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Optimal Taxation and the Equal-Sacrifice Social Welfare Function

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(Preliminary and incomplete: any comment is welcome!)

Abstract

A prominent principle of fairness in taxation is the "equal-sacrifice principle:" the tax burden ought to be shared so that each tax-payer makes the same sacrifice. J.S. Mill (1848) immediately realized that equality of sacrifice, "like other standards of perfection, cannot be completely realized:" in second-best settings, it would lead to inefficient policies. In this paper, we propose and axiomatically characterize a family of welfare criteria that captures both the fairness principle of equal sacrifice as well as efficiency concerns. Society ought to maximize the sum of specific indices of each individual's well-being, which depend on how society measures sacrifice. We then apply the criterion to the standard Mirrlees optimal taxation problem and highlight how the second-best tax implications of the "equal-sacrifice social welfare function" differ from those of utilitarianism.

JEL: D60, D63, H21, H23, I38. Keywords: equal-sacrifice principle; optimal income taxation; welfare criterion.

1 Introduction

The equal-sacrifice principle is a central principle of fairness in taxation. Following Mill (1848), the equal-sacrifice principle requires sharing the tax burden so that each tax-payer makes the same sacrifice. This principle has a strong intuitive appeal and is supported by a large share of the population (Weinzierl, 2014). Unfortunately, the principle has shown evasive for the purpuse of characterizing the optimal income tax schedule.

Two issues seem crucial. The first relates to the measurement of sacrifice (see Musgrave, 1959). One possibility is to measure sacrifice in terms of absolute or proportional loss of consumption due to taxation.

Unfortunately, this disregards labor supply choices and, thus, does not respect individuals' preferences. Another possibility is to measure sacrifice in terms of utility losses. However, these raise ethical concerns for their interpersonal comparability and empirical concerns for their real-world applicability.[1] The second issue is the incapacity of the criterion to trade off equity—embodied by the equal-sacrifice principle—and efficiency. Said differently, unequal sacrifices are not permissible independently of their cost for society and individuals. As a result, equal-sacrifice taxation is generally inefficient (see Berliand and Gouveia, 1993; da Costa and Pereira, 2014).

In this paper, we introduce axioms of fairness inspired by the equal-sacrifice principle. These axioms are then used to characterize a family of welfare criteria—named the *equal-sacrifice social welfare functions*—that address the above issues. We show that a society adopting our axioms ought to maximize the sum of specific indices of each individual's well-being. These indices satisfy the following properties: *(i)* they are consistent with the preferences of individuals, i.e., individuals are beffer off if and only if their index of well-being is larger; *(ii)* they are interpersonally comparable with respect to the level of sacrifice, i.e., when two individuals make the same sacrifice, the marginal social benefit of increasing their consumption is equal; and *(iii)* they are concave, i.e., society prioritizes individuals with largest sacrifices.

The workhorse model of modern optimal income taxation is proposed by Mirrlees (1971). The social welfare function is "utilitarian," i.e., it is a concave transformation of individuals' utility function. The drawbacks of this welfare criterion are well-known. Already Edgeworth (1987) highlighted its extreme egalitarian implications: with inelastic labor supply choices, society would tax earnings at a 100% rate and equalize after-tax incomes. Implicitly, the utilitarian criterion considers individuals not responsible for their heterogeneous skills and wants to correct for the differences in utilities these determine. However, this correction is often excessive. In fact, utilitarianism leads to the slavery of the talented: more skilled individuals need to work more to achieve the same consumption and are thus made worse off than the less skilled individuals (Mirrlees, 1974; Fleurbaey and Maniquet, 2018). Nevertheless, these extreme egalitarian implications do not emerge in the Mirrleesian setting due to the combination of asymmetric information—society cannot condition the tax on individuals' skills—and incentive compatibility—endogenous labor supply responses. As highlighted by Weinzierl (2014), the utilitarian criterion seems to recommend policies that are inconsistent with the ethical views held by the majority of the population.[2] Notwithstanding these fundamental objections to the

[1] Young (1988) has characterized a version of the equal-sacrifice principle that does not require utility information. In this case, the utility function adopted to evaluate the sacrifices of individuals is selected by society for the purpose of the evaluation of sacrifices and need not be related to individuals' preferences.

[2] If one recognizes proportionality as a special case of equal-sacrifice, the experimental evidence by Cappelen et al. (2007) corroborates the importance of equal-sacrifice ideals.

utilitarian approach, few applicable alternatives for optimal taxation have been developed.[3]

In contrast to utilitarianism, without asymmetric information the equal-sacrifice social welfare function equalizes the sacrifice incurred by each individual, as advocated by the proponents of the equal-sacrifice principle. In a second-best Mirrleesian setting, the first-best outcome remains available when the government budget requirement is low and with not too large elasticity of labor supply. More generally, even if the first-best outcome is not available due to asymmetric information, the main different with utilitarianism is that the equal-sacrifice social welfare function assigns a higher social marginal welfare weight to individuals incurring higher levels of sacrifice. Since for same after-tax consumption, higher skill individuals incur a larger sacrifice, this is equivalent to assigning a higher social marginal welfare weight to high-skill individuals. This effect limits the redistributive tendency of utilitarianism.

2 A simple illustration

We illustrate our results in a Mirrleesian model with quasi-linear utilities. Individuals' preferences over consumption c and labor supply ℓ are represented by a utility function $u(c, \ell) = c - v(l)$ with $v', v'' > 0$. Individuals are heterogeneous with respect to labor market productivity: each individual i is characterized by the wage rate $w_i > 0$.

2.1 The welfare criterion

We next introduce a simple version of our welfare criterion. The first step is to define how to measure and compare the sacrifice of any two pairs of individuals i and j. A natural starting point is the **laissez-faire allocation**. At the laissez-faire, no taxes are levied and each individual i maximizes her utility over the budget set $B_i \equiv \{(c_i, \ell_i) \,|c_i \leq w_i \ell_i\}$. Let $(\bar{c}_i, \bar{\ell}_i)$ denote the **laissez-faire bundle** of i. Clearly, at the reference laissez-faire allocation no individual makes any sacrifice.[4]

Assume instead that individual i consumes c_i and works the laissez-faire labor supply $\bar{\ell}_i$. Then, her **proportional sacrifice** is $\frac{\bar{c}_i - c_i}{\bar{c}_i}$, i.e., the ratio between the (implicit) tax contribution $\bar{c}_i - c_i$ and the before-tax income $\bar{c}_i = w_i \bar{\ell}_i$.[5] The main characterization result is more general and allows measures of sacrifice that differ from the proportional one presented here.

[3] See Fleurbaey and Maniquet (2018) for a recent overview.

[4] Piketty and Saez (2012) have emphasized how utilitarianism fails to ensure that laissez-faire prevails even when all agents have the same productivity level (see also Jacquet and Van de Gaer, 2011, and Fleurbaey and Maniquet, 2018).

[5] Note that if this definition of sacrifice was extended to changes in labor supply, it would necessarily be independent of the utility cost of working and, thus, would lead to violations of the Pareto principle.

The key fairness idea is the following. When two individuals incur the same level of sacrifice, society ought to be indifferent between assigning a marginal increase in consumption to either of them. This ethical stand leads to the **proportional-sacrifice social welfare function**, formally characterized as a special case in Section 4. For each individual i, define the consumption-equivalent at (c_i, ℓ_i) as the level of consumption k that makes the individual indifferent between the bundle (c_i, ℓ_i) and consuming k while working the laissez-faire labor supply $\bar{\ell}_i$. Formally, $e(c_i, \ell_i) = k$ if and only if $u(k, \bar{\ell}_i) = u(c_i, \ell_i)$. Then, the proportional-sacrifice social welfare function is defined as:

$$W^p \equiv \sum_i \bar{c}_i^\gamma \frac{[e(c_i, \ell_i)]^{1-\gamma}}{1-\gamma},$$

where $\gamma > 0$. The consumption equivalent is a representation of the preferences of individual i: formally, $e(c_i, \ell_i) = u(c_i, \ell_i) + v(\bar{\ell}_i)$. It follows that the consumption equivalent of a bundle $(c_i, \bar{\ell}_i)$ is exactly c_i. Society maximizes the weighted and transformed consumption equivalents of individuals.

The weight attached to the utility of each individual depends, through the laissez-faire bundle, on her skill level. This is crucial to ensure equal consideration for all individuals when they incur the same sacrifice. To see this, compute the social marginal welfare weight of an individual at bundle $(c_i, \bar{\ell}_i)$:

$$\frac{\partial W^p}{\partial c_i} = \frac{\partial}{\partial c_i}\left(\bar{c}_i^\gamma \frac{[e(c_i, \ell_i)]^{1-\gamma}}{1-\gamma}\right) = \left(\frac{c_i}{\bar{c}_i}\right)^{-\gamma}.$$

Thus, when two individuals i and j incur the same sacrifice $\frac{\bar{c}_i - c_i}{\bar{c}_i} = \frac{\bar{c}_j - c_j}{\bar{c}_j}$, it follows that $\frac{c_i}{\bar{c}_i} = \frac{c_j}{\bar{c}_j}$ and society is indifferent between allocating a marginal increase in consumption to either i or j. The factor $\bar{c}_i^\gamma$—placing a larger weight on the utilities of high-skilled individuals—is then explained by the need to counterbalance the effect of decreasing marginal utility (when $\gamma > 0$).

The parameter γ measures the willingness of society to avoid inequalities in the level of sacrifice incurred by individuals. At the limit for $\gamma = 0$, society is indifferent to such inequalities and social welfare simplifies into the simple sum of individuals' utilities (social marginal welfare weights are constant). As γ increases, society is less and less willing to trade-off inequalities in sacrifice against a larger sum of distributed consumption. At the limit for $\gamma \to \infty$, society attributes full priority to the individual with largest sacrifice.

2.2 A comparison with utilitarianism: first best

Let us first assume away the asymmetric information problem: the goverment covers the budget R by levying an individual-specific lump-sum tax T_i. By the additive separability of the utility function, lump-sum taxes will not distort the labor supply decision of individuals, which are then set to the laissez-faire level. Thus, the maximization problem of a (generalized) utilitarian society (with iso-elastic inequality aversion $\rho \geq 0$) simplifies to:

$$\begin{aligned} \max_{\{T_i\}} \quad & \sum \frac{\left[u\left(w_i \bar{\ell}_i - T_i, \bar{\ell}_i\right)\right]^{1-\rho}}{1-\rho} \\ \text{s.t.} \quad & \sum_i T_i \geq R. \end{aligned}$$

Similarly, the first-best maximization problem for the proportional-sacrifice social welfare function simplifies to:

$$\begin{aligned} \max_{\{T_i\}} \quad & -\frac{\bar{c}_i^{\gamma} T_i^{1+\gamma}}{1+\gamma} \\ \text{s.t.} \quad & \sum_i T_i \geq R. \end{aligned}$$

The first-best optimum for the utilitarian criterion is instructive. If $\rho = 0$, the distribution of consumption is irrelevant and the lump-sum taxes are undefined. However, when $\rho > 0$ and small (formally at the limit for $\rho \to 0$), the optimal lump sum taxes are set to equalize the levels of consumption. This egalitarian implication of utilitarianism was already highlighted by Edgeworth (see also Mirrlees, 1974; Saez and Stantcheva, 2016; Fleurbaey and Maniquet, 2018). This also leads to the slavery of the talented. All individuals consume the same, but higher-skill individuals supply more labor. Thus, the first best allocation for the utilitarian society "forces" high-skill individuals to produce for the sake of providing more consumption to the lower-skilled individuals. Note that this redistribution is quite extreme as, at the optimum, higher-skilled individuals will achieve a lower level of utility. Only at the limit for $\rho \to \infty$, when the criterion is "Rawlsian," utilities are equalized. This is summarized by the following remark.

Remark 1. Utilitarianism leads to the slavery of the talented: the utilitarian optimum demands high-skill individuals to work more than low-skill individuals and only partially compensates the high-skill ones with higher consumption. Thus, high-skill individuals achieve a lower utility than low-skill individuals. This emerges even if society needs not raise any revenues, i.e., $R = 0$.

In contrast, the first-best optimum for the proportional-sacrifice social welfare function requires the lump-sum tax to be a fixed proportion of the gross income.[6] Combining the first order conditions on the lump-sam taxes of i and j leads to

$$\frac{T_i^*}{T_j^*} = \frac{w_i \bar{\ell}_i}{w_j \bar{\ell}_j} = \frac{\bar{c}_i}{\bar{c}_j} = \frac{c_i^*}{c_j^*}.$$

As a consequence, the higher-skilled individuals combine a larger labor supply with a larger consumption. In contrast, lower-skilled individuals work less and consume less. This correlation between consumption and labor supply emerges from the proportional-sacrifice social welfare function attributing relatively more weight

[6] The reader should be reminded that $\gamma = 0$ is excluded and emerges only as a limit case. The reason is mostly technical. When $\gamma = 0$, the criterion is insensitive to the distribution of individuals' sacrifice and, thus, the notion of sacrifice itself cannot be singled out from the axioms.

to the high-skilled individuals.

Remark 2. The optimum for the proportional-sacrifice criterion ensures that high-skilled individuals be compensated for the larger labor supply with higher consumption. All individuals will contribute to the tax burden so that they incur the same level of sacrifice, here measured by the proportional sacrifice.

Nevertheless, the proportional-sacrifice optimum does not ensure that the utility of high-skilled individuals is higher than that of low-skilled individuals. This should not come as a surprise and depends on the budget of the government R. To illustrate with an extreme case, when the budget is the gross income of the entire economy ($R = \sum_i w_i \bar{\ell}_i$), each individual will be taxed with their entire gross income: then, consumption is zero and equal across individuals, while the labor supply is unchanged and penalizes (in terms of utility) those individuals with a higher skill who supply more labor. At the other extreme when $R = 0$, the utilitarian optimum equalizes consumptions, while the proportional-sacrifice optimum requires society to avoid any intervention.[7] Then, higher-skilled individuals are better off than lower-skilled ones.

2.3 A comparison with utilitarianism: second best

Assume there are two types of individuals, h and l, with $w_h > w_l$. Let $y_i \equiv w_i \ell_i$ and let P_i be a real-valued function for $i = h, l$, named **Pareto function**. Then, both the utilitarian and sacrifice-based ethical views are captured by the following sum of utility social welfare function:

$$W = \sum_{i=h,l} P_i \left(u \left(c_i, \frac{y_i}{w_i} \right) \right).$$

The utilitarian criterion emerges when the Pareto functions P_h and P_l are equal across individuals, increasing, and concave. The proportional-sacrifice social welfare function emerges when the Pareto functions $P_i(z) = \bar{c}_i^{\gamma} \frac{(z+v(\bar{l}_i))^{1-\gamma}}{1-\gamma}$ for each $z \in \mathbb{R}$ and $i = h, l$. It follows that the government's problem has the same structure as in Stiglitz (1982).

The government maximizes the social welfare function W subject to the budget revenue requirement:

$$RC: \sum_{i=h,l} T(y_i) = \sum_{i=h,l} (y_i - c_i) \leq R,$$

[7] The absence of a pure redistributive objective is sometimes considered a drawback of the equal-sacrifice principle. This is a valid objection, in particular for countries with a large presence of poverty or where education opportunities significantly vary across individuals and skills cannot be considered responsibility of individuals. Partially addressing this issue, the fathers of the equal-sacrifice principle always assumed that equal-sacrifice is restricted to the consumption/utility exceeding a subsistence level. Another possibility is to assume that the distribution of benefits take care of extreme poverty. We leave to future work the generalization of the equal-sacrifice principle as to accommodate such concern for redistribution.

and to the incentive compatibility constraints:

$$ICC_h: \; u\left(c_h, \frac{y_h}{w_h}\right) \geq u\left(c_l, \frac{y_l}{w_h}\right),$$

$$ICC_l: \; u\left(c_l, \frac{y_l}{w_l}\right) \geq u\left(c_h, \frac{y_h}{w_l}\right).$$

Two cases are possible, depending on the Pareto functions P_h and P_l.

Case 1. $\boldsymbol{ICC_h}$ **binds**. When the incentive compatibility constraint of the high-skill type is binding, the optimal tax schedule requires $T'(y_h) = 0$ and $T'(y_l) > 0$. The labor supply choice of the high-skill type is undistorted, while the labor supply choice of the low-skill type is distorted downwards. The government trades off the efficiency cost of labor-supply distorsions with the information rent of the high-skilled type.

Case 2. **Both** $\boldsymbol{ICC_h}$ **and** $\boldsymbol{ICC_l}$ **do not bind**. The optimal tax schedule requires $T'(y_h) = T'(y_l) = 0$. Labor supply choices are not distorted and the first-best allocation is implemented.

The implications of the welfare criteria for the optimal non-linear taxes are insightful:

1. with a utilitarian government, only Case 1 can emerge. The government would like the high-skill type to achieve a lower utility than the low-skill type (see Remark 1). However, the high-skill type can always mimic the low-skill and achieve a higher utility (the utility cost of earning y_l is smaller). Thus, the incentive compatibility constraint of the high-skill type is always binding.

2. with the equal-sacrifice government, both cases can emerge. There exists a budget requirement level $\bar{R} > 0$ that distinguishes these cases. When the government budget requirement is small, i.e., when $R \leq \bar{R}$, the incentive compatibility constraints are not binding and the first-best equal-sacrifice allocation can be implemented. When the government budget requirement is large, i.e., when $R > \bar{R}$, the incentive compatibility constraint of the high-skill type binds.

These results lead to the following remarks.

Remark 3. With asymmetric information, the incentive compatibility constraints significantly limit the extent of redistribution that the government can implement. Then, the first-best goal of "favoring" the low-skilled individuals in unachievable and, thus, is only indirectly reflected in the second-best optimum. This reduces the gap between the implications of utilitarianism and common views on the extent of income redistribution.

Remark 4. The differences in second-best policies between welfare criteria might not be very large. This depends on a combination of factors. First, a sufficiently large government budget requirement ensures that the incentive compatibility constraint of the high type is binding for both criteria. Second, a large elasticity of labor supply limits the space of feasible redistributive policies for the government and, thus, forces the policies to be similar. Third, on the definition of inequality attitudes towards utility (for the utilitarian criterion) and towards sacrifice (for the proportional-sacrifice criterion).

2.4 A comparison with utilitarianism: a simulation exercise

Finally, we highlight the difference with utilitarianism by replicating the Mankiw, Weinzierl, and Yagan (2009) simulation exercise. We set the utility function as $u(c_i, \ell_i) = \frac{c_i^{1-\rho}-1}{1-\rho} - \frac{\alpha}{\sigma}\ell_i^{\sigma}$, where the parameters are $\rho = 1.5$, $\alpha = 2.55$, and $\sigma = 3$. We use the same data and methodology and compare the log utilitarian criterion:

$$W^U = \sum_i \ln u(c_i, \ell_i)$$

with the log proportional sacrifice social welfare function (emerging when $\gamma = 1$):

$$W^P = \sum_i \bar{c}_i \ln\left[e(c_i, \ell_i)\right].$$

The results are summarized by the following graphs, representing the marginal tax rate, the average tax rate, and the after-tax utility emerging for the two criteria. The main difference is that the proportional sacrifice social welfare function is less redistributive than the utilitarian criterion. Crucially, this does not imply that the equal sacrifice criterion is insensitive to the utilities of low income individuals. Rather, the proportional sacrifice criterion reduces the willingness of society to transfer to the worst-off to a level that appears more consistent with real-world income taxes. In particular, the utilitarian second best policy supports marginal tax rates above 60% for all individuals. In contrast, the proportional-sacrifice criterion supports about 20% lower marginal tax rates. Utilitarianism suggests subsidies (negative average taxes) ought to be give to the bottom 35% of the population, while the proportional-sacrifice criterion does so only for the bottom 15% of the population. Utilitarianism sets a lump sum transfer that is more than 5 times larger than the proportional-sacrifice criterion. These implications are summarized by a flatter utility schedule for utilitarianism, rather than the proportional-sacrifice principle.

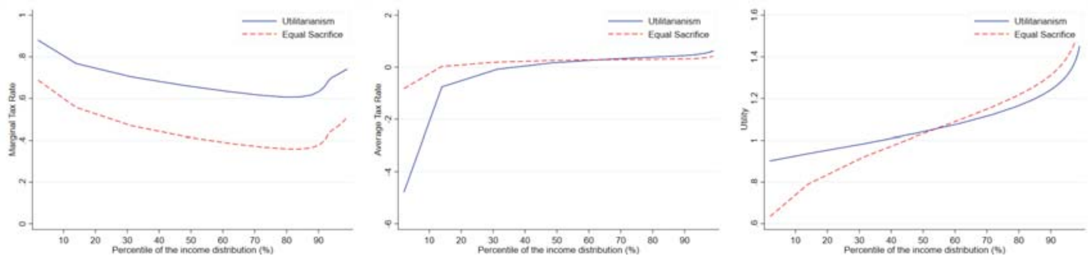

Figure 1: A comparison of marginal tax rate, the average tax rate, and individuals' utility.

3 Model and axioms

3.1 Model

Society consists of I individuals, with $|I| \geq 3$. Individuals differ by their labor skills, which are reflected by their wage rate: for each $i \in I$, let $w_i > 0$ denote the wage rate of individual i.

Each individual $i \in I$ supplies labor $\ell_i \geq 0$, earns income $y_i \equiv w_i \ell_i$, and consumes $c_i \geq 0$. Her preferences are represented by a utility function $u(c_i, \ell_i)$, which is continuous, increasing in c_i, decreasing in ℓ_i, and strictly concave. We assume that consumption is an essential good, i.e. $\lim_{c \to 0} u_c = \infty$.

An allocation $a \equiv \left(\{c_i, \ell_i\}_{i \in I}\right)$ specifies a bundle (c_i, ℓ_i) for each individual $i \in I$. Let A be the set of all allocations.

The government collects taxes to cover an exogenous budget R. Since skills/wages are private information of individuals, taxes can only depend on individuals' incomes. Let the tax function be denoted by $T : \mathbb{R} \to \mathbb{R}$. For each $i \in I$, after-tax income is $y_i - T(y_i)$.

The goal of the government is to set the tax function to maximize social preferences. Social preferences are a complete, transitive, and continuous preference relation $\succsim$ on the set of allocations A. For each pair $a, a' \in A$, $a \succsim a'$ means that a is socially at least as desirable as a'. The asymmetric and symmetric counterparts of $\succsim$ are denoted $\succ$ and $\sim$. Social welfare can be represented by a continuous social welfare function $W : A \to \mathbb{R}$. Thus, for each pair $a, a' \in A$, $a \succsim a'$ if and only if $W(a) \geq W(a')$.

As standard, we require social preferences to satisfy the Pareto principle. Said differently, if individuals are better off, social welfare is higher.

Efficiency: *For each pair $a, a' \in A$, if $u(c_i, \ell_i) \geq u(c'_i, \ell'_i)$ for each $i \in I$ and $u(c_i, \ell_i) > u(c'_i, \ell'_i)$ for some $i \in I$, then $a \succsim a'$.*

Next, we impose that social preferences are averse to inequalities. Social attituds to inequality are very similar to individual attitudes to risk. Building on this analogy, we here impose social preferences to be strictly convex.[8]

Inequality aversion: *For each pair* $a, a' \in A$ *and each* $\beta \in (0,1)$, $a \sim a'$ *implies* $\beta a + (1-\beta)\, a' \succ a$.

Finally, we impose that social welfare comparisons do not depend on the bundle assigned to an unconcerned individual. For the sake of simplicity, denote by (a_i, a_{-i}) the allocation $a \in A$ that assigns $a_i \equiv (c_i, \ell_i)$ to individual i and $a_{-i} \equiv (c_j, \ell_j)_{j \in I \setminus \{i\}}$ to the other individuals.

Separability: *For each* $a, a' \in A$, *each* $i \in I$, *and each* $\bar{a}_i = \left(\bar{c}_i, \bar{\ell}_i\right)$, $(a_i, a_{-i}) \succsim \left(a_i, a'_{-i}\right)$ *if and only if* $(\bar{a}_i, a_{-i}) \succsim \left(\bar{a}_i, a'_{-i}\right)$.

Efficiency, *inequality aversion*, and *separability* imply that the social welfare function is a very general type of utilitarian criterion. By *efficiency*, society evaluates individuals through their own preferences: society is not paternalistic and, thus, W can be written as a function of the utilities achieved by each individual. By *inequality aversion*, social preferences are strictly convex with respect to the allocation and, thus, W is strictly concave in its arguments. By *separability*, the assignment of individual i does not matter for how society trades off the utility of individuals j and k; thus, W is additively separable.

Let $(f_i)_{i \in I}$ be real-valued, continuous, and strictly increasing functions such that, for each $i \in I$, $f_i\left(u\left(c_i, \ell_i\right)\right)$ is strictly concave. A social welfare function $W : A \to \mathbb{R}$ is **sum-of-utilities** if for each $a \in A$:

$$W(a) = \sum_{i \in I} f_i\left(u\left(c_i, \ell_i\right)\right). \tag{1}$$

Note that this family of social welfare functions is significantly more general than what is typically assumed in the literature, in particular if one includes the limit case of infinite and zero concavity. Differently from Mirrlees (1971), the functions $(f_i)_{i \in I}$ need not be equal across individuals. Differently from weighted utilitarianism (see Maskin, 1978; d'Aspremont and Gevers, 1977), the functions $(f_i)_{i \in I}$ need not be increasing affine transformations (thus, specifying Pareto weights). As we discuss in the following, this degree of freedom is necessary to incorporate principles of equal-sacrifice. Before doing so, we formalize the implication of the above axioms. This result is well-known and the proof is omitted (see Piacquadio, 2017).

[8] Note that convexity is significantly weaker than what is standardly assumed in the literature, where this condition is supplemented with some form of symmetry or anonymity. In fact, most social welfare functions satisfy convexity. Here, strict convexity avoids a techical issue: when social preferences are linear, inequalities are irrelevant and the axioms cannot identify how to measure inequalities in sacrifice. Convex social preferences then emerge as a limit case.

Lemma 1. *Social preferences $\succsim$ satisfy efficiency, inequality aversion, and separability if and only if $\succsim$ can be represented by a sum-of-utilities social welfare function.*

3.2 Averting unequal sacrifices

The principle of equal-sacrifice tells that all individuals ought to contribute to the tax burden by incurring the same sacrifice. There is no consensus, however, on how to measure such sacrifice. In the following, we argue that some distributions of tax burdens are unfair and propose three axioms that discipline how social preferences should deal with such inequalities.

To start with, define the **laissez-faire allocation** $\bar{a}$. At the laissez-faire allocation, each individual freely chooses how much labor to supply and consumes her entire income. Formally, the laissez-faire bundle of each individual $i \in I$ is $\left(\bar{c}_i, \bar{\ell}_i\right)$ such that $u\left(\bar{c}_i, \bar{\ell}_i\right) \geq u\left(c_i, \ell_i\right)$ for each $\left(c_i, \ell_i\right)$ with $c_i \leq y_i \equiv \ell_i w_i$. Crucially, since individuals entirely appropriate the returns from their own work, individuals make no sacrifice.

Let individual $i's$ **(implicit) tax burden** at the bundle $\left(c_i, \bar{\ell}_i\right)$ be measured by the difference in consumption with the laissez-faire allocation, i.e. $b_i \equiv \bar{c}_i - c_i$. By definition of laissez-faire, an individual working $\bar{\ell}_i$ has a gross income of $\bar{y}_i \equiv \bar{\ell}_i w_i$. At the laissez-faire allocation, individual i would consume the entire income $\bar{c}_i = \bar{y}_i$. At the bundle $\left(c_i, \bar{\ell}_i\right)$, instead, individual i works the same time $\bar{\ell}_i$, but consumes c_i. Then, the tax burden is the difference between these consumption levels.[9] The basic intuition is that each individual's sacrifice increases the larger her tax burden.

Our first principle of equal sacrifice tells that society should avert situations at which one individual makes a sacrifice, while another individual does not. Said differently, individuals should solidarily bear the cost of taxation. We state this ideal in the form of a transfer principle. More precisely, assume that at allocation $a \in A$, individual i has a positive tax burden $b_i > 0$ (i consumes less than at the laissez-faire bundle), while individual j has a negative tax burden $b_j < 0$ (j consumes more than at the laissez-faire bundle). This distribution of the tax burden is unfair according to the equal-sacrifice principle. Then, *ceteris paribus*, a transfer of consumption from i to j increases further the tax burden of i, while decreasing that of j. This distribution of the tax burden is even more unfair and, thus, social welfare cannot be higher.

Tax solidarity: *For each pair $a, a' \in A$, each pair $i, j \in I$, and each $\varepsilon > 0$, such that:*

- $b'_i + \varepsilon = b_i \geq 0 \geq b_j = b'_j - \varepsilon$;

[9] The fact that the labor supply is fixed can be interpreted as a *ceteris paribus* assumption. Its importance is easily explained: when charging a positive income tax, individuals may adjust labor supply upwards to compensate for the lost income. If labor supply is allowed to vary, the extent of this income effect will matter for the measurement of the tax burden. In some situations, the individual might end up consuming more than at the laissez-faire (when leisure is a Giffen good).

- $\ell_i = \ell'_i = \bar{\ell}_i$ *and* $\ell_j = \ell'_j = \bar{\ell}_j$*; and*
- $(c_k, \ell_k) = (c'_k, \ell'_k) = (\bar{c}_k, \bar{\ell}_k)$ *for each* $k \in I / \{i, j\}$*;*

then, $a \succsim a'$*.*

Tax solidarity implies that (together with *efficiency*) no taxation is optimal when the government's budget is $R = 0$. The intuition is immediate. The $R = 0$ budget condition means that taxation cannot provide efficiency and can only have a redistributive welfare effect. However, *tax solidarity* tells that redistribution away from the laissez faire allocation cannot improve social welfare. Thus, when $R = 0$, the laissez faire allocation is optimal and no taxation should be introduced. Thus, when $R = 0$, no individual makes any sacrifice and equal-sacrifice obtains.

The next ethical principle deals with a different type of unfairness. Without loss of generality, assume individual i's consumption at the laissez-faire allocation is larger than j's, i.e. $\bar{c}_i \geq \bar{c}_j$. At allocation $a \in A$, individual i has a smaller tax burden than j, i.e. $0 \leq b_i < b_j$; labor supply is that of the laissez-faire allocation. Individuals earn incomes $w_i \bar{\ell}_i = \bar{c}_i \geq \bar{c}_j = w_j \bar{\ell}_j$ and consume $c_i > c_j$. Crucially, $b_i < b_j$ implies that the difference in earnings is smaller than the difference in consumption: the tax burden imposed on the individuals exacerbates inequality. Consider now increasing further the tax burden of j, while further reducing that of i. This transfer of consumption makes the allocation more unfair and, thus, cannot improve social welfare.

Fair burden: *For each pair* $a, a' \in A$*, each pair* $i, j \in I$ *with* $\bar{c}_i \geq \bar{c}_j$*, and each* $\varepsilon > 0$*, such that:*

- $0 \leq b'_i - \varepsilon = b_i < b_j = b'_j + \varepsilon$*;*
- $\ell_i = \ell'_i = \bar{\ell}_i$ *and* $\ell_j = \ell'_j = \bar{\ell}_j$*; and*
- $(c_k, \ell_k) = (c'_k, \ell'_k) = (\bar{c}_k, \bar{\ell}_k)$ *for each* $k \in I / \{i, j\}$*;*

then, $a \succsim a'$*.*

Fair burden deals with situations in which the sacrifice of individual i (who is better-off at the laissez-faire allocation) is too small relative to some other individual. Next, we disclipline how social welfare deals with situations in which the sacrifice of individual i is too large.

As before, assume individual i's consumption at the laissez-faire allocation is larger than j's, i.e. $\bar{c}_i \geq \bar{c}_j$. At allocation $a \in A$, individual i's consumption is smaller than j's, i.e. $c_i < c_j$; labor supply is that of the laissez-faire allocation. Individuals earn incomes $w_i \bar{\ell}_i = \bar{c}_i \geq \bar{c}_j = w_j \bar{\ell}_j$ and consume $c_i \leq c_j$. The sacrifice of i is so large that, net of the sacrifice, the consumption of i is now smaller than that of j. Consider now making the sacrifice of i even harscher by reducing his consumption for the benefit of j. This change makes the allocation more unfair and cannot improve social welfare.

Fair reward: *For each pair $a, a' \in A$, each pair $i, j \in I$ with $\bar{c}_i \geq \bar{c}_j$, and each $\varepsilon > 0$, such that:*

- $c_i' + \varepsilon = c_i < c_j = c_j' - \varepsilon$;
- $\ell_i = \ell_i' = \bar{\ell}_i$ *and* $\ell_j = \ell_j' = \bar{\ell}_j$; *and*
- $(c_k, \ell_k) = (c_k', \ell_k') = \left(\bar{c}_k, \bar{\ell}_k\right)$ *for each* $k \in I/\{i, j\}$;

then, $a \succsim a'$.

The three above-introduced axioms can be graphically summarized in the following graph. On the Cartesian plane, the consumptions of individuals i and j are represented on the axes. Consumption levels $\bar{c}_i$ and $\bar{c}_j$ are those of the laissez-faire allocation. Without loss of generality, here i is again the individual that consumes more at the laissez-faire allocation, i.e., $\bar{c}_i \geq \bar{c}_j$. Not represented, the labor supplies are set to the those at the laissez-faire allocation.

The north-west and south-east areas from the laissez-faire consumptions are those where *tax solidarity* applies. These areas are characterized by one individual making a sacrifice, while the other does not. The arrow towards the laissez-faire consumptions represents the direction of increasing social welfare.

South-west of the laissez-faire consumptions is the area where both individuals make a sacrifice. *Fair burden* applies below the 45 degree line from the laissez-faire consumptions. In this area, the tax burden of j is larger than that of i. Again, the arrow suggests that transferring consumption from i to j increases social welfare.

Finally, the portion of the area above the 45 degree line from the origin is such that the consumption of j is larger than that of i. In this area, *fair reward* applies. In this area, the tax burdn of i is so large that her consumption is now smaller than j's. The arrow points at reducing j's consumption for the benefit of i's. This is what *fair reward* suggests doing to increase social welfare.

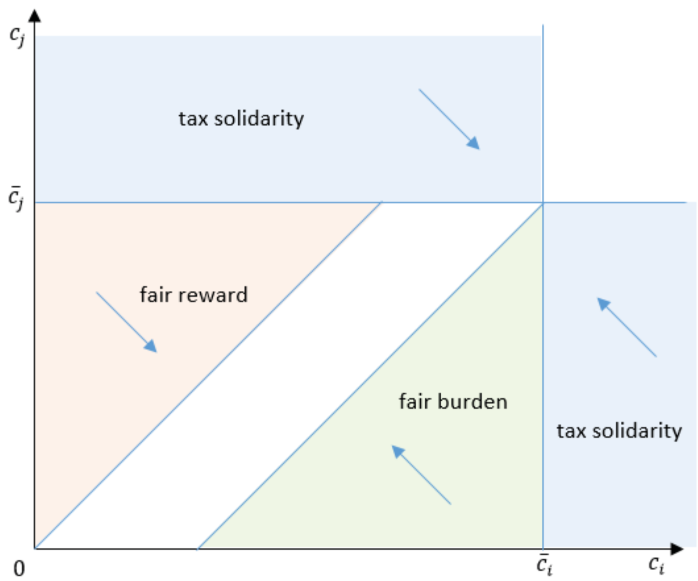

Figure 2: Equal-sacrifice principles.

4 The sacrifice-based welfare criteria

4.1 Comparisons of sacrifice

First, for each allocation $a \in A$ and each individual $i \in I$, let the **equivalent consumption of i at a** be the level of consumption $e_i(c_i, \ell_i)$ such that:

$$e_i(c_i, \ell_i) = \lambda \iff u(c_i, \ell_i) = u\left(\lambda, \bar{\ell}_i\right),$$

where $\bar{\ell}_i$ is the labor supply at the laissez-faire allocation. The function $e_i(c_i, \ell_i)$ tells the level of consumption that, when combined with the laissez-faire labor supply, gives the same utility as the bundle (c_i, ℓ_i).

Next, we define the **sacrifice function** $S : \mathbb{R}_+ \times \mathbb{R}_{++} \to \mathbb{R}$. This function measures the sacrifice made by each individual in a way that is interpersonally comparable. Let individual $i \in I$ be assigned the bundle $\left(c_i, \bar{\ell}_i\right)$; then, i's sacrifice is given by $S(c_i; \bar{c}_i)$. More generally, i's sacrifice at bundle (c_i, ℓ_i) is given by $S(e(c_i, \ell_i); \bar{c}_i)$. Let the sacrifice function S be decreasing in the first argument, increasing in the second argument, and continuous. Furthermore, it satisfies the following restrictions:

- [*zero sacrifice normalization*] $x = y$ implies $S(x; y) = 0$;
- [*slope bound for positive sacrifice*] whenever $S(x; y) = S(x'; y') > 0$, then $|x - x'| \leq |y - y'|$.

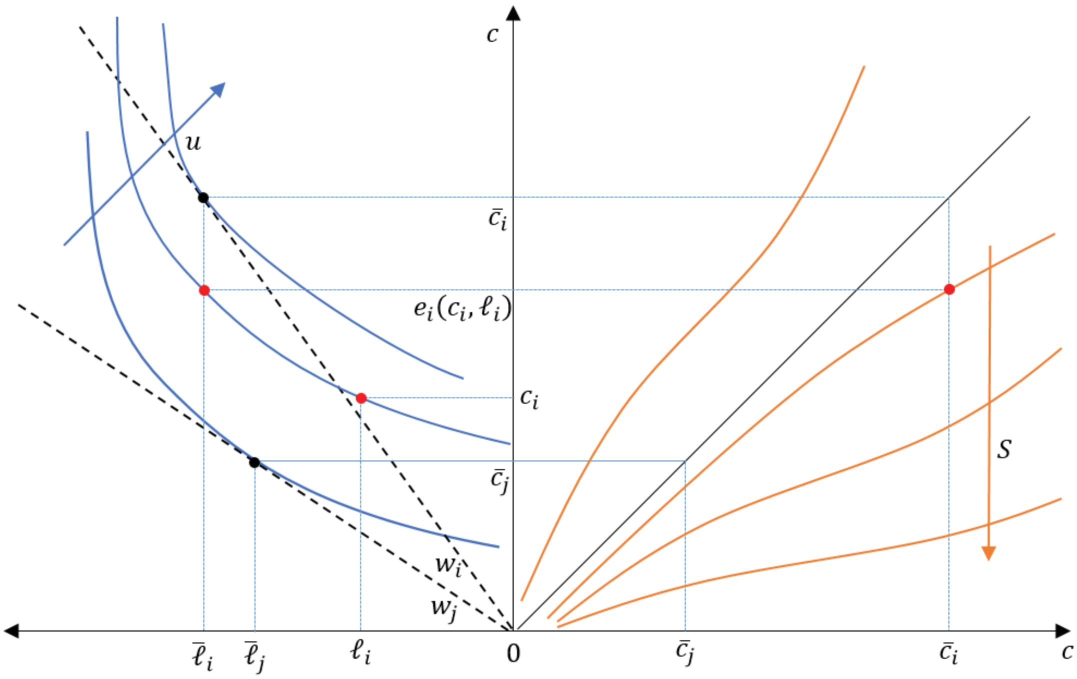

Figure 3: Equal-sacrifice principles.

Let $\mathcal{S}$ be the domain of these functions. Importantly, the sacrifice function is ordinal, as it only represents an ordering of levels of sacrifice incurred by any two individuals. In the following figure, we illustrate how the sacrifice function works. On the left part of the Cartesian plane, individual i faces the wage level w_i and chooses the utility-maximizing bundle $\left(\bar{c}_i, \bar{\ell}_i\right)$. Similarly, individual j with wage level w_j chooses the utility maximizing bundle $\left(\bar{c}_j, \bar{\ell}_j\right)$. These bundles are those corresponding to the laissez-faire allocation. The corresponding levels of consumption $\bar{c}_i$ and $\bar{c}_j$ are reported to the horizontal axis of the right part of the Cartesian plane, where we represent the sacrifice function through its **iso-sacrifice curves**. On the 45 degree line, the level of sacrifice is 0. The level of sacrifice decreases with the assigned consumption and increases with the laissez-faire consumption, making the iso-sacrifice curves increasing. The slope bound implies that, for positive levels of sacrifice, the slope of the sacrifice function cannot exceed 1.

Let individual i be assigned the bundle (c_i, ℓ_i). Her level of utility is the same as if she was assigned her equivalent consumption $e_i\left(c_i, \ell_i\right)$ and the laissez-faire labor supply $\bar{\ell}_i$. The implicit tax burden of i is given by the difference between $\bar{c}_i$ and $e_i\left(c_i, \ell_i\right)$. Interpersonal comparisons of sacrifice are made through the iso-sacrifice curve of level $S\left(e_i\left(c_i, \ell_i\right); \bar{c}_i\right)$. Individual i makes a larger sacrifice than j whenever $S\left(e_i\left(c_i, \ell_i\right); \bar{c}_i\right) \geq S\left(e_j\left(c_j, \ell_j\right); \bar{c}_j\right)$.

4.2 The measurement of each individuals' welfare

For each individual i, let the **Pareto function of i** be denoted by $P_i : \mathbb{R}^2_+ \to \mathbb{R}$. Then, $P_i(u(c_i, \ell_i))$ denotes the **Pareto-transformed welfare** of i at (c_i, ℓ_i). As we shall discuss, society maximizes the sum of individuals' Pareto transformed welfares. Before that, we introduce the restrictions on the Pareto functions.

For each i and each bundle (c_i, ℓ_i), denote by $\beta_i(c_i, \ell_i)$ the **social marginal welfare weight** of i at bundle (c_i, ℓ_i); formally:

$$\beta_i(c_i, \ell_i) \equiv \frac{\partial}{\partial c_i} P_i(u(c_i, \ell_i)).$$

Then, we impose that there exists a sacrifice function $S \in \mathcal{S}$ and a real-valued increasing function g such that for each i and each $c_i \in \mathbb{R}_+$:

1. $\beta_i\left(c_i, \bar{\ell}_i\right) = g(S(c_i; \bar{c}_i)) > 0$; and

2. $P_i(u(c_i, \ell_i))$ is strictly concave in its arguments.

To explain, since the social marginal welfare weights are positive, the Pareto functions are increasing. Thus, the Pareto-transformed welfare of individuals are all ordinally-equivalent representations of individuals' preferences.Condition 1 also imposes equality of social marginal welfare weights when the level of sacrifice incurred by individuals is the same, as identified by the sacrifice function S. Furthermore, since g is increasing, the social marginal welfare weights are higher for those individuals incurring a larger level of sacrifice. Condition 2 ensures concavity of the Pareto-transformed welfares and, thus, inequality aversion of society.

When the above conditions are satisfied, we say that a set of Pareto functions $(P_i)_{i \in I}$ is **consistent** with a sacrifice function S.

4.3 The welfare criterion

The **equal-sacrifice social welfare function** $W : A \to \mathbb{R}$ is defined by setting for each $a \in A$:

$$W(a) \equiv \sum_{i \in I} P_i(u(c_i, \ell_i)),$$

where $(P_i)_{i \in I}$ are Pareto functions consistent with a sacrifice function $S \in \mathcal{S}$.

Our main result shows that the above ethical principles single out the family of equal-sacrifice social welfare functions.

Theorem 1. *Social welfare $\succsim$ satisfies efficiency, inequality aversion, separability, tax solidarity, fair burden, and fair reward if and only if it can be represented by an equal-sacrifice social welfare function.*

Proof. **Part 1.** We first show that the equal-sacrifice social welfare function satisfies the axioms. Let the sacrifice function be $S \in \mathcal{S}$ and let the individual Pareto functions $(P_i)_{i \in I}$ be consistent with S. Then, the equal-sacrifice social welfare function is:

$$W(a) \equiv \sum_{i \in I} P_i \left(u \left(c_i, \ell_i \right) \right).$$

Efficiency. Since social marginal welfare weights are positive, the Pareto functions are increasing. Then, for each $i \in I$ and each pair $(c_i, \ell_i), (c'_i, \ell'_i)$, $u(c_i, \ell_i) \geq u(c'_i, \ell'_i)$ if and only if $P_i(u(c_i, \ell_i)) \geq P_i(u(c'_i, \ell'_i))$. Consider a pair of allocations $a, a' \in A$ such that $u(c_i, \ell_i) \geq u(c'_i, \ell'_i)$ for each $i \in I$ and $u(c_i, \ell_i) > u(c'_i, \ell'_i)$ for some $i \in I$. Thus, also $P_i(u(c_i, \ell_i)) \geq P_i(u(c'_i, \ell'_i))$ for each $i \in I$ and $P_i(u(c_i, \ell_i)) > P_i(u(c'_i, \ell'_i))$ for some $i \in I$. Then, $\sum_{i \in I} P_i(u(c_i, \ell_i)) = W(a) > W(a') = \sum_{i \in I} P_i(u(c'_i, \ell'_i))$ and $a \succ a'$. This proves that the equal-sacrifice social welfare function satisfies *efficiency*.

Inequality aversion. By construction, for each $i \in I$, $P_i(u(c_i, \ell_i))$ is strictly concave in its arguments. It follows that $W(a)$ is strictly concave in its arguments and *inequality aversion* holds.

Separability. *Separability* follows from the additivity of the function W: the bundle of an unconcerned individual is irrelevant for the ranking of two allocations.

Tax solidarity. Consider a pair of allocations $a, a' \in A$ satisfying the requirements in the definition of *tax solidarity*. These allocations are such that for some pair of individuals $i, j \in I$ and some $\varepsilon > 0$: $b'_i + \varepsilon = b_i \geq 0 \geq b_j = b'_j - \varepsilon$; $\ell_i = \ell'_i = \bar{\ell}_i$ and $\ell_j = \ell'_j = \bar{\ell}_j$; and $(c_k, \ell_k) = (c'_k, \ell'_k) = \left(\bar{c}_k, \bar{\ell}_k\right)$ for each $k \in I / \{i, j\}$. By definition, $b_i \equiv \bar{c}_i - c_i$ and $b'_i \equiv \bar{c}_i - c'_i$. Thus, $c'_i = \bar{c}_i - b'_i = \bar{c}_i - b_i - \varepsilon$ and, substituting for b_i, $c'_i = c_i - \varepsilon$. Similarly, $c'_j = c_j + \varepsilon$. Substituting and using the fact that individuals $k \in I / \{i, j\}$ are unaffected, we can write:

$$W(a) - W(a') = P_i\left(u\left(c_i, \bar{\ell}_i\right)\right) - P_i\left(u\left(c_i - \varepsilon, \bar{\ell}_i\right)\right) + P_j\left(u\left(c_j, \bar{\ell}_j\right)\right) - P_j\left(u\left(c_j + \varepsilon, \bar{\ell}_j\right)\right).$$

Now, by first degree Taylor expansion and concavity of the Pareto-transformed welfare functions:

$$P_i\left(u\left(c_i - \varepsilon, \bar{\ell}_i\right)\right) \leq P_i\left(u\left(c_i, \bar{\ell}_i\right)\right) - \varepsilon \beta_i\left(c_i, \bar{\ell}_i\right),$$

where, for memory, $\beta_i\left(c_i, \bar{\ell}_i\right) = \frac{\partial}{\partial c_i} P_i\left(u\left(c_i, \bar{\ell}_i\right)\right)$; and

$$P_j\left(u\left(c_j + \varepsilon, \bar{\ell}_j\right)\right) \leq P_j\left(u\left(c_j, \bar{\ell}_j\right)\right) + \varepsilon \beta_j\left(c_j, \bar{\ell}_j\right),$$

where $\beta_j\left(c_j,\bar{\ell}_j\right)=\frac{\partial}{\partial c_j}P_j\left(u\left(c_j,\bar{\ell}_j\right)\right)$.

Thus,

$$W(a)-W(a')\geq\varepsilon\left[\beta_i\left(c_i,\bar{\ell}_i\right)-\beta_j\left(c_j,\bar{\ell}_j\right)\right].$$

Finally, since $S(c_i;\bar{c}_i)>0>S(c_j;\bar{c}_j)$ and since g is increasing, $\beta_i\left(c_i,\bar{\ell}_i\right)=g\left(S\left(c_i;\bar{c}_i\right)\right)>g\left(S\left(c_j;\bar{c}_j\right)\right)=\beta_j\left(c_j,\bar{\ell}_j\right)$. Thus, $W(a)\geq W(a')$ and $a\succsim a'$. This proves that *tax solidarity* holds.

Fair burden. Consider a pair of allocations $a,a'\in A$ satisfying the requirements in the definition of *fair burden*. These allocations are such that for some pair of individuals $i,j\in I$ with $\bar{c}_i\geq\bar{c}_j$ and some $\varepsilon>0$: $0\leq b_i'-\varepsilon=b_i<b_j=b_j'+\varepsilon$; $\ell_i=\ell_i'=\bar{\ell}_i$ and $\ell_j=\ell_j'=\bar{\ell}_j$; and $(c_k,\ell_k)=(c_k',\ell_k')=\left(\bar{c}_k,\bar{\ell}_k\right)$ for each $k\in I/\{i,j\}$. Substituting for b_i, b_i', b_j, and b_j', $c_i'=c_i+\varepsilon$ and $c_j'=c_j-\varepsilon$. Substituting and using the fact that individuals $k\in I/\{i,j\}$ are unaffected, we can write:

$$W(a)-W(a')=P_i\left(u\left(c_i,\bar{\ell}_i\right)\right)-P_i\left(u\left(c_i+\varepsilon,\bar{\ell}_i\right)\right)+P_j\left(u\left(c_j,\bar{\ell}_j\right)\right)-P_j\left(u\left(c_j-\varepsilon,\bar{\ell}_j\right)\right).$$

Now, by first degree Taylor expansion and concavity of the Pareto-transformed welfare functions:

$$P_i\left(u\left(c_i+\varepsilon,\bar{\ell}_i\right)\right)\leq P_i\left(u\left(c_i,\bar{\ell}_i\right)\right)+\varepsilon\beta_i\left(c_i,\bar{\ell}_i\right),$$

and

$$P_j\left(u\left(c_j-\varepsilon,\bar{\ell}_j\right)\right)\leq P_j\left(u\left(c_j,\bar{\ell}_j\right)\right)-\varepsilon\beta_j\left(c_j,\bar{\ell}_j\right).$$

Thus,

$$W(a)-W(a')\geq\varepsilon\left[\beta_j\left(c_j,\bar{\ell}_j\right)-\beta_i\left(c_i,\bar{\ell}_i\right)\right].$$

Finally, since $c_i\leq\bar{c}_i$, $S(c_i;\bar{c}_i)\geq 0$. Furthermore, $c_i-c_j\geq\bar{c}_i-\bar{c}_j$. Thus, by the slope bound restriction on S, $S(c_i;\bar{c}_i)\leq S(c_j;\bar{c}_j)$. Then, since g is increasing, $\beta_i\left(c_i,\bar{\ell}_i\right)=g\left(S\left(c_i;\bar{c}_i\right)\right)\leq g\left(S\left(c_j;\bar{c}_j\right)\right)=\beta_j\left(c_j,\bar{\ell}_j\right)$. Thus, $W(a)\geq W(a')$ and $a\succsim a'$. This proves that *fair burden* holds.

Fair reward. Consider a pair of allocations $a,a'\in A$ satisfying the requirements in the definition of *fair reward*. These allocations are such that for some pair of individuals $i,j\in I$ with $\bar{c}_i\geq\bar{c}_j$ and some $\varepsilon>0$: $c_i'+\varepsilon=c_i<c_j=c_j'-\varepsilon$; $\ell_i=\ell_i'=\bar{\ell}_i$ and $\ell_j=\ell_j'=\bar{\ell}_j$; and $(c_k,\ell_k)=(c_k',\ell_k')=\left(\bar{c}_k,\bar{\ell}_k\right)$ for each $k\in I/\{i,j\}$. Using the fact that individuals $k\in I/\{i,j\}$ are unaffected, we can write:

$$W(a) - W(a') = P_i\left(u\left(c_i, \bar{\ell}_i\right)\right) - P_i\left(u\left(c_i - \varepsilon, \bar{\ell}_i\right)\right) + P_j\left(u\left(c_j, \bar{\ell}_j\right)\right) - P_j\left(u\left(c_j + \varepsilon, \bar{\ell}_j\right)\right).$$

Now, by first degree Taylor expansion and concavity of the Pareto-transformed welfare functions:

$$P_i\left(u\left(c_i - \varepsilon, \bar{\ell}_i\right)\right) \leq P_i\left(u\left(c_i, \bar{\ell}_i\right)\right) - \varepsilon\beta_i\left(c_i, \bar{\ell}_i\right),$$

and

$$P_j\left(u\left(c_j + \varepsilon, \bar{\ell}_j\right)\right) \leq P_j\left(u\left(c_j, \bar{\ell}_j\right)\right) + \varepsilon\beta_j\left(c_j, \bar{\ell}_j\right).$$

Thus,

$$W(a) - W(a') \geq \varepsilon\left[\beta_i\left(c_i, \bar{\ell}_i\right) - \beta_j\left(c_j, \bar{\ell}_j\right)\right].$$

Finally, S is decreasing in the first argument and increasing in the second: $c_i < c_j$ and $\bar{c}_i \geq \bar{c}_j$ imply that $S(c_i; \bar{c}_i) > S(c_j; \bar{c}_j)$. Thus, since g is increasing, $\beta_i\left(c_i, \bar{\ell}_i\right) = g\left(S\left(c_i; \bar{c}_i\right)\right) > g\left(S\left(c_j; \bar{c}_j\right)\right) = \beta_j\left(c_j, \bar{\ell}_j\right)$. Thus, $W(a) \geq W(a')$ and $a \succsim a'$. This proves that *fair reward* holds.

Part 2. We now show that social preferences satisfying the axioms admit a representation by means of an equal-sacrifice social welfare function.

The proof is divided in several steps.

Step 1. *Assume social preferences $\succsim$ satisfy the axioms. Then, there exists real-valued increasing and strictly concave functions $(h_i)_{i \in I}$ such that social welfare W representing $\succsim$ is defined by setting for each $a \in A$*

$$W(a) = \sum_{i \in I} h_i\left(e_i\left(c_i, \ell_i\right)\right). \tag{2}$$

Proof. Lemma 1 establishes that social preferences $\succsim$ can be represented by a social welfare function of the sum-of-utilities type. Formally, there exist real-valued increasing functions $(f_i)_{i \in I}$ such that $f_i\left(u\left(c_i, \ell_i\right)\right)$ is strictly concave for each $i \in I$ and such that, for each pair $a, a' \in A$, $a \succsim a'$ if and only if:

$$W(a) = \sum_{i \in I} f_i\left(u\left(c_i, \ell_i\right)\right) \geq \sum_{i \in I} f_i\left(u\left(c'_i, \ell'_i\right)\right) = W(a').$$

Next, for each $i \in I$, $e_i\left(c_i, \ell_i\right)$ is the consumption-equivalent representation of preferences of i. Thus, there exists a real-valued increasing function h_i such that $h_i(e_i(c_i, \ell_i)) = f_i(u(c_i, \ell_i))$ for each (c_i, ℓ_i). This shows that social preferences can be represented by a social welfare function (2).

It is left to show that $(h_i)_{i\in I}$ are strictly concave. By definition of the consumption equivalent representation of preferences, for each $i \in I$ and each $c_i \in \mathbb{R}_+$, $h_i(c_i) = f_i\left(u\left(c_i, \bar{\ell}_i\right)\right)$. Since $f_i(u(c_i, \ell_i))$ is strictly concave, also h_i is strictly concave. □

Next, for each $c_i \in \mathbb{R}_+$, denote $h_i'\left(c_i^-\right)$ and $h_i'\left(c_i^+\right)$ the left and right first-order derivatives of h_i at c_i. Let $\bar{A}$ be the set of allocations $a \in A$ such that $\ell_i = \bar{\ell}_i$ for each $i \in I$. Then, for each $a \in \bar{A}$, $W(a) = \sum_{i\in I} h_i(c_i)$. Let the choice correspondence C be defined as follows: for each $k \geq 0$, $C(k)$ is the set of consumption vectors $(c_i)_{i\in I}$ with $\sum_{i\in I} c_i \leq k$ that maximize W. Let $\bar{k} = \sum_{i\in I} \bar{c}_i$. The following steps characterize the properties of C (abusing notation, we shall use C to denote the choice function).

Step 2. *The choice correspondence C satisfies the following properties:*

1. *it is non-empty, single-valued, and continuous with respect to k;*
2. *it is strictly monotonic, $k > k'$ implies $C(k) \gg C(k')$;*
3. $C\left(\bar{k}\right) = (\bar{c}_i)_{i\in I}$;
4. $(c_i)_{i\in I} = C(k)$ *implies* $c_i > c_j \Longleftrightarrow \bar{c}_i > \bar{c}_j$ *for each* $i, j \in I$;
5. *for* $k \leq \bar{k}$, $(c_i)_{i\in I} = C(k)$ *implies* $c_i - c_j < \bar{c}_i - \bar{c}_j$ *for each* $i, j \in I$.

Proof. 1. Since W is increasing, continuous, and striclty concave, also $\sum_{i\in I} h_i(c_i)$ is. Thus, the choice correspondence C is non-empty, single-valued, and continuous with respect to k.

2. Let $(c_i)_{i\in I} = C(k)$ and $(c_i')_{i\in I} = C(k')$. By contradiction assume $k > k'$ and $C(k) \not\gg C(k')$. Then, there exists a pair of individuals $i, j \in I$ such that $c_i' \leq c_i$ and $c_j' > c_j$. At the optima, $h_i'\left(c_i^-\right) \geq h_j'\left(c_j^+\right)$ and $h_i'\left(c_i^+\right) \leq h_j'\left(c_j^-\right)$ and, similarly, $h_i'\left(c_i'^-\right) \geq h_j'\left(c_j'^+\right)$ and $h_i'\left(c_i'^+\right) \leq h_j'\left(c_j'^-\right)$. By strict concavity, $h_i'\left(c_i'^-\right) \geq h_i'\left(c_i'^+\right) \geq h_i'\left(c_i^-\right) \geq h_i'\left(c_i^+\right)$ and $h_j'\left(c_j^-\right) \geq h_j'\left(c_j^+\right) > h_j'\left(c_j'^-\right) \geq h_j'\left(c_j'^+\right)$. Combining these conditions leads to the following impossibility:

$$h_i'\left(c_i^-\right) \geq h_j'\left(c_j^+\right) > h_j'\left(c_j'^-\right) \geq h_i'\left(c_i'^+\right) \geq h_i'\left(c_i^-\right).$$

3. By contradiction, assume $(\bar{c}_i)_{i\in I} \neq C\left(\bar{k}\right) \equiv (c_i)_{i\in I}$. Then, $\sum_{i\in I} h_i(c_i) > \sum_{i\in I} h_i(\bar{c}_i)$. At $(c_i)_{i\in I}$, the tax burden of each individual $i \in I$ is $b_i \equiv \bar{c}_i - c_i$. Since $\bar{k} = \sum_i \bar{c}_i$, $\sum_{i\in I} b_i = 0$. Let $\overrightarrow{b}$ be the reordered vector of tax burdens of individuals: $\overrightarrow{b} \equiv \left(b_{(1)}, ..., b_{(|I|)}\right)$ is such that $b_{(1)} \leq b_{(2)} \leq ... \leq b_{(|I|)}$, where (i) is the

individual that, after permutation, occupies the i'th place in the order of tax burdens. Since $(\bar{c}_i)_{i\in I} \neq (c_i)_{i\in I}$, $\overrightarrow{b} \neq 0$.

Next, we apply *tax solidarity* a finite number of times to show that $(\bar{c}_i)_{i\in I}$ is socially at least as desirable as $(c_i)_{i\in I}$, leading to a contradiction.

For each $t = 1, ..., (|I|)$. Define $c^t \equiv (c_i^t)_{i\in I}$ and let $\overrightarrow{b}^t$ be the corresponding reordered vector of tax burdens. Compare the lowest and largest tax burdens. Two cases can emerge: either $\left|b_{(1)}^t\right| \leq \left|b_{(|I|)}^t\right|$ or $\left|b_{(1)}^t\right| > \left|b_{(|I|)}^t\right|$. If $\left|b_{(1)}^t\right| \leq \left|b_{(|I|)}^t\right|$, construct a new consumption allocation $c^{t+1} \equiv \left(c_i^{t+1}\right)_{i\in I}$ by introducing a consumption transfer $\varepsilon^t \equiv b_{(1)}^t$ from individual (1) to $(|I|)$: after the transfer, individual (1) with have consumption $c_{(1)}^{t+1} = \bar{c}_{(1)}$ and zero tax burden, while individual $(|I|)$ will have consumption $c_{(|I|)}^{t+1} = c_{(|I|)}^t + \varepsilon$. If instead $\left|b_{(1)}^t\right| > \left|b_{(|I|)}^t\right|$, perform a consumption transfer $\varepsilon^t \equiv b_{(I)}^t$ from individual (1) to $(|I|)$: after the transfer, individual (1) with have consumption $c_{(1)}^{t+1} = c_{(1)}^t - \varepsilon$, while individual $(|I|)$ with have consumption $c_{(|I|)}^{t+1} = \bar{c}_{(|I|)}$ and zero tax burden. Let $c_i^{t+1} = c_i^t$ for all $i \in I$ such that $(i) \neq (1), (|I|)$.

Let $c^1 \equiv (c_i)_{i\in I}$. Then, $c^{(|I|)} = (\bar{c}_i)_{i\in I}$. For each $t = 1, ..., (|I|)$, let the allocation $a^t \in A$ assign to each individual the consumption defined by c^t and labor supply defined by $\left(\bar{\ell}_i\right)_{i\in I}$. Then, for each $t = 2, ..., (|I|)$, *tax solidarity* establishes that $a^t \succsim a^{t-1}$. By the representation result in Step 1, this implies that $\sum_{i\in I} h_i\left(c_i^t\right) \geq \sum_{i\in I} h_i\left(c_i^{t-1}\right)$. Thus, also $\sum_{i\in I} h_i\left(\bar{c}_i\right) \geq \sum_{i\in I} h_i\left(c_i\right)$. This is a contradiction.

4. The proof is similar to that of 3, where *fair reward* is applied.

5. The proof is similar to that of 3, where *fair burden* is applied. □

We next constructs the function $S : \mathbb{R}_+ \times \mathbb{R}_{++} \to \mathbb{R}$ and then verify that it is a sacrifice function.

First, for each $i \in I$ and each $k \geq 0$, let $S\left(c_i; \bar{c}_i\right) = \bar{k} - k$ if and only if $(c_i)_{i\in I} = C(k)$.

Second, we complete the sacrifice function "linearly" for non-observed levels of laissez-faire consumption (when $y \neq \bar{c}_i$ for each i). Reorder individuals in increasing order of laissez-faire consumption, i.e., $(i) \leq (j)$ if $\bar{c}_i \leq \bar{c}_j$. Let $\bar{c}_0 = 0$. Set $y \in \mathbb{R}_{++}$. Then, two cases can emerge: either there exists $i \in I$ such that $\bar{c}_{(i-1)} \leq y \leq \bar{c}_{(i)}$ or $y > \bar{c}_{(|I|)}$.

Case 1. Let $i \in I$ be such that $\bar{c}_{(i-1)} \leq y \leq \bar{c}_{(i)}$ and let $\alpha \in [0, 1]$ be such that $y = \alpha\bar{c}_{(i-1)} + (1 - \alpha)\,\bar{c}_{(i)}$. Then, for each $x \in \mathbb{R}_+$, $S(x; y) = \bar{k} - k$ if and only if $x = \alpha c_{(i-1)} + (1 - \alpha)\, c_{(i)}$ where $S\left(c_{(i-1)}; \bar{c}_{(i-1)}\right) = S\left(c_{(i)}; \bar{c}_{(i)}\right) = \bar{k} - k$.

Case 2. Let (i) be such that $\bar{c}_{(|I|)} - \bar{c}_{(i)}$ is smallest but positive (this is generally $(|I| - 1)$, but could be different if $\bar{c}_{(|I|)} = \bar{c}_{(|I|-1)}$). Let $\alpha > 1$ be such that $\left(y - \bar{c}_{(i)}\right) = \alpha\left(\bar{c}_{(|I|)} - \bar{c}_{(i)}\right)$. Then, for each $x \in \mathbb{R}_+$, $S(x; y) = \bar{k} - k$ if and only if $\left(x - c_{(i)}\right) = \alpha\left(c_{(|I|)} - c_{(i)}\right)$ where $S\left(c_{(i)}; \bar{c}_{(i)}\right) = S\left(c_{(|I|)}; \bar{c}_{(|I|)}\right) = \bar{k} - k$.

<u>Step 3.</u> *The function S is a sacrifice function. That is, S satisfies the following conditions:*

1. *a) decreasing in the first argument, b) increasing in the second argument, and c) continuous;*
2. *$x = y$ implies $S(x; y) = 0$; and*
3. *$S(x; y) = S(x'; y') > 0$ implies $|x - x'| \leq |y - y'|$.*

Proof. 1a) For each i, the function $S(c_i; \bar{c}_i)$ is decreasing in c_i by strict monotonicity of $C(k)$: more precisely, let $k < k'$; then, $(c_i)_{i \in I} = C(k) \ll C(k') = (c_i')_{i \in I}$, $c_i < c_i'$, and $S(c_i; \bar{c}_i) = \bar{k} - k > S(c_i'; \bar{c}_i) = \bar{k} - k'$. For each $y \in \mathbb{R}_+$, $S(x; y)$ is decreasing in x as it is constructed as a linear combination of functions $(S(c_i; \bar{c}_i))_{i \in I}$ which are decreasing in the first variable.

1b) Property 4 of Step 3 states that: $(c_i)_{i \in I} = C(k)$ implies $c_i > c_j \Longleftrightarrow \bar{c}_i > \bar{c}_j$ for each $i, j \in I$. By construction of S, this implies that $S(x, y) = S(x', y')$ with $y < y'$ if and only if $x < x'$. Since S is decreasing in the first argument, $S(x, y) < S(x, y')$.

1c) Since $C(k)$ is continuous in k, for each i, the function $S(c_i; \bar{c}_i)$ is continuous in c_i. Continuity of S then follows by contruction.

2) By construction, $S(\bar{c}_i, \bar{c}_i) = \bar{k} - \bar{k} = 0$ for each $i \in I$. Now, for each $y \in \mathbb{R}_{++}$, either there exists $i \in I$ such that $\bar{c}_{(i-1)} \leq y \leq \bar{c}_{(i)}$ or $y > \bar{c}_{(|I|)}$. In the first case, $S(y, y) = 0$ since, by definition of S, $S\left(\bar{c}_{(i-1)}; \bar{c}_{(i-1)}\right) = S\left(\bar{c}_{(i)}; \bar{c}_{(i)}\right) = 0$ and $y = \alpha \bar{c}_{(i-1)} + (1 - \alpha) \bar{c}_{(i)}$ for some $\alpha \in [0, 1]$. In the second case, let (i) be such that $\bar{c}_{(|I|)} - \bar{c}_{(i)}$ is smallest but positive. Then, $S(y; y) = 0$ since, by definition of S, $S\left(\bar{c}_{(i)}; \bar{c}_{(i)}\right) = S\left(\bar{c}_{(|I|)}; \bar{c}_{(|I|)}\right) = 0$ and $\left(y - \bar{c}_{(i)}\right) = \alpha\left(\bar{c}_{(|I|)} - \bar{c}_{(i)}\right)$ for some $\alpha > 1$.

3) By contradiction, let $k \equiv S(x; y) = S(x'; y') > 0$ and $|x - x'| > |y - y'|$. Without loss of generality, let $x > x'$ and $y > y'$. By construction, the implicit function $S(x, y) = k$ is piecewise linear: it may change slope only in correspondence to $y = \bar{c}_i$ with $(i) = 2, ..., (|I|) - 1$. By the mean value theorem, $x - x' > y - y'$ implies that there exists $i, j \in I$ such that $c_i - c_j > \bar{c}_i - \bar{c}_j$ with $S(c_i; \bar{c}_i) = S(c_j; \bar{c}_j) = k$. Clearly, c_i and c_j belong to $(c_m)_{m \in I} = C(k)$. Thus, $c_i - c_j > \bar{c}_i - \bar{c}_j$ is a violation of *fair reward* (as shown above). □

The proof is completed by the following step, which shows that the Pareto functions $P_i = f_i$ are consistent with the sacrifice function S.

<u>*Step 4.*</u> *For each $i \in I$, let the Pareto function of i be P_i such that $P_i(u(c_i, \ell_i)) \equiv h_i(e_i(c_i, \ell_i))$. The Pareto functions $(P_i)_{i \in I}$ are consistent with S. That is, for each $i \in I$, the social marginal welfare weights β_i satisfy $\beta_i\left(c_i, \bar{\ell}_i\right) = g(S(c_i; \bar{c}_i)) > 0$, where g is a real-valued increasing function, equal across individuals, and such that the Pareto-transformed welfares of individuals $(P_i(u(\cdot, \cdot)))_{i \in I}$ are strictly concave in their arguments.*

Proof. Strict concavity of $P_i(u(\cdot,\cdot))$ immediately follows from Step 1. Strict concavity and efficiency also imply that g is increasing and such that social marginal welfare weights are positive.

Finally, we show that g is equal across individuals. By contradiction, assume not. Then, there exists $i, j \in I$ and $c_i, c_j \in \mathbb{R}_+$ with $S(c_i; \bar{c}_i) = S(c_j; \bar{c}_j)$ such that $\beta_i(c_i, \bar{\ell}_i) \neq \beta_j(c_j, \bar{\ell}_j)$. By construction, c_i and c_j belong to $(c_m)_{m \in I} = C(k)$ for some $k \geq 0$. However, $\beta_i(c_i, \bar{\ell}_i) \neq \beta_j(c_j, \bar{\ell}_j)$ imply that $\frac{\partial h_i(c_i, \bar{\ell}_i)}{\partial c_i} \neq \frac{\partial h_j(c_j, \bar{\ell}_j)}{\partial c_j}$. This contradicts that $C(k)$ maximizes social welfare W (from Step 1) among the vectors $(c'_m)_{m \in I}$ such that $\sum_{m \in I} c'_m \leq k$. □

□

4.4 The proportional-sacrifice social welfare function

In Section 2, we discussed a special case of the family of equal-sacrifice social welfare function. Two restrictions characterize this family. First, the sacrifice function S is "proportional": $S(c; \bar{c}) = \frac{\bar{c} - c}{\bar{c}}$. Second, the social attitude towards inequality in sacrifice is captured by the parameter $\gamma > 0$. The proportional-sacrifice social welfare function (with constant relative inequality aversion) is characterized next.

The following principle strenghtens *fair burden* to deal with situations in which the sacrifice of individual i (who is better-off at the laissez-faire allocation) is relatively too small as compared to some other individual j. In these cases, a regressive transfer from j to i cannot improve social welfare.

Fair relative burden: *For each pair $a, a' \in A$, each pair $i, j \in I$ with $\bar{c}_i \geq \bar{c}_j$, and each pair $\alpha, \varepsilon > 0$, such that:*

- $c'_i + \varepsilon = c_i > \alpha \bar{c}_i$ *and* $\alpha \bar{c}_j > c_j = c_j - \varepsilon$;
- $\ell_i = \ell'_i = \bar{\ell}_i$ and $\ell_j = \ell'_j = \bar{\ell}_j$; and
- $(c_k, \ell_k) = (c'_k, \ell'_k) = (\bar{c}_k, \bar{\ell}_k)$ *for each* $k \in I / \{i, j\}$;

then, $a \succsim a'$.

The next principle strengthens *fair reward* and disciplines situations in which the sacrifice of individual i (who is better-off at the laissez-faire allocation) is relatively too large as compared to some other individual j. In these cases, a regressive transfer from i to j cannot improve social welfare.

Fair relative reward: *For each pair $a, a' \in A$, each pair $i, j \in I$ with $\bar{c}_i \geq \bar{c}_j$, and each pair $\alpha, \varepsilon > 0$, such that:*

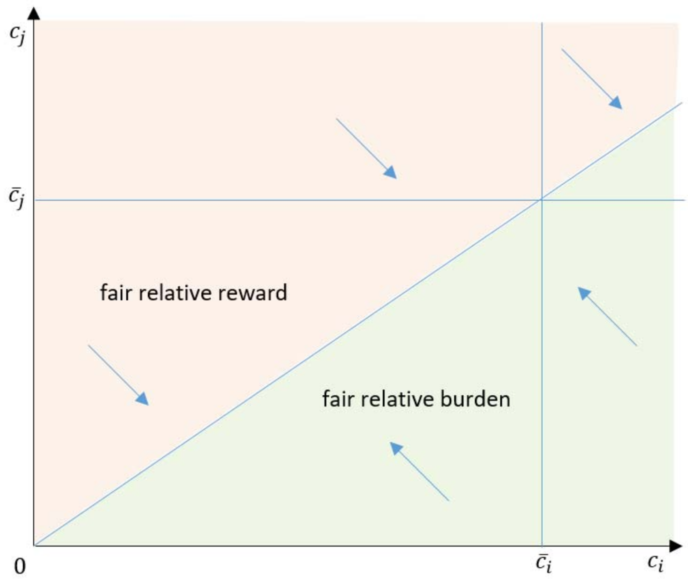

Figure 4: Relative equal-sacrifice principles.

- $c_i' + \varepsilon = c_i < \alpha \bar{c}_i$ *and* $\alpha \bar{c}_j < c_j = c_j - \varepsilon$;
- $\ell_i = \ell_i' = \bar{\ell}_i$ and $\ell_j = \ell_j' = \bar{\ell}_j$; and
- $(c_k, \ell_k) = (c_k', \ell_k') = (\bar{c}_k, \bar{\ell}_k)$ *for each* $k \in I / \{i, j\}$;

then, $a \succsim a'$.

We can represent these principles in the following graph.

Finally, we introduce a weak form of ratio-scale invariance with respect to the tax consumption of individuals. Assume all individuals work at laissez-faire labor supply. Then, proportional changes in the consumptions does not affect how society ranks two allocations.

Ratio-scale invariance: *For each* $a, a', a'', a''' \in A$ *and each* $\kappa > 0$ *such that:*

- $\ell_i = \ell_i' = \ell_i'' = \ell_i''' = \bar{\ell}_i$ for each $i \in I$;
- $c_i = \kappa c_i''$ *and* $c_i' = \kappa c_i'''$ for each $i \in I$;

then, $a \succsim a'$ *if and only if* $a'' \succsim a'''$.

The relationships between the above axioms are summarized by the following result.

Lemma 2. *For a social welfare* $\succsim$*:*

1. *fair relative burden implies fair burden;*
2. *fair relative reward implies fair reward;*
3. *fair relative burden and fair relative reward imply tax solidarity;*
4. *fair burden and ratio-scale invariance imply fair relative burden;*
5. *fair reward and ratio-scale invariance imply fair relative reward;*
6. *tax solidarity and ratio-scale invariance imply fair relative burden and fair relative reward.*

We can now characterize the proportional-sacrifice social welfare function (with constant relative inequality aversion). The proof is similar to Piacquadio (2019) and is omitted.

Theorem 2. *Social welfare $\succsim$ satisfies efficiency, inequality aversion, separability, tax solidarity and ratio-scale invariance if and only if it can be represented by a social welfare function W^p such that for each $a \in A$:*

$$W^p(a) \equiv \sum_{i \in I} P_i(u(c_i, \ell_i)),$$

where, for each $i \in I$, P_i satisfies

$$\frac{\partial}{\partial c_i} P_i\left(u\left(c_i, \bar{\ell}_i\right)\right) \equiv \beta_i\left(c_i, \bar{\ell}_i\right) = \left(\frac{c_i}{\bar{c}_i}\right)^{-\gamma},$$

for some $\gamma > 0$.

This result states that, for the proportional-sacrifice social welfare function, the Pareto functions $(P_i)_{i \in I}$: *(i)* need to be consistent with respect to the proportional sacrifice function $S(c, \bar{c}) = \left(\frac{\bar{c} - c}{\bar{c}}\right)$; and *(ii)* their derivatives at laissez-faire labor supply (defining the social marginal welfare weights) need to be a power transformation of $S(c, \bar{c}) - 1$.

5 Conclusions

The choice of the optimal income redistribution is a key and open question in public economics. The answer requires combining a positive model of the economy—capturing the behavioral choices of individuals—with normative considerations—reflecting ethical principles about how to compare benefits and losses of individuals. However, since the seminal contribution of Mirrlees (1971), the literature has mostly advanced by considering richer and more complex models of the economy, while the normative criterion was generally set to be the utilitarian social welfare function.

The utilitarian criterion is subject to a number of criticisms, even in the simple Mirrleesian framework where individuals have the same preferences.[10] Among those, Edgeworth (1987) highlights that the utilitarian criterion leads to large incentives to redistribute: with inelastic earnings, the optimal taxation policy is to tax income at 100% and redistribute the tax revenues equally across individuals. Relatedly, Mirrlees (1974) showed that the first best policies lead to the slavery of the talented: high-skill individuals are taxed so much as to enjoy a lower utility than low-skill individuals. A partial solution has been to introduce flexible Pareto weights. However, we share Piketty and Saez (2013)'s view that "the Pareto weight approach is too general to deliver practical policy prescriptions in most cases" (p.393). Another recent solution is to abandon the social welfare function approach and directly postulate ethical views in terms of social marginal welfare weights, defining the importance that society attaches to a dollar of consumption change for each individual (Saez and Stantcheva, 2016).

In this paper, we revisit an old and well-known theory of fairness in taxation. As Mills (1848, Ch.2) writes: "Equality of taxation ... as a maxim of politics, means equality of sacrifice. It means apportioning the contribution of each person towards the expenses of government so that he shall feel neither more nor less inconvenience for his share of the payment than every other person experiences from his. This standard, like other standards of perfection, cannot be completely realized; but the first object in every practical discussions should be to know what perfection is." Earlier contributions have characterized equality of sacrifice as a standard of perfection (Young, 1988, 1990). However, as a standard of perfection, its application to second-best settings—such as the Mirrleesian model—leads to violations of the Pareto principle (Berliant and Gouveia, 1993; da Costa and Pereira, 2014).

Our results show that it is possible to construct a social welfare function that flexibly combines fairness considerations inspired by the equal-sacrifice principle together with concerns for efficiency. The main result of the paper is the axiomatic characterization of the *equal-sacrifice social welfare function*. When abiding by our axioms, society needs to choose: *(i)* how to measure the sacrifice of individuals; and *(ii)* how much to avert inequality of sacrifices.

We show that these choices single out social marginal welfare weights that are directly expressed in terms of the level of sacrifice of individuals and are increasing with respect to the skill level of individuals. This reduces the counterintuitive instances of redistributive motive of utilitarian first-best policies and, thus, may have a potentially large impact on the optimal taxation policy. We show that, for a two-types model, the

[10] This assumption is generally used to assume away the issue of interpersonal comparisons of utilities.

second-best policies differ most from the utilitarian ones when the budget requirement of the government is not too large and when the elasticity of labor supply is small.

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References

[1] Berliant, M., and Gouveia, M., 1993. Equal sacrifice and incentive compatible income taxation. *The Journal of Public Economics*, 51(2), 219-240.

[2] Cappelen, A.W., Hole, A.D., Sørensen, E.Ø. and Tungodden, B., 2007. The pluralism of fairness ideals: An experimental approach. *The American Economic Review*, 97(3), pp.818-827.

[3] d'Aspremont, C. and Gevers, L., 1977. Equity and the informational basis of collective choice. *The Review of Economic Studies*, 44(2), pp.199-209.

[4] da Costa, C.E. and Pereira, T., 2014. On the efficiency of equal sacrifice income tax schedules. *The European Economic Review*, 70, pp.399-418.

[5] Edgeworth, F. 1897. "The Pure Theory of Taxation." *The Economic Journal* 7: 46–70.

[6] Fleurbaey, M. and Maniquet, F., 2018. Optimal income taxation theory and principles of fairness. *The Journal of Economic Literature*, 56(3), pp.1029-79.

[7] Jacquet, L. and van de Gaer, D., 2011. A comparison of otpimal tax policies when compensation and responsibility matter," *The Journal of Public Economics*, 95, 1248-1262.

[8] Mankiw, N. G., Weinzierl, M., & Yagan, D. (2009). Optimal taxation in theory and practice. *Journal of Economic Perspectives*, 23(4), 147-74.

[9] Maskin, E., 1978. A theorem on utilitarianism. *The Review of Economic Studies*, 45(1), pp.93-96.

[10] Mill, J. S., 1848. *Principles of Political Economy with Some of their Applications to Social Philosophy*. Reprint, Ashley, W. J., ed., 1929, London: Longmans, Green and Co.

[11] Mirrlees, J.A., 1971. An exploration in the theory of optimum income taxation. *The Review of Economic Studies*, 38(2), pp.175-208.

[12] Mirrlees, J.A., 1974. Notes on welfare economics, information and uncertainty," in *Essays in Equilibrium Behavior under Uncertainty* (M. Balch, D. McFadden, and S. Wu, Eds.). Amsterdam: North-Holland, 243-258.

[13] Musgrave, R. A. 1959. *Theory of public finance; a study in public economy*. New York: McGraw-Hill.

[14] Piacquadio, P.G., 2017. A fairness justification of utilitarianism. *Econometrica*, 85: 1261-1276.

[15] Piacquadio, P.G., 2019. The Ethics of Intergenerational Risk, *mimeo*.

[16] Piketty, T. and Saez, E., 2013. Optimal labor income taxation. In *Handbook of public economics* (Vol. 5, pp. 391-474). Elsevier.

[17] Saez, E. and Stantcheva, S., 2016. Generalized social marginal welfare weights for optimal tax theory. *The American Economic Review*, 106(1), pp.24-45.

[18] Stiglitz, J. E. 1982. Self-selection and Pareto efficient taxation. *The Journal of Public Economics*, 17(2), 213-240.

[19] Weinzierl, M., 2014. The promise of positive optimal taxation: normative diversity and a role for equal sacrifice. *The Journal of Public Economics*, 118, pp.128-142.

[20] Young, H.P., 1988. Distributive justice in taxation. *The Journal of Economic Theory*, 44(2), pp.321-335.

[21] Young, H.P., 1990. Progressive taxation and equal sacrifice. *The American Economic Review*, 80(1), pp.253-266.