in ability to delay tax debt collection actions. The heterogeneity could exist either because different collection tools are applicable or because people have different abilities to battle in courts, etc. When there is such a heterogeneity, the collateral tax sanction has some advantage over the monetary fine because the collateral tax sanction affects immediately even those debtors who can substantially delay their tax debt payment. Moreover, the longer the tax debt is postponed the longer the collateral tax sanction remains in effect.

The time when a punishment instrument becomes effective is an important factor that influences taxpayers' and the tax authority's decisions. However, the consequences of delays and variation in punishment effective time has not been well examined in the existing literature. The tax enforcement literature usually assumes that punishment happens immediately after evasion is conducted. One exception is the paper by Andreoni (1992) who considers settings when an audit and punishment (monetary fine) for tax evasion occur one period after evasion is undertaken. In these settings, tax evasion may be a high-risk substitute for a loan. As a result, when individuals face binding borrowing constraints, some evasion may be socially desirable. So, Andreoni argues that the government can increase welfare by playing

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<sup>4</sup>It is not, however, socially costless, which is discussed later.

<sup>5</sup>See press-release "Governor Cuomo Announces Initial Results of Tax Scofflaw Driver License Suspension Initiative", March 17, 2014, available at <http://www.governor.ny.gov/press/03172014-drivers-license-suspension-initiative>

<sup>6</sup>Of course, there is always a concern that some people may break the law and drive without their driver's license.

the role of 'loan shark' to people whose borrowing is constrained on the private market. Additionally, he shows that in his model it is not optimal to use non-monetary penalties (collateral sanctions).

In this paper, I develop a dynamic model where risk-averse individuals have a tax debt and decide whether to pay it in the first period or postpone paying until future. The individuals are heterogeneous in two different respects. First, they differ in their income in the first period. In the subsequent periods, they have identical incomes. Second, they differ in how likely the tax authority can collect the tax debt by using a collection tool in the second period (or any other subsequent period). This probability represents debtors' abilities to postpone their tax debts. So, the debtors are heterogeneous not only in how much they are income constrained, but also in how long they can delay their payments.

To enforce tax debt collection, the tax authority can impose a monetary fine and a collateral tax sanction (a driver's license suspension). The tax authority cares not only about tax revenue, but about the welfare of taxpayers. Therefore, when a taxpayer fails to pay her past-due tax liabilities because of being cash constrained, the tax authority may choose to wait for the repayment of the tax debt till the next period. There is, however, another reason for why the tax debt may not be paid in the first period. The taxpayer may expect to escape paying tax debt for a sufficiently long period of time. The tax authority does not want to dismiss from punishment those taxpayers who do not pay their tax debt because of the second reason. But, the monetary fine is favoring those who have a better ability to delay their debt payment because the monetary fine is collected at the moment when the debt is collected. In contrast, the collateral tax sanction can be applied immediately and remains in effect until the debt is paid. Therefore, it affects stronger those debtors who postpone paying their debt for a longer time.

I show that when individuals are homogeneous in ability to postpone paying tax debt, it is optimal to use only the monetary fine. In this case, while both the monetary fine and the collateral tax sanction reduce private welfare, the monetary fine has an advantage over the collateral tax sanction because the monetary fine generates additional income for the tax authority. This resembles Andreoni's (1992) result.<sup>7</sup> However, when individuals are heterogeneous in ability to postpone paying the tax debt, it may be optimal to supplement the monetary fine with the collateral tax sanction. The collateral sanction helps to discourage deliberate procrastination of paying the tax debt by those who have higher abilities to delay tax debt payment.

To gain further insights, I consider the case when utility has constant relative risk aversion and the distribution of income in period 1 and the distribution of ability to postpone tax debt

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<sup>7</sup>In Andreoni (1992), individuals are only characterized by the extent of being income constrained.

are uniform. In this case, I find that it is optimal to impose the collateral tax sanction when the upper bound of income in the first period is relatively large. This means that initially not all taxpayers are poor and income constrained; some taxpayers are relatively rich. Thus, when the population of tax debtors consists not just from people who are simply income constrained but also from high-income individuals, it is optimal to impose the collateral tax sanction.

The paper proceeds as follows. Section 2 explains how enforcement of tax evasion and tax debt collection is organized in practice and describes collateral tax sanctions. Section 3 presents a dynamic model of the tax debt repayment behavior when risk-averse individuals are heterogeneous to the extent of being income constrained and in ability to delay tax debt payment. Section 4 characterizes the optimal choice of a monetary fine and a collateral tax sanction for the tax debt enforcement purpose in the cases when taxpayers are homogeneous and heterogeneous in ability to delay tax debt payment. Section 5 concludes.

## 2 Enforcement in Practice

Tax enforcement can be considered as a two-stage process. The first stage is to discover tax evasion and avoidance and to assess the due amount of taxes. The next stage is to collect the assessed due amounts. To enforce tax evasion and avoidance, the tax authority conducts mail, office, and field audits as well as collects and cross-checks information reports. As a result of a tax examination, the IRS may discover additional tax liabilities and impose tax fines. The IRS notifies taxpayers about reassessed tax liabilities and imposed tax fines and asks the taxpayer to pay the owed amount. Tax evasion constitutes a serious problem for the IRS. Therefore, it is not surprising that a great body of the tax enforcement literature concentrates on tax evasion.<sup>8</sup> It is important to note, that an underlying assumption in this literature is that once evasion is discovered taxpayers fully pay their additional taxes and fines. However, people not only evade taxes, they also do not pay their tax debts.<sup>9</sup> According to the U.S. Department of Treasury (2012), delinquent taxes composed more than 20 percent of the total U.S. gross tax gap in 2006.<sup>10</sup>

### 2.1 Tax Debt Collection Tools

The tax debt collection procedure is multi-step. The first step taken by the tax authority is to notify a delinquent taxpayer by letter assessing the tax amount owed and demanding

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<sup>8</sup>See the surveys in Cowell 1990, Andreoni et al. 1998, Slemrod and Yitzhaki 2002.

<sup>9</sup>See Galmarini et al. 2014.

<sup>10</sup>For more details, see Perez-Truglia and Troiano (2015).

payment of that amount. If after that no actions are taken to pay the tax debt or to achieve a settlement agreement, then the second step is exerted, which is to place a lien on the taxpayer's assets. It is placed in order to secure payment of taxes. A tax lien is the government's claim to taxpayer assets. This means if the taxpayer tries to sell that asset the IRS will be able to take its cut of the funds before he does. If still no actions are taken to settle or pay the taxes owed, the third step will be taken. The IRS will proceed by implementing a tax levy. Depending upon the taxpayer's financial situation, the IRS may implement any of the following forms of levy: wage garnishment, bank levy, and asset seizure.

The duration of tax debt collection process substantially depends on the type of levy that can be applied to a tax delinquent. Usually, collection through wage garnishment takes a shorter time than through property seizure. The cases of property seizure involves court examinations which, depending on the situation, may take years. Because of this, 60 percent of identified tax debts are never collected (Burman (2003)). That is, half of tax debtors are able to escape imposed fines. This implies that in many cases fines could be postponed or not paid, which makes monetary fines less effective for enforcement of tax debt.

## 2.2 Collateral Tax Sanctions

The use of collateral sanctions for the enforcement of tax debt collection has grown over the last decade. On February 2018, the IRS began to deny new or renewed passports for taxpayers who have more than \$51,000 of overdue tax debt.<sup>11</sup> Specifically, the IRS notifies the State Department of taxpayers the IRS has certified as owing a seriously delinquent tax debt and then the State Department denies their passport application or denies renewal of their passport. In some cases, the State Department may revoke their passport. A taxpayer with a seriously delinquent tax debt is generally someone who owes the IRS more than \$51,000 in back taxes, penalties and interest for which the IRS has filed a Notice of Federal Tax Lien and the period to challenge it has expired or the IRS has issued a levy.

Another recent example of the introduction of a collateral tax sanction is a driver's license suspension program established by New York State Tax Department in August 2013. This program aids in the collection of past-due state tax liabilities by suspending the drivers' licenses of taxpayers with past-due tax liabilities of \$10,000 or more. Other states, where

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<sup>11</sup>See news-releases "IRS Urges Travelers Requiring Passports to Pay Their Back Taxes or Enter into Payment Agreements; People Owing \$51,000 or More Covered", January 16, 2018, available at <https://www.irs.gov/newsroom/irs-urges-travelers-requiring-passports-to-pay-their-back-taxes-or-enter-into-payment-agreements-people-owing-51000-or-more-covered>. See also <https://www.irs.gov/businesses/small-businesses-self-employed/revocation-or-denial-of-passport-in-case-of-certain-unpaid-taxes>



similar programs have been introduced, are California and Louisiana. In California, the Delinquent Taxpayer Accountability Act (Assembly Bill 1424) signed in 2011 (effective in 2012) requires to suspend occupational, professional, and drivers' licenses from the top 500 debtors. It also prohibits taxpayers on the lists from entering into contracts to provide goods and services to state agencies. In Louisiana, the Department of Revenue has adopted Suspension and Denial of Renewal of Drivers' Licenses (LAC 61:I.1355) act in 2003 (effective in 2004), which initiated the suspension, revocation, or denial of a taxpayer's driver's license if the taxpayer owes more than \$1,000 in Louisiana individual income tax. The same act requires the Department to initiate the suspension, revocation, or denial of a taxpayer's hunting and fishing licenses when the taxpayer owes more than \$500 in Louisiana individual income tax.

The implementation of a driver's license suspension is a quick and cheap procedure. If a delinquent taxpayer does not respond to the tax department notification to pay taxes, the tax department can notify the department of motor vehicles to proceed with the suspension of driver's license. The actual time needed for the revocation of driver's license to affect the taxpayer is short. The suspension will remain in effect until the department of motor vehicles receives the tax department notification that the taxpayer has satisfied his or her past-due tax liabilities. Therefore, suspension of a driver's license may be an effective instrument for tax debt collection enforcement. The same considerations apply to the denial of new or renewed passports.

### 3 Model of Tax Debt

This section considers a simple dynamic model with risk-averse individuals who differ in their ability and willingness to pay tax debt. The model introduces a collateral tax sanction in addition to a monetary fine as means to enforce tax debt collection. It draws attention to features that distinguish these two instruments.

Assume that there is an infinite number of periods. In period 1, individuals are characterized by their income,  $w$ , which is distributed on  $[w_l, w_h]$  according to  $F(w)$ . In the subsequent periods, all individuals receive identical income,  $I$ . Each individual starts period 1 with debt equal to  $\delta$  (such that  $w_l \leq \delta$ ) and decides whether to pay tax debt in period 1 or to postpone paying until future.

I assume that individuals cannot save or borrow, so that their consumption is equal to their income. The assumption that individuals cannot borrow is important because it enables me to model a situation when some individuals are income constrained in the first period. The assumption that individuals cannot save is imposed for simplicity; in the appendix, I

show that relaxation of this assumption does not substantially affect the analysis.

The preferences are assumed to be identical across individuals and in each period are characterized by a utility function  $u(\cdot)$ , which is an increasing and convex function. Individuals discount their utility in future periods by discount factor  $\beta$ . If a taxpayer chooses to pay her tax debt in period 1, then her lifetime utility is

$$u(w - \delta) + \sum_{t=1}^{\infty} \beta^t u(I) = u(w - \delta) + \frac{\beta}{1 - \beta} u(I). \quad (1)$$

### 3.1 Monetary Fine

If the tax debt is not paid in period 1, the tax authority starts collection actions. Depending on the collection tool that can be applied to a delinquent taxpayer, the duration of time until the tax debt is collected can differ among taxpayers. For example, it is much faster to collect tax debt through wage garnishment, rather than through property seizure. Assume that the probability that the tax debt is collected from a taxpayer in a given period of time,  $p$ , is distributed across the population with distribution  $G(p)$ . To discourage delaying of tax debt payment, the tax authority can impose monetary fine,  $(\pi - 1) > 0$ , that is proportional to tax debt amount. The monetary fine is paid by a delinquent taxpayer only at the time when the tax debt is collected.

When the tax debt is not paid in period 1 and a collection tool is applied, the expected utility of a tax debtor in period 2 is

$$E[u|\pi]_{t=2} = p \left[ u(I - \pi\delta) + \frac{\beta}{1 - \beta} u(I) \right] + (1 - p) [u(I) + \beta E[u|\pi]_{t=2}]. \quad (2)$$

Use equation (2) to solve for  $E[u|\pi]_{t=2}$ , which is

$$E[u|\pi]_{t=2} = \frac{p}{1 - \beta + \beta p} u(I - \pi\delta) + \frac{\frac{\beta p}{1 - \beta} + 1 - p}{1 - \beta + \beta p} u(I). \quad (3)$$

Note that  $\frac{p}{1 - \beta + \beta p} + \frac{\frac{\beta p}{1 - \beta} + 1 - p}{1 - \beta + \beta p} = \frac{1}{1 - \beta}$ . Then, the tax debtor's lifetime utility, when she chooses not to pay her tax debt in period 1, is

$$u(w) + \beta E[u|\pi]_{t=2} = u(w) + qu(I - \pi\delta) + (\gamma - q)u(I), \quad (4)$$

where for simplicity of notations I denote  $q = \frac{\beta p}{1 - \beta + \beta p}$  and  $\gamma = \frac{\beta}{1 - \beta}$ .

As equation (4) suggests,  $q$  can be interpreted as the probability with which the tax debt and the monetary fine are collected. Given that  $p \in [0, 1]$  is distributed across the population with probability  $G(p)$ ,  $q$  is distributed on  $[0, \beta]$  with probability  $H(q) = G\left(\frac{1 - \beta}{\beta} \frac{q}{1 - q}\right)$ .

**Assumption.** Assume also that the monetary fine is bounded from the above, so that  $\pi \leq \frac{I}{\delta}$ . This assumption guarantees that individuals do not have negative income (i.e,  $I - \pi\delta \geq 0$ ).

### 3.2 A Collateral Tax Sanction

In addition to the monetary fine, the tax authority can impose a collateral tax sanction to enforce payment of the tax debt in period 1. In contrast to the monetary fine, the collateral tax sanction affects the tax debtor right away and remains in effect until the tax debt is paid. As a result, the longer the tax debt payment is postponed the longer the collateral tax sanction is applied. An imposition of the collateral tax sanction reduces the utility by  $c$  in each period when it is in effect. The lifetime utility, when the tax debt is not paid in period 1 and both the monetary fine and the collateral tax sanction are imposed, is  $u(w) - c + \beta E[u|\pi, c]$ , where  $E[u|\pi, c]$  is determined by

$$E[u|\pi, c] = p \left[ u(I - \pi\delta) + \frac{\beta}{1 - \beta} u(I) \right] + (1 - p) [u(I) - c + \beta E[u|\pi, c]]. \quad (5)$$

Using equation (5), solve for  $E[u|\pi, c]$  to obtain

$$E[u|\pi, c] = \frac{p}{1 - \beta + \beta p} u(I - \pi\delta) + \frac{\frac{\beta p}{1 - \beta} + 1 - p}{1 - \beta + \beta p} u(I) - \frac{1 - p}{1 - \beta + \beta p} c. \quad (6)$$

The tax debtor's lifetime utility, when she chooses not to pay her tax debt in period 1 and both the monetary fine and the collateral tax sanction are imposed, is

$$u(w) - c + \beta E[u|\pi, c] = u(w) + qu(I - \pi\delta) + (\gamma - q)u(I) - \theta(1 - q)c, \quad (7)$$

where  $\theta = \frac{1}{1 - \beta} > 1$ .

As equation (7) shows, the utility cost of the collateral tax sanction is proportional to  $(1 - q)$ , which reflects that the longer the tax debtor postpones paying her tax debt the longer she is subject to the collateral tax sanction. So, while those who have lower  $q$  are affected less by monetary fine, they are affected more by the collateral tax sanction.

