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ABSTRACT

We study contracting between a public good provider and users with private valuations of the good. We show that, once the provider extracts the users' private information, she benefits from manipulating the information received from one user when communicating with another user. We derive conditions under which such manipulation determines the direction of distortions in public good provision. If the provider is non-manipulative, the public good is always underprovided, whereas overprovision occurs with a manipulative provider. With overprovision, not only high-valuation users, but also low-valuation users may obtain positive rents—users may prefer facing a manipulative provider.

JEL Classification: D82, D86, H41

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1 Introduction

Since Samuelson's pioneering work (1954), analyzing distortions in the provision of public goods has become a classical topic in economics. The conventional understanding is that the users' incentives result in an "underprovision"—the direction of distortion in the public good provision is downward. For instance, Comes and Sandler (1996) note that "the tendency for public goods to be provided at suboptimal levels is a celebrated result in public economics." Casual observations, however, indicate that, in real life, some public goods and services are often "overprovided"—the direction of distortion in the public good provision is upward, instead of downward.

As an example, consider the class-action lawsuits in which a group of victims consolidate their claims into a single lawsuit. A criticism against such collective litigation procedures is that they lead to an excessive amount of litigation.[1] This, in fact, is one of the main reasons that led the United States Congress to pass the Class Action Fairness Act of 2005, removing the class-action lawsuits from the jurisdiction of state courts which are deemed to be over-friendly to lawyers representing plaintiffs. Industrial lobbying is another instance for over-provision. As pointed out by studies in political science, the amount of lobbying can exceed what would be efficient for the industry.[2]

The services in the examples above are public goods, since a single service by the provider (a lawyer or a lobbyist) benefits multiple users (clients) who are in the same group. From an economic perspective, these excessive public good provisions are rather puzzling, because according to the standard theory, public goods are expected to be underprovided.

The objective of this paper is to identify a new economic mechanism that results in overprovision of a public good rather than underprovision. We study this mechanism in an agency framework of public good provision with private information. In our model, a provider (the principal) produces a public good for the consumption of multiple users (agents) in exchange for monetary payments from them. Each user's valuation for the public good is his private information, and after all users report their valuations to the provider, she produces the good according to the collective valuation reported by the users.

As in the standard model of screening, a user with a high valuation for the good receives an information rent not to misrepresent his true valuation. In order to reduce this information rent, the provider's second-best contract distorts the size of the public good

[1] See Ulen (2011) among others.

[2] See Ainsworth (1993) for examples of trade association lobbying efforts which do not necessarily reflect the demand from their general membership.

downward. Except for the case where every user claims that his valuation of the public good is high, the provider lowers provision of the public good from the efficient level in the optimal contract. This is in line with the traditional result in public good provision—the public good is underprovided.

This result, however, is under the assumption that, while the users of the public good are opportunistic, the provider is not. Such an assumption seems to be naive. While each user has private information about his valuation of the public good, at the point of producing the public good, the provider is the only party that has information about all users' *collective valuation* of the good. If possible (and profitable), the public good provider may seize the opportunity to misrepresent the collective information sent by the users, by falsifying the information received from one user when communicating with another user.

We take the public good provider's incentive to manipulate the collective information into account in our model. We identify that, when the provider announces the collective valuations to the users, she has an incentive to exaggerate it—in particular, the provider has an incentive to make a user with a low value for the public good think that the other users' valuations are high. The public good provider's incentive to manipulate information is anticipated by the users when contracting with her. Such an anticipation provides a high-valuation user with a stronger incentive to misrepresent his own valuation. In other words, there is a tension between a user's incentive to misrepresent his private information and the provider's incentive to misrepresent her collective information. To ease this tension, the public good provider must convince the users that she would not manipulate the collective information reported by them.

One way to convince the users that the public provider will not falsify their reports is designing a *bunching* contract that pools different collective information at the same level of the public good. The provider will not have a reason to manipulate if the public good and the payment levels do not change with her manipulation. More interestingly, the provider can also eliminate her incentive to manipulate by inflating the provision of the public good and leaving a positive rent to users with low valuations. Our result shows that, depending on the likelihood of different user valuations, it is optimal for the provider to implement bunching or overprovision to convince the users that she will not manipulate.

Manipulability of information may have unexpected winners and losers. The provider must convince the users that she will not falsify the reported collective information, and this consideration imposes an additional constraint on the provider's design problem on top of the standard incentive conditions. Modifying the second-best contract to satisfy this new constraint entails a lower payoff for the provider. An examination of how the provider modifies the second-best contract also reveals the effects of the manipulation opportunities on the users' payoffs. Larger public good sizes lead to larger information rents for high-valuation users. In addition, even low-valuation users may end up with positive rent under the optimal manipulation-proof contract. In other words, while the public good provider is worse off for having the opportunity to manipulate, the users themselves may benefit from the provider's manipulability of the information that they report to the provider. This implies that the users want to deal with a manipulative public good provider.