### 3.3 The Tax Debtor's Problem

In this model, a tax debtor makes only one decision: she decides whether to pay tax debt today in the first period or not.<sup>12</sup> By combining equation (1) and equation (7), we can express the tax debtor's problem as

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<sup>12</sup>An extension of the model when individuals can make savings in period 1 is considered in the appendix.

$$\max_{x \in \{0,1\}} u(w - x\delta) + qu(I - (1-x)\pi\delta) + (\gamma - q)u(I) - (1-x)\theta(1-q)c.$$

A tax debtor chooses to pay tax debt ( $x = 1$ ) iff  $u(w - \delta) + \gamma u(I) > u(w) + qu(I - \pi\delta) + (\gamma - q)u(I) - \theta(1 - q)c$ . This condition can be simplified and expressed as

$$u(w) - u(w - \delta) < q[u(I) - u(I - \pi\delta)] + \theta(1 - q)c. \quad (8)$$

For a given  $q$ , let us define  $\tilde{w}(q)$  such that

$$u(\tilde{w}(q)) - u(\tilde{w}(q) - \delta) = q[u(I) - u(I - \pi\delta)] + \theta(1 - q)c. \quad (9)$$

This expression determines the income level,  $\tilde{w}(q)$ , at which the tax debtor is indifferent between paying or not paying the tax debt in the first period. However, some qualifications concerning  $\tilde{w}(q)$  are needed to be made. Because  $u(w - \delta)$  is undefined for  $w < \delta$  the solution of equation (9) may be undefined for some  $q$ . Specifically, because  $\tilde{w}(q)$  is decreasing in  $q$  (as Proposition 1 shows later) it may be that there exist  $q_h < \beta$  such that  $\tilde{w}(q_h) = \delta$ . Then for  $q \in (q_h, \beta]$  the solution of equation will be undefined. In such a case, it is natural to supplement a definition of the threshold income by defining it equal to  $\delta$  for  $q \in (q_h, \beta]$ . In addition to this, we want to ensure that income threshold does not exceed  $w_h$ , which could happen for some small  $q$ ,  $q < q_l$ , where  $q_l$  is defined so that  $w(q_l) = w_h$ . Therefore, let us supplement a definition of the threshold income by defining it equal to  $w_h$  for  $q \in [0, q_l]$ . Given all these subtle aspects, let us define

$$\hat{w}(q) = \max\{\delta, \min\{\tilde{w}(q), w_h\}\}. \quad (10)$$

As Proposition 1 shows, individuals with income above  $\hat{w}(q)$  choose to pay tax debt, and individuals with income below  $\hat{w}(q)$  choose not to pay the tax debt.

**Proposition 1.** *Assume that  $u(\cdot)$  is strictly convex. Let  $\hat{w}(q)$  be defined by (10).*

*i) If the tax debtor's income in period 1 is  $w \in [w_l, \hat{w}(q)]$ , then the tax debtor chooses not to pay tax debt in period 1 ( $x^* = 0$ ). If the tax debtor's income in period 1 is  $w \in (\hat{w}(q), w_h]$ , then the tax debtor chooses to pay tax debt in period 1 ( $x^* = 1$ ).*

*ii) Assume that  $u(I) - u(I - \pi\delta) - \theta c > 0$ <sup>13</sup>, then  $\hat{w}(q)$  decreases with  $q$  (i.e.,  $\frac{\partial \hat{w}(q)}{\partial q} \leq 0$ ). Specifically,  $\frac{\partial \hat{w}(q)}{\partial q} = \frac{\partial \tilde{w}(q)}{\partial q} < 0$  for  $q \in [q_l, q_h]$ , and  $\frac{\partial \hat{w}}{\partial q} = 0$  for*

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<sup>13</sup>Note that condition  $u(I) - u(I - \pi\delta) - \theta c > 0$  is satisfied if  $c = 0$  or  $c$  is not too large.

$q \in [0, q_l]$  and  $q \in (q_h, \beta]$ .

iii) The cut-off income  $\hat{w}(q)$  decreases with  $\pi$  and  $c$ . Specifically,  $\frac{\partial \hat{w}(q)}{\partial \pi} < 0$  and  $\frac{\partial \hat{w}(q)}{\partial c} < 0$  for  $q \in [q_l, q_h]$ , and  $\frac{\partial \hat{w}}{\partial \pi} = \frac{\partial \hat{w}}{\partial c} = 0$  for  $q \in [0, q_l]$  and  $q \in (q_h, \beta]$ .

*Proof.* In the appendix.

Proposition 1 characterizes the behavior of the tax debtor by using the income threshold,  $w(q)$ , which depends on  $q$ . Alternatively, the tax debtor's problem could be characterized by  $q$ -threshold function  $\tilde{q}(w)$ , which depends on  $w$  and is equal to the inverse function of  $\tilde{w}(q)$  (i.e.,  $\tilde{q}(\cdot) = \tilde{w}^{-1}(\cdot)$ ). This alternative description would be equivalent to the used one.

To sum up, there are two reasons why a tax debtor may not pay the tax debt. First, she may be income constrained ( $w < \hat{w}(q)$ ). Second, she may be able to postpone paying tax debt for a long period of time (has a low  $q$ ). This second reason is responsible for the fact that  $\frac{\partial \hat{w}(q)}{\partial q} < 0$ . The lower is  $q$ , the less likely the person will choose to pay the tax debt. An imposition of the collateral tax sanction reduces the motivation for people with low  $q$  to postpone paying the tax debt.

### 3.4 Tax Revenue

When the tax debt is paid in period 1 (i.e.,  $w \in (\hat{w}(q), w_h]$ ), the tax authority receives  $\delta$ . When the tax debt is not paid in period 1 (i.e.,  $w \in [w_l, \hat{w}(q)]$ ), the tax authority collects  $\beta E[TR]$ , which is determined by

$$E[TR] = p\pi\delta + (1 - p)\beta E[TR]. \quad (11)$$

From this equation,  $E[TR] = \frac{p\pi\delta}{1 - \beta + \beta p} = \frac{q}{\beta}\pi\delta$ . Hence, the tax revenue is equal to

$$\begin{aligned} TR &= \int_0^\beta \left[ \int_{w_l}^{\hat{w}(q)} q\pi\delta dF(w) + \int_{\hat{w}(q)}^{w_h} \delta dF(w) \right] dH(q) \\ &= \int_0^\beta [q\pi\delta F(\hat{w}(q)) + \delta(1 - F(\hat{w}(q)))] h(q) dq. \end{aligned} \quad (12)$$

The tax authority cares not only about tax revenue, but also about welfare of taxpayers, which is equal to

$$\begin{aligned} W &= \int_0^\beta \int_{w_l}^{\hat{w}(q)} [u(w) + qu(I - \pi\delta) + (\gamma - q)u(I) - \theta(1 - q)c] dF(w) dH(q) + \\ &\quad + \int_0^\beta \int_{\hat{w}(q)}^{w_h} [u(w - \delta) + \gamma u(I)] dF(w) dH(q). \end{aligned} \quad (13)$$

The tax authority maximizes the social welfare that is a weighted average of the tax revenue and private welfare:

$$\max_{1 \leq \pi \leq \frac{1}{\delta}, 0 \leq c} \alpha TR + (1 - \alpha)W, \quad (14)$$

where  $\alpha \in (0, 1)$ .

As can be seen from (12), both the monetary fine and the collateral tax help to raise the tax revenue by encouraging more people to pay their tax debt in the first period (i.e., by reducing the income threshold,  $\hat{w}(q)$ ). The monetary fine, however, generates additional revenue, because fines are added to the tax debt. As (13) reveals, both the monetary fine and the collateral tax sanction reduces private welfare of those individuals who choose not to pay the tax debt in the first period. But, these two instruments differ in how likely those welfare costs apply to a  $q$ -type individual.

## 4 Optimal policy

The analysis of the tax authority's problem proceeds by considering first the case when individuals are homogeneous in  $q$  and then when individuals are heterogeneous in  $q$ .

### 4.1 Optimal policy when taxpayer are homogeneous in ability to postpone paying tax debt

Assume that all individuals are characterized by the same  $q$ . Then, there is the same income threshold  $\hat{w}$  for everyone. Debtors with income above it choose to pay their tax debts. Debtors with income below it choose to delay paying.

The tax revenue, in this case, reduces to

$$TR = \delta(1 - F(\hat{w})) + q\pi\delta F(\hat{w}). \quad (15)$$

The formula (13), describing the welfare of taxpayers, reduces to

$$W = \int_{w_l}^{\hat{w}} [u(w) + qu(I - \pi\delta) + (\gamma - q)u(I) - \theta(1 - q)c] dF(w) + \int_{\hat{w}}^{w_h} [u(w - \delta) + \gamma u(I)] dF(w). \quad (16)$$

The FOCs for the tax authority's maximization problem (14) w.r.t  $\pi$  and  $c$  are

$$FOC_{\pi} : \alpha q \delta F(\hat{w}) + \alpha \delta (1 - q \pi) f(\hat{w}) \left( -\frac{\partial \hat{w}}{\partial \pi} \right) - (1 - \alpha) q \delta u'(I - \pi \delta) F(\hat{w}) \stackrel{\leq}{\geq} 0, \quad (17)$$

$$FOC_c : \alpha\delta(1 - q\pi)f(\hat{w}) \left( -\frac{\partial\hat{w}}{\partial c} \right) - (1 - \alpha)\theta(1 - q)F(\hat{w}) \leq 0, \quad (18)$$

where (17) holds with equality if  $1 < \pi < \frac{I}{\delta}$ , is negative if  $\pi = 1$  and is positive if  $\pi = \frac{I}{\delta}$ , and (18) holds with equality if  $c > 0$ .

When  $q \in [0, q_l]$  or  $q \in (q_h, \beta]$ , according to Proposition 1  $\frac{\partial\hat{w}}{\partial\pi} = \frac{\partial\hat{w}}{\partial c} = 0$ , which immediately implies that  $FOC_c < 0$ . When  $q \in [q_l, q_h]$  by substituting  $\frac{\partial\hat{w}(q)}{\partial\pi} = -\frac{q\delta u'(I - \pi\delta)}{u'(\hat{w}(q) - \delta) - u'(\hat{w}(q))} < 0$  and  $\frac{\partial\hat{w}(q)}{\partial c} = -\frac{\theta(1 - q)}{u'(\hat{w}(q) - \delta) - u'(\hat{w}(q))} < 0$  into equations (17) and (18), we can simplify those equations and express them as

$$FOC_\pi : \frac{F(\hat{w})}{u'(I - \pi\delta)} + \frac{\delta(1 - q\pi)f(\hat{w})}{u'(\hat{w} - \delta) - u'(\hat{w})} - \frac{(1 - \alpha)}{\alpha}F(\hat{w}) \stackrel{\leq}{\geq} 0, \quad (19)$$

$$FOC_c : \frac{\delta(1 - q\pi)f(\hat{w})}{u'(\hat{w} - \delta) - u'(\hat{w})} - \frac{(1 - \alpha)}{\alpha}F(\hat{w}) \leq 0. \quad (20)$$

As can be seen, the expression for  $FOC_\pi$  differs from  $FOC_c$  only by the first term,  $\frac{F(\hat{w})}{u'(I - \pi\delta)}$ , which arises because the monetary fine generates extra revenue for each dollar of delayed tax debt. Because this term is positive, whenever  $FOC_\pi$  is equal to zero,  $FOC_c$  is negative. Moreover, if  $FOC_c$  was zero,  $FOC_\pi$  would be positive. As a result, at the optimum  $c^* = 0$  whenever  $\pi^* < \frac{I}{\delta}$ . That is, in this case, when it is optimal to use not the highest monetary fine (i.e.,  $\pi^* < \frac{I}{\delta}$ ), the tax authority does not use collateral tax sanction and relies only on the monetary fine. This is true for any  $\alpha \in (0, 1)$ . The reason is the following. The only dimension, in which individuals are different, is the amount of income in the first period. Therefore, the only motivation for not paying tax debt is being income constrained. Hence, to maximize social welfare (i.e., to balance the tax revenue gain against private welfare loss), the tax authority only needs to select a uniform income threshold, which can be done either by using the monetary fine or the collateral tax sanction. While both the monetary fine and the collateral tax sanction impose private welfare costs, the monetary fine, however, is preferable because it generates additional revenue.

The following proposition summarizes the result of this section.

**Proposition 2.** *Assume that all individuals are characterized by the same  $q$  and differ only in their income in period 1. Then, at the optimum  $c^* = 0$  whenever  $\pi^* < \frac{I}{\delta}$ , where the optimal monetary fine,  $\pi^*$ , is the solution of FOC (19).*

## 4.2 Optimal policy when taxpayers are heterogeneous in $q$

Return now to the case when individuals differ in ability to postpone paying tax debt (i.e., heterogeneous in  $q$ ). Recall that in this case debtors may have two motives for not paying the tax debt in period 1. While some debtors may not pay tax debt because they are income constrained, others may do this because they expect to delay in the tax debt collection for a long period of time.

The effect of the monetary fine and the collateral tax sanction depends on debtor's ability to postpone paying the tax debt. The later the tax debt is collected, the later the monetary fine is paid. However, the collateral tax sanction cannot be delayed, it affects the debtor immediately and is applied as long as the tax debt is not paid. Therefore, while a collateral tax sanction does not generate additional revenue (as a monetary fine does), it has an advantage over monetary fine as it allows a stronger punishment for those debtors who can delay paying taxes for longer.

To determine the conditions, when it is optimal to impose the collateral tax sanction, consider the tax authority's maximization problem (14). The FOCs for this problem w.r.t  $\pi$  and  $c$  are

$$FOC_{\pi} : \alpha \int_0^{\beta} \left[ qF(\hat{w}(q)) + (1 - q\pi)f(\hat{w}(q)) \left( -\frac{\partial \hat{w}(q)}{\partial \pi} \right) \right] dH(q) - (1 - \alpha) \int_0^{\beta} qu'(I - \pi\delta)F(\hat{w}(q))dH(q) \stackrel{\leq}{\geq} 0, \quad (21)$$

$$FOC_c : \alpha \int_0^{\beta} \delta(1 - q\pi)f(\hat{w}(q)) \left( -\frac{\partial \hat{w}(q)}{\partial c} \right) dH(q) - (1 - \alpha) \int_0^{\beta} \theta(1 - q)F(\hat{w}(q))dH(q) \leq 0, \quad (22)$$

where (21) holds with equality if  $1 < \pi < \frac{I}{\delta}$ , is negative if  $\pi = 1$  and is positive if  $\pi = \frac{I}{\delta}$ , and (22) holds with equality if  $c > 0$ . Let  $(\pi^*, c^*)$  represent the solution to (21) and (22).

By using  $\frac{\partial \hat{w}(q)}{\partial \pi} = -\frac{q\delta u'(I - \pi\delta)}{u'(\hat{w}(q) - \delta) - u'(\hat{w}(q))} < 0$  and  $\frac{\partial \hat{w}(q)}{\partial c} = -\frac{\theta(1 - q)}{u'(\hat{w}(q) - \delta) - u'(\hat{w}(q))} < 0$  for  $q \in [q_l, q_h]$  and  $\frac{\partial \hat{w}}{\partial \pi} = \frac{\partial \hat{w}}{\partial c} = 0$  for  $q \in [0, q_l]$  and  $q \in (q_h, \beta]$ , we can simplify the above expressions:

$$FOC_{\pi} : \int_0^{\beta} q\Psi(q, \pi)dH(q) + \frac{1}{u'(I - \pi\delta)} \int_0^{\beta} qF(\hat{w}(q))dH(q) \stackrel{\leq}{\geq} 0, \quad (23)$$

$$FOC_c : \int_0^{\beta} (1 - q)\Psi(q, \pi)dH(q) \leq 0, \quad (24)$$

where  $\Psi(q, \pi) = \frac{\delta(1 - q\pi)f(\hat{w}(q))}{u'(\hat{w}(q) - \delta) - u'(\hat{w}(q))}\chi(q) - \frac{(1 - \alpha)}{\alpha}F(\hat{w}(q))$  and  $\chi(q) = \begin{cases} 1 & \text{if } q \in [q_l, q_h] \\ 0 & \text{if } q \in [0, q_l] \text{ and } q \in (q_h, \beta] \end{cases}$ .

Let  $\tilde{\pi}$  denote the level of the monetary fine that solves equation (23) which is held as equality when  $c$  is constrained to be zero. Then the necessary and sufficient condition for



$c^* > 0$  is that the left-hand side of (24) evaluated at  $(\tilde{\pi}, c = 0)$  is positive. The following proposition summarizes this result.

**Proposition 3.** *Assume that individuals are heterogeneous in  $q$  and as well as in their income in period 1. Then, the collateral tax sanction is used in the optimum if  $\int_0^\beta (1 - q)\Psi(q, \tilde{\pi})dH(q) > 0$ , where  $\Psi(q, \pi) = \frac{\delta(1-q\pi)f(\hat{w}(q))}{u'(\hat{w}(q)-\delta)-u'(\hat{w}(q))}\chi(q) - \frac{(1-\alpha)}{\alpha}F(\hat{w}(q))$  and  $\tilde{\pi}$  the level of monetary fine that solves (23) when  $c$  is constrained to be zero.*

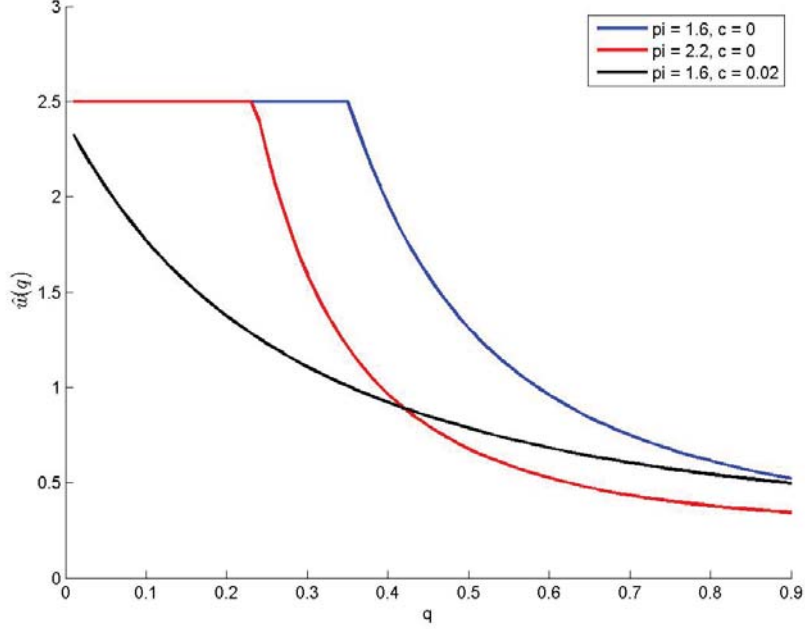
Let us discuss the meaning of  $\Psi(q, \pi) = \frac{\delta(1-q\pi)f(\hat{w}(q))}{u'(\hat{w}(q)-\delta)-u'(\hat{w}(q))}\chi(q) - \frac{(1-\alpha)}{\alpha}F(\hat{w}(q))$ . In what follows, I show that it measures the difference between the tax revenue gain and the private welfare loss caused by a unit reduction in the expected utility of  $q$ -type taxpayers who did not pay tax debt in the first period. If the expected utility of  $q$ -type taxpayers who did not pay tax debt in the first period is exogeneously decreased by 1 then the income threshold is decreases so that  $-\Delta\hat{w}(q) = \frac{1}{u'(\hat{w}(q)-\delta)-u'(\hat{w}(q))} \cdot \chi(q)$ . The numerator of the first term in  $\Psi(q, \pi)$ ,  $\delta(1 - q\pi)f(\hat{w}(q))$ , is equal to the change in the tax revenue received from  $q$ -type taxpayers given that the income threshold,  $\hat{w}(q)$ , decreases by 1 (i.e.,  $\frac{\Delta TR(q)}{\Delta\hat{w}(q)}$ ). Thus, the first term  $(\frac{\delta(1-q\pi)f(\hat{w}(q))}{u'(\hat{w}(q)-\delta)-u'(\hat{w}(q))} \cdot \chi(q))$  shows the tax revenue gain when the expected utility of  $q$ -type taxpayers who did not pay tax debt in the first period is decreased by 1. The second term,  $\frac{(1-\alpha)}{\alpha}F(\hat{w}(q))$ , is equal to the mass of the  $q$ -type debtors, who did not to pay tax debt in the first period and are exposed to a unit reduction in the expected utility, multiplied by relative weight of private welfare,  $\frac{1-\alpha}{\alpha}$ . This term represents the change in the private welfare. Thus,  $\Psi(q, \pi)$  is the difference between the tax revenue gain and the private welfare loss caused by a unit reduction in the expected utility of  $q$ -type taxpayers who did not pay tax debt in the first period. In view of this,  $\Psi(q, \pi)$  can be called the marginal tax revenue net of private welfare loss from the imposition of a sanction causing a unit reduction in the expected utility of  $q$ -type taxpayers.

Note that because the monetary fine produces the expected utility cost proportional to  $q$ , FOC (23) includes  $\Psi(q, \pi)$  multiplied by  $q$ . In its turn, the expected utility cost of the collateral tax sanction is proportional to  $(1 - q)$ , which results in FOC (24) that includes  $\Psi(q, \pi)$  multiplied by  $(1 - q)$ .

The reason why an imposition of a collateral tax sanction may become optimal when tax debtors are heterogeneous in ability to postpone paying tax debt in addition to being heterogeneous in income is that the collateral tax sanction helps to differentially affect different types of tax debtors. That is, the tax debtors with low  $q$  and the tax debtors high  $q$ . This is illustrated by Figure 1<sup>14</sup>, which shows how the graph of the income threshold function,  $\hat{w}(q)$ ,

<sup>14</sup>The graph is constructed for the parameters values used in the numerical calculations discussed later.

Figure 1: The income threshold function and its shifts as a result of an introduction of the monetary fine and of the collateral tax sanction.



shifts as a result of an introduction of the monetary fine and of the collateral tax sanction. An introduction of the monetary fine shifts the graph down (it is illustrated by the red line), with the largest shift occurring for high values of  $q$ . On the other hand, an introduction of the collateral tax sanction also shifts the graph down (it is illustrated by the black line), but with the largest shift occurring for low values of  $q$ . As can be seen from the graph, because the collateral tax sanction affects mostly taxpayers with low  $q$ , the magnitude of the highest income  $w_h$  is critical for whether the collateral tax sanction is effective or not. When  $w_h$  is low, the value of  $q_l$  is relatively large and the

Note also that for  $\int_0^\beta (1-q)\Psi(q, \pi)dH(q)$  to be positive it is sufficient that  $\int_0^\beta \Psi(q, \pi)dH(q)$  is positive. Indeed, if that  $\tilde{\pi}$  is less than  $\frac{I}{\delta}$  then it follows from (23) that  $\int_0^\beta q\Psi(q, \pi)dH(q)$  is negative. Because  $\int_0^\beta (1-q)\Psi(q, \pi)dH(q) = \int_0^\beta \Psi(q, \pi)dH(q) - \int_0^\beta q\Psi(q, \pi)dH(q)$  and  $\int_0^\beta q\Psi(q, \pi)dH(q)$  is negative, we have  $\int_0^\beta \Psi(q, \pi)dH(q) > 0$  is sufficient for  $\int_0^\beta (1-q)\Psi(q, \pi)dH(q) > 0$ . The following corollary summarizes this result.

**Corollary 1.** *Assume that individuals are heterogeneous in  $q$  and as well as in their income in period 1. Then, the collateral tax sanction is used in the optimum if  $\int_0^\beta \Psi(q, \tilde{\pi})dH(q) > 0$ , where  $\Psi(q, \pi) = \frac{\delta(1-q\pi)f(\hat{w}(q))}{u'(\hat{w}(q)-\delta)-u'(\hat{w}(q))} - \frac{(1-\alpha)}{\alpha}F(\hat{w}(q))$  and  $\tilde{\pi}$  the level of monetary fine that solves (23) when  $c$  is constrained to be zero.*

This corollary shows that it is optimal to introduce the collateral tax sanction when the average marginal tax revenue net of private welfare loss is positive.

As the following proposition shows, under additional assumptions we can make some inferences about the sign of  $\int_0^\beta \Psi(q, \pi) dH(q)$ . Specifically, when  $w_h$  converges to the size of debt (i.e.,  $\delta$ ),  $\int_0^\beta \Psi(q, \pi) dH(q)$  is negative, while when  $w_h$  converges to infinity,  $\int_0^\beta \Psi(q, \pi) dH(q)$  is positive.

**Proposition 4.** *Assume that individuals are heterogeneous in  $q$  and as well as in their income in period 1. Assume that  $u(w) = \frac{w^\sigma - 1}{\sigma}$  where  $\sigma < 1$  and assume that  $w$  is uniformly distributed on  $[0, w_h]$  and  $q$  is uniformly distributed on  $[0, \beta]$ . Then,  $\lim_{w_h \rightarrow \delta} \int_0^\beta \Psi(q, \pi) dH(q) = -\frac{1-\alpha}{\alpha} \frac{\delta^\sigma}{((I)^\sigma - (I-\pi\delta)^\sigma)\beta} < 0$  and  $\lim_{w_h \rightarrow +\infty} \int_0^\beta \Psi(q, \pi) dH(q) = \frac{\sigma\delta}{((I)^\sigma - (I-\pi\delta)^\sigma)\beta} > 0$ .*

*Proof.* In the appendix.

This proposition implies that for some high values of  $w_h$  the value of  $\int_0^\beta \Psi(q, \pi) dH(q)$  is positive, which is according to Corollary 1 is sufficient for the  $c^*$  to be positive at the optimum. Thus, when  $w_h$  is high and therefore the population of tax debtors includes some high-income individuals, it becomes optimal to impose the collateral tax sanction.

#### 4.2.1 Numerical Calculation

To illustrate the above result, I run a numerical calculation for the following case. I assume that  $u(c) = \frac{c^{0.5} - 1}{0.5}$  and that  $w$  is uniformly distributed on  $[0, w_h]$  and  $q$  is uniformly distributed on  $[0, \beta]$ . I assume also that  $\beta = 0.8$ ,  $I = 1$ . Table 1 provides results for  $\alpha = 0.4$ , Table 2 for  $\alpha = 0.5$ , and Table 3 for  $\alpha = 0.6$ . The rows of the tables correspond to different values of tax debt,  $\delta$ . The columns of the tables correspond to different values of upper bound of income,  $w_h$ . The pair (1.67;<0) within a table indicates that  $\tilde{\pi} = 1.67$ ,  $FOC_c < 0$ .

As Table 1 shows, when  $w_h \leq 2.1$  the  $FOC_c$  is negative, when  $w_h \geq 2.2$  the  $FOC_c$  is positive. This means that it becomes optimal to impose a collateral tax sanction ( $c^* > 0$ ) once  $w_h$  is greater than some value that lies between 2.1 and 2.2. Note that small value of  $w_h$  (relative to  $I = 1$ ) implies that in period 1 all taxpayers are relatively poor. So, when initially all taxpayers are relatively income constrained, it is non-optimal to use the collateral tax sanction. When the spread of income in period 1 is sufficiently large, it becomes optimal to impose the collateral tax sanction.

As can be seen from Table 2 and Table 3, similar pattern takes place for  $\alpha = 0.5$  and  $\alpha = 0.6$ . When  $\alpha = 0.5$ , the value of  $w_h$ , at which it becomes optimal to use the collateral

Table 1: Numerical Example for  $\alpha = 0.4$

	$w_h = 1.8$	$w_h = 2.0$	$w_h = 2.1$	$w_h = 2.2$	$w_h = 2.3$	$w_h = 2.5$
$\delta = 0.1$	(1.67; < 0)	(1.78; < 0)	(1.84; < 0)	(1.9; > 0)	(.96; > 0)	(2.07; > 0)
$\delta = 0.3$	(1.36; < 0)	(1.44; < 0)	(1.48; < 0)	(1.52; > 0)	(1.57; > 0)	(1.64; > 0)
$\delta = 0.5$	(1.12; < 0)	(1.18; < 0)	(1.22; < 0)	(1.24; > 0)	(1.27; > 0)	(1.33; > 0)

where pair (1.67;<0) indicates that  $\tilde{\pi} = 1.67$ ,  $FOC_c < 0$ .

Table 2: Numerical Example for  $\alpha = 0.5$

	$w_h = 1.2$	$w_h = 1.4$	$w_h = 1.5$	$w_h = 1.6$	$w_h = 1.8$	$w_h = 2.0$
$\delta = 0.1$	(2.2; < 0)	(2.4; < 0)	(2.5; < 0)	(2.65; > 0)	(2.85; > 0)	(3.12; > 0)
$\delta = 0.3$	(1.71; < 0)	(1.44; < 0)	(1.9; > 0)	(1.95; > 0)	(2.1; > 0)	(2.22; > 0)
$\delta = 0.5$	(1.32; < 0)	(1.18; < 0)	(1.42; < 0)	(1.5; > 0)	(1.58; > 0)	(1.64; > 0)

where pair (2.2;<0) indicates that  $\tilde{\pi} = 2.2$ ,  $FOC_c < 0$ .

Table 3: Numerical Example for  $\alpha = 0.6$

	$w_h = 0.9$	$w_h = 1.1$	$w_h = 1.2$	$w_h = 1.3$	$w_h = 1.4$	$w_h = 1.6$
$\delta = 0.1$	(2.94; < 0)	(3.35; < 0)	(3.56; < 0)	(3.78; > 0)	(4.0; > 0)	(4.5; > 0)
$\delta = 0.3$	(2.11; < 0)	(2.31; < 0)	(2.4; > 0)	(2.5; > 0)	(2.57; > 0)	(2.71; > 0)
$\delta = 0.5$	(1.52; < 0)	(1.64; < 0)	(1.69; > 0)	(1.74; > 0)	(1.77; > 0)	(1.84; > 0)

where pair (2.94;<0) indicates that  $\tilde{\pi} = 2.94$ ,  $FOC_c < 0$ .

tax sanction, lies between 1.4 and 1.6. When  $\alpha = 0.6$ , the threshold value of  $w_h$  lies between 1.1 and 1.3.

Comparison of Tables 1, 2 and 3 allows us to make two additional observations. First, the threshold value of  $w_h$  seems to depend on the size of the debt,  $\delta$ , non-monotonically. Some further investigation of this behavior is required. Second, as  $\alpha$  rises, that is as the tax revenue receives a higher weight in social welfare, the threshold value of  $w_h$ , at which it becomes optimal to use the collateral tax sanction, declines. This indicates that when the tax authority cares less about private welfare, it is willing to impose the collateral tax sanction at a lower spread of income in period 1.

## 5 Conclusion

In the United States, the size of unpaid tax debt is substantial: more than half of identified tax debts are never collected. Hence, it is critical to have the strategy to enforce tax debt collection that is effective and does not harm debtor's welfare. However, the problem of tax debt enforcement has not been well studied in the existing literature. This paper explores under which conditions the optimal strategy to enforce tax debt collection involves using collateral tax sanctions (i.e., suspension of a driver's license or denial of a passport).

The reasons for why people do not pay their tax debt could be different. It is possible that some people just cannot afford to pay their tax debts because they are income constrained. Other taxpayers do not pay their tax debt in the hope to escape from their liabilities by, for example, prolonging court hearings. This diversity in motivations for not paying the tax debt makes the enforcement of tax debt collection a complex task.

To discourage tax debt, the tax authority imposes monetary fines (i.e., an above-market interest rate on the debt amount). Additionally, to collect tax debt the tax authority uses such collection tools as wage garnishment, bank levy, or asset seizure. Depending on which collection tool is applicable, the collection process may significantly vary in its length. As a result, it may take a long time until the tax debt and monetary fines are collected.

Other enforcement instruments that some state tax authorities have recently used are the denial of a passport and the suspension of driver's license, which are examples of a collateral tax sanction. In contrast to the monetary fine, the collateral tax sanction can be quickly applied and starts to influence the tax debtor immediately. Moreover, it remains in effect until the tax debt is paid and, therefore, affects stronger those debtors who postpone paying tax debt for a longer period of time.

In this paper, I develop a simple dynamic model describing tax debtors' behavior when they are heterogeneous in income and ability to escape tax debt payment. I consider the tax

authority that maximizes the weighted sum of the revenue from collecting tax debt and tax debtor's welfare. I characterize the optimal strategy of the tax authority that chooses the size of the monetary fine and the collateral tax sanction.

I show that when debtors are homogeneous in their ability to escape tax debt payment, it is optimal to use only monetary fine. However, when debtors differ in their ability to escape tax debt payment, it may be optimal to use the collateral tax sanction in addition to the monetary fine. Specifically, in the case when utility has constant relative risk aversion and the distribution of income and the distribution of ability to escape tax debt are uniform, it is optimal to use the collateral tax sanction if the upper bound of income distribution is sufficiently large, that is, if the population of taxpayers includes not only income constrained individuals, but also some high-income individuals.

It is important to note that there are, however, other properties of collateral tax sanctions that have not been considered in this paper. Some of these other properties are discussed by Blank (2014) and Kuchumova (2018). When a policy implementing a collateral sanction is designed, it is critical to take into consideration all features of the collateral tax sanctions.

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## Appendix

### Model Extension with Saving Decisions

Consider a model when individuals in addition to deciding whether to pay or not to pay their tax debt can also choose how much to save in the first period. Then, the tax debtor’s problem is

$$\max_{x \in \{0,1\}, s \geq 0} u(w - x\delta - s) + qu(I - (1 - x)\pi\delta + s) + (\gamma - q)u(I + s). \quad (25)$$

Let  $s_0$  be the optimal savings when the debtor decides not to pay tax debt ( $x = 0$ ) and  $s_1$  be the optimal savings when the debtor decides to pay tax debt ( $x = 1$ ). The FOCs for  $s_0$  and  $s_1$  are

$$u'(w - s_0) - qu'(I - \pi\delta + s_0) - (\gamma - q)u'(I + s_0) \geq 0, \quad (26)$$

$$u'(w - \delta - s_1) - \gamma u'(I + s_1) \geq 0, \quad (27)$$

where (26) holds with equality if  $s_0 > 0$  and (27) holds with equality if  $s_1 > 0$ .

Note that both  $s_0(w)$  and  $s_1(w)$  are increasing with  $w$  because  $u(\cdot)$  is concave.

Let us define  $\hat{w}$  such that the tax debtor is indifferent between paying and not paying tax debt in period 1, that is

$$u(\hat{w} - \hat{s}_0) + qu(I - \pi\delta - \hat{s}_0) + (\gamma - q)u(I + \hat{s}_0) = u(\hat{w} - \delta - \hat{s}_1) + \gamma u(I + \hat{s}_1), \quad (28)$$

where  $\hat{s}_0 = s_0(\hat{w})$  and  $\hat{s}_1 = s_1(\hat{w})$ .

Note that if individuals would be able to borrow (i.e.,  $s_i$  could be negative) it would be always better to pay tax debt in the first period and  $\hat{w}$  would be undetermined.

To insure that for  $w \geq \hat{w}$  we have  $x^* = 1$  and for  $w < \hat{w}$  we have  $x^* = 0$ , we need that the right-hand side of (28) ( $RHS = u(w - \delta - \hat{s}_1) + \gamma u(I + \hat{s}_1)$ ) rises faster with  $w$  than the

left-hand side of (28) ( $LHS = u(\hat{w} - \hat{s}_0) + qu(I - \pi\delta - \hat{s}_0) + (\gamma - q)u(I + \hat{s}_0)$ ). That is, it is necessary that  $\frac{\partial RHS}{\partial w}\Big|_{w=\hat{w}} > \frac{\partial LHS}{\partial w}\Big|_{w=\hat{w}}$ , where

$$\frac{\partial RHS}{\partial w}\Big|_{w=\hat{w}} = u'(\hat{w} - \delta - \hat{s}_1)\left(1 - \frac{\partial s_1(\hat{w})}{\partial w}\right) - \gamma u'(I + \hat{s}_1)\frac{\partial s_1(\hat{w})}{\partial w} = u'(\hat{w} - \delta - \hat{s}_1), \quad (29)$$

$$\frac{\partial LHS}{\partial w}\Big|_{w=\hat{w}} = u'(\hat{w} - \hat{s}_0)\left(1 - \frac{\partial s_0(\hat{w})}{\partial w}\right) - (qu'(I - \pi\delta + \hat{s}_0) + (\gamma - q)u'(I + \hat{s}_0))\frac{\partial s_1(\hat{w})}{\partial w} = u'(\hat{w} - \hat{s}_0), \quad (30)$$

where the final expression for (29) is achieved because of (27) for  $\hat{s}_1 > 0$  and because of  $\frac{\partial s_1(\hat{w})}{\partial w} = 0$  for  $\hat{s}_1 = 0$  and the final expression for (30) is achieved because of (26) for  $\hat{s}_0 > 0$  and because of  $\frac{\partial s_0(\hat{w})}{\partial w} = 0$  for  $\hat{s}_0 = 0$ .

The condition  $u'(\hat{w} - \delta - \hat{s}_1) > u'(\hat{w} - \hat{s}_0)$  is true if  $\hat{s}_0 < \delta + \hat{s}_1$ . In its turn,  $\hat{s}_0 < \delta + \hat{s}_1$  is true because  $\hat{s}_0$  should be less than  $\delta$  when  $\frac{q\pi}{\gamma} \geq 1$ . If we assume the opposite ( $\hat{s}_0 \geq \delta$ ) then the debtor would be better off by paying tax debt and saving  $\tilde{s} = \hat{s}_0 - \delta \geq 0$ , because  $\frac{q}{\gamma}u(I - \pi\delta + \hat{s}_0) + \frac{\gamma - q}{\gamma}u(I + \hat{s}_0) < u(I + \hat{s}_0 - \delta) = u(I + \tilde{s})$  by convexity of  $u(\cdot)$  when  $\frac{q\pi}{\gamma} \geq 1$ .

Thus, the behavior of a tax debtor in this model with savings is described by  $x^* = 1$  for  $w \geq \hat{w}$  and  $x^* = 0$   $w < \hat{w}$  and resembles the behavior of a tax debtor in the main model without saving.

### ***Proof of Proposition 1***

i) For  $q \in [q_l, q_h]$ , income  $\tilde{w}(q)$  is well defined and  $\hat{w}(q) = \tilde{w}(q)$ . Because  $u(\cdot)$  is strictly convex, the left-hand side of equation (8) is decreasing in  $w$ . The right-hand side of equation (8) is constant (w.r.t  $w$ ). Therefore, given the definition of  $\tilde{w}(q)$ , when  $w \in (\hat{w}(q), w_h]$ , condition (8) is satisfied and hence  $x = 1$ . When  $w \in [w_l, \hat{w}(q)]$ , the condition (8) is not satisfied and hence  $x = 0$ . For  $q \in [0, q_l)$ ,  $\tilde{w}(q) > w_h$  implying that all individuals ( $w \in [w_l, w_h]$ ) will prefer not to pay tax debt in period 1. Because in this case  $\hat{w}(q) = w_h$ , this is indeed what part (i) of the proposition states. For  $q \in (q_h, \beta]$ , the solution of (8) is not well defined. In this region, it is strictly better to pay the tax debt in the first period unless the taxpayer is income constrained such that  $w < \delta$ . Given that in this case  $\hat{w}(q) = \delta$ , we indeed have that the tax debtor chooses not to pay tax debt in period 1 when  $w \in [w_l, \delta]$ , and he tax debtor chooses to pay tax debt in period 1 when  $w \in [w_l, w_h]$ .

ii) When  $c = 0$ ,  $\frac{\partial \hat{w}(q)}{\partial q} = -\frac{u(I) - u(I - \pi\delta)}{u'(\hat{w}(q) - \delta) - u'(\hat{w}(q))} < 0$ , because  $u(\cdot)$  is convex. When  $c > 0$ ,  $\frac{\partial \hat{w}(q)}{\partial q} = -\frac{[u(I) - u(I - \pi\delta)] - \theta c}{u'(\hat{w}(q) - \delta) - u'(\hat{w}(q))}$ , which is negative if  $u(I) - u(I - \pi\delta) - \theta c > 0$ .

iii) For  $q \in [q_l, q_h]$ , we have  $\frac{\partial \hat{w}(q)}{\partial \pi} = \frac{\partial \tilde{w}(q)}{\partial \pi} = -\frac{q\delta u'(I - \pi\delta)}{u'(\hat{w}(q) - \delta) - u'(\hat{w}(q))} < 0$ , and  $\frac{\partial \hat{w}(q)}{\partial c} = \frac{\partial \tilde{w}(q)}{\partial c} = -\frac{\theta(1 - q)}{u'(\hat{w}(q) - \delta) - u'(\hat{w}(q))} < 0$ . For  $q \in [0, q_l)$  and  $q \in (q_h, \beta]$ ,  $\hat{w}(q)$  does not depend on  $\pi$  and  $c$ ,



hence  $\frac{\partial \hat{w}}{\partial \pi} = \frac{\partial \hat{w}}{\partial c} = 0$ .

**Q.E.D.**

**Proof of Proposition 4**

By definition

$$\Psi(q, \tilde{\pi}) = \frac{\delta(1 - q\tilde{\pi})f(\hat{w}(q))}{u'(\hat{w}(q) - \delta) - u'(\hat{w}(q))} \chi(q) - \frac{(1 - \alpha)}{\alpha} F(\hat{w}(q)),$$

where  $\chi(q) = \begin{cases} 1 & \text{if } q \in [q_l, q_h] \\ 0 & \text{if } q \in [0, q_l] \text{ and } q \in (q_h, \beta] \end{cases}$ .

Note that for  $q \in [0, q_l)$ , we have  $\Psi(q, \tilde{\pi}) = -\frac{(1-\alpha)}{\alpha} F(w_h) = -\frac{(1-\alpha)}{\alpha}$  and for  $q \in (q_h, \beta]$ , we have  $\Psi(q, \tilde{\pi}) = -\frac{(1-\alpha)}{\alpha} F(\delta)$ . When  $q \in [q_l, q_h]$ ,

$$\Psi(q, \tilde{\pi}) = \frac{\delta(1 - q\tilde{\pi})f(\hat{w}(q))}{u'(\hat{w}(q) - \delta) - u'(\hat{w}(q))} - \frac{(1 - \alpha)}{\alpha} F(\hat{w}(q)).$$

Note that if  $q \geq \frac{1}{\tilde{\pi}}$  then expression for  $\Psi(q, \tilde{\pi})$  is negative.

Under the assumptions that  $u(w) = \frac{w^{\sigma-1}}{\sigma}$  and  $w$  is uniformly distributed on  $[0, w_h]$  we have that  $u'(w) = w^{\sigma-1}$ ,  $f(w) = \frac{1}{w_h}$ ,  $F(w) = \frac{w}{w_h}$  and thus for  $q \in [q_l, q_h]$ ,

$$\Psi(q, \tilde{\pi}) = \frac{\delta(1 - q\tilde{\pi})}{[(\hat{w}(q) - \delta)^{\sigma-1} - (\hat{w}(q))^{\sigma-1}] \cdot w_h} - \frac{(1 - \alpha)}{\alpha} \frac{\hat{w}(q)}{w_h}. \quad (31)$$

For clarification purposes, let us denote the expression above by  $\varphi(q)$ , that is,

$$\varphi(q) \triangleq \frac{\delta(1 - q\tilde{\pi})}{[(\hat{w}(q) - \delta)^{\sigma-1} - (\hat{w}(q))^{\sigma-1}] \cdot w_h} - \frac{(1 - \alpha)}{\alpha} \frac{\hat{w}(q)}{w_h},$$

where  $q \in [0, \beta]$

Note that the first term  $(\frac{\delta(1-q\tilde{\pi})}{[(\hat{w}(q)-\delta)^{\sigma-1}-(\hat{w}(q))^{\sigma-1}]\cdot w_h})$  is decreasing in  $q$  and therefore its maximum is achieved when  $q$  is minimal. The denominator of this term could be equal to zero if  $\hat{w}(q)$  is equal to infinity.

Now consider the integral  $\int_0^\beta \Psi(q, \tilde{\pi}) dH(q)$ . It is equal to

$$\int_0^\beta \Psi(q, \tilde{\pi}) dH(q) = \int_0^{q_l} \left( -\frac{(1-\alpha)}{\alpha\beta} \right) dq + \int_{q_l}^{q_h} \varphi(q) \frac{1}{\beta} dq + \int_{q_h}^\beta \left( -\frac{(1-\alpha)}{\alpha\beta} \frac{\delta}{w_h} \right) dq. \quad (32)$$

Let us explore the convergence of this integral when  $w_h$  converges to infinity. Given that  $w_h = \hat{w}(q_l)$ , when  $w_h$  converges to infinity,  $q_l$  converges to zero. When  $q_l$  converges to zero

the first term of the converges to zero. The third term also converges to zero. We only left to explore the convergence of the second term,  $\lim_{q_l \rightarrow 0} \int_{q_l}^{q_h} \varphi(q) \frac{1}{\beta} dq$ . This integral has a singular point at  $q_l = 0$  and therefore deserves a special attention there. Because of this, lets consider  $q_l$  that lies in a neighborhood of zero and break this integral into two parts:

$$\int_{q_l}^{q_h} \varphi(q) \frac{1}{\beta} dq = \int_{q_l}^{q_0} \varphi(q) \frac{1}{\beta} dq + \int_{q_0}^{q_h} \varphi(q) \frac{1}{\beta} dq, \quad (33)$$

where  $q_0 > 0$  also lies in a neighborhood of zero and  $q_l < q_0 < q_h$ .

To estimate the first integral, let us use the Taylor approximation formulas. When  $q$  lies in a neighborhood of zero we have

$$\hat{w}(q)|_{\pi=\tilde{\pi}, c=0} \sim \left( \frac{\Delta}{q} \right)^{\frac{1}{1-\sigma}},$$

where  $\Delta = \frac{\sigma\delta}{(I)^\sigma - (I-\pi\delta)^\sigma}$ , and thus

$$\varphi(q) \sim \frac{(1-q\pi)}{(1-\sigma)w_h} \left( \frac{\Delta}{q} \right)^{\frac{2-\sigma}{1-\sigma}} - \frac{(1-\alpha)}{\alpha w_h} \left( \frac{\Delta}{q} \right)^{\frac{1}{1-\sigma}}.$$

Then,

$$\begin{aligned} \int_{q_l}^{q_0} \varphi(q) \frac{1}{\beta} dq &= \frac{\Delta^{\frac{1}{1-\sigma}}}{w_h} \int_{q_l}^{q_0} \left[ \frac{\Delta}{1-\sigma} \left( \frac{1}{q} \right)^{\frac{2-\sigma}{1-\sigma}} - \left( \frac{\Delta\pi}{1-\sigma} + \frac{1-\alpha}{\alpha} \right) \left( \frac{1}{q} \right)^{\frac{1}{1-\sigma}} \right] dq = \\ &= \frac{\Delta^{\frac{1}{1-\sigma}}}{\beta w_h} \left[ \Delta \left( \left( \frac{1}{q_l} \right)^{\frac{1}{1-\sigma}} - \left( \frac{1}{q_0} \right)^{\frac{1}{1-\sigma}} \right) - \left( \frac{\Delta\pi}{\sigma} + \frac{(1-\alpha)(1-\sigma)}{\alpha\sigma} \right) \left( \left( \frac{1}{q_l} \right)^{\frac{\sigma}{1-\sigma}} - \left( \frac{1}{q_0} \right)^{\frac{\sigma}{1-\sigma}} \right) \right]. \end{aligned}$$

When  $w_h \rightarrow +\infty$ , then  $q_l \simeq \frac{\Delta}{(w_h)^{1-\sigma}} \rightarrow 0$ , and

$$\lim_{w_h \rightarrow +\infty} \int_{q_l}^{q_0} \varphi(q) \frac{1}{\beta} dq = \lim_{q_l \rightarrow 0} \frac{1}{\beta} \left[ \Delta \left( 1 - \left( \frac{q_l}{q_0} \right)^{\frac{1}{1-\sigma}} \right) - \left( \frac{\Delta\pi}{\sigma} + \frac{(1-\alpha)(1-\sigma)}{\alpha\sigma} \right) \left( q_l - q_l \left( \frac{q_l}{q_0} \right)^{\frac{\sigma}{1-\sigma}} \right) \right] = \frac{\Delta}{\beta}.$$

Let us now consider the second part of the integral in (33),

$$\int_{q_0}^{q_h} \varphi(q) \frac{1}{\beta} dq = \frac{1}{w_h \beta} \left( \int_{q_0}^{q_h} \frac{\delta(1-q\tilde{\pi})}{[(\hat{w}(q) - \delta)^{\sigma-1} - (\hat{w}(q))^{\sigma-1}]} - \frac{(1-\alpha)}{\alpha} \hat{w}(q) \right) dq.$$

Because the integrated function in the above expression is bounded on  $[q_0, q_h]$ , its integral is finite. Hence, when  $w_h \rightarrow +\infty$  the integral  $\int_{q_0}^{q_h} \varphi(q) \frac{1}{\beta} dq$  converges to zero.

Putting now all pieces together, we can conclude that when  $w_h \rightarrow +\infty$  the integral  $\int_0^\beta \Psi(q, \tilde{\pi}) dH(q)$  converges to  $\frac{\Delta}{\beta}$ .

On the other hand, when  $w_h$  converges to  $\delta$ ,  $q_l = (\hat{w})^{-1}(w_h)$  converges to  $q_h = (\hat{w})^{-1}(\delta)$ . And then when  $q_l$  converges to  $q_h$  the second term of equation (32) converges to zero, the first and the third terms are both negative. Hence, the integral  $\int_0^\beta \Psi(q, \tilde{\pi}) dH(q)$  converges to a negative value:

$$\begin{aligned} \lim_{w_h \rightarrow \delta} \int_0^\beta \Psi(q, \tilde{\pi}) dH(q) &= \lim_{w_h \rightarrow \delta} \left[ \int_0^{q_h} \left( -\frac{(1-\alpha)}{\alpha\beta} \right) dq + \int_{q_h}^\beta \left( -\frac{(1-\alpha)}{\alpha\beta} \frac{\delta}{w_h} \right) dq \right] = \\ &= \int_0^\beta \left( -\frac{(1-\alpha)}{\alpha\beta} \right) dq = -\frac{(1-\alpha)}{\alpha} \frac{q_h}{\beta} = -\frac{(1-\alpha)}{\alpha} \frac{\delta^\sigma}{((I)^\sigma - (I - \pi\delta)^\sigma)\beta}. \end{aligned}$$

***Q.E.D.***

### ***Numerical Calculation***

To perform numerical calculations, it is important to evaluate the uniform upper bound on the value of the collateral tax sanction,  $c$ . For this purpose, we will require that the expected utility of the lowest income tax debtor, when she did not pay the debt in the first period and both the monetary fine and the collateral tax sanctions are applied, is not less than  $\frac{u(w_l)}{1-\beta} = \theta u(w_l)$ , which is equal to the expected utility a person who in all periods receives the lowest possible utility level. That is, we require

$$\begin{aligned} u(w_l) + qu(I - \pi\delta) + (\gamma - q)u(I) - \theta(1 - q)c &\geq \theta u(w_l) \\ \Leftrightarrow c &\leq \frac{qu(I - \pi\delta) + (\gamma - q)u(I)}{\theta(1 - q)} \end{aligned} \quad (34)$$

Finally, to obtain a uniform upper bound on  $c$  we take the minimum of the right-hand side in equation (34). Specifically,

$$\begin{aligned} c &\leq c_h \equiv \min_{\pi, q} \frac{qu(I - \pi\delta) + (\gamma - q)u(I) - \gamma u(w_l)}{\theta(1 - q)} = \\ &= \frac{qu(I - \pi\delta) + (\gamma - q)u(I) - \gamma u(w_l)}{\theta(1 - q)} \Big|_{\pi=I/\delta, q=0} = \beta(u(I) - u(w_l)). \end{aligned}$$