The remainder of the paper is organized as follows. The next section discusses the related literature. A model of public good provision is presented in Section 3. In Section 4, we outline the second-best contract that maximizes the provider's expected payoff in the absence of her manipulation opportunities and show that the public good is underprovided in this case. In Section 5, we show that the second-best contract may be prone to manipulation by the provider and we state our results that characterize the optimal manipulation-proof contract. We discuss the welfare effects of the public good provider's manipulability of information in Section 6. Section 7 provides a numerical example of our results. Section 8 concludes with some remarks. Proofs are relegated to Appendix.

2 Related Literature

It is a classical result that public goods are underprovided. Under symmetric information and voluntary contributions, this underprovision result obtains when comparing the non-cooperative equilibrium outcome to the cooperative one (e.g. Bergstrom et al. 1986).[3] Under asymmetric information and voluntary participation, the underprovision arises from a trade-off between efficiency and information rents (e.g. Green and Laffont 1977 or Laffont 1988).[4]

Consequently, the literature views overprovision as an 'anomaly'. The theoretical literature has studied this anomaly mainly under symmetric information. This paper's contribution is to provide a rationale for overprovision that is due to 'endogenous private

[3] Cheikbossian and Sand-Zantman (2011) show that the underprovision result even persists with repeated interactions that involve imperfect monitoring, while Teoh (1997) shows that information disclosure worsens the free-riding problem that underlies the underprovision.

[4] With forced, involuntary participation and asymmetric information, Clarke (1971), and Groves (1973) demonstrate that the efficient level of the public good can be implemented in dominant strategies. Under Bayesian incentive compatibility conditions, d'Aspremont and Gerard-Varet (1979) show that, with forced participation, such an outcome can even be achieved with budget-balanced transfers.

information'—information manipulation by a public good provider.[5]

Focusing on the preferences of economic agents, Buchanan and Kafoglis (1963), Diamond and Mirrlees (1973) and Sadka (1977) discuss necessary conditions on those preferences for an overprovision to arise. There are studies considering strategic tax policies, demonstrating that overprovision may arise when there is tax exporting (e.g. Gerking and Mutti 1981), when public goods are inputs in production (e.g. Dhillon et al. 2007), or when policy makers have Leviathan tendencies (e.g., Mintz and Tulkens, 1996). All explanations in these studies abstract from private information.

Our modeling of manipulation is similar to Dequiedt and Martimort (2016), Akbarpour and Li (2018), and Celik et al. (2019). All these papers focus on manipulation in environments with private goods. Considering a principal who can falsify received information in a multi-agent framework with correlated private information, Dequiedt and Martimort (2016) point out that full rent extraction through yardstick competition is no longer possible.[6] As a result, simple sell-out contracts are optimal in a vertical framework of an upstream manufacturer dealing with a retailing network under a wide range of settings.

Akbarpour and Li (2018) study manipulation-proof auction design. They demonstrate that the sealed-bid second-price auction is susceptible to undetectable manipulation, because the auctioneer can overstate the second-highest bid to increase the payment from the winner. In contrast, no such manipulating incentive arises for the first-price auction. They, moreover, develop a general formalization of undetectable manipulation by a mechanism designer, which also provides a micro foundation of the manipulation-proofness constraints that we apply in our framework of public goods. The manipulation opportunities in a public good setting are, however, more limited than under private consumption, because the provided level of a public good is naturally observed and consumed by all users in the group. As we show, this limited form of manipulability has nevertheless an adverse effect on the principal's abilities to extract rents. They are economically significant in that, depending on parameter constellations, they lead to an overprovision of public goods.

[5] Bierbrauer and Winkelmann (2019) study public good overprovision from a mechanism design perspective. They do not, however, consider the public good provider's endogenous private information.

[6] Crémer and McLean (1985, 1988) show that, when the agents' types are correlated, a non-manipulative principal could fully extract the agents' information rents by conditioning her transaction with one agent to the information transmitted by another agent.

3 Public Good Provision Model

We present a model of public good provision with a provider (the principal) and two users (the agents). The provider's cost of producing size $q \geq 0$ of the public good is given by $c(q)$, where $c(\cdot)$ is a continuously differentiable, strictly increasing, and strictly convex function. We assume that $c(\cdot)$ satisfies the Inada conditions: $c(0) = 0$, $\lim_{q\to 0} c'(q) = 0$, and $\lim_{q\to\infty} c'(q) = \infty$. User $k \in \{1, 2\}$ values q units of the public good by $\theta^k q$. The size of the public good q is verifiable and contractible, whereas each user's valuation parameter θ^k is his private information (his type). The types are independently and identically distributed. Specifically, a user has the high valuation θ_h for the public good with probability $\varphi \in (0, 1)$, and the low valuation $\theta_l > 0$ with probability $1 - \varphi$, where $\Delta\theta \equiv \theta_h - \theta_l > 0$.

In line with the examples in the introduction, we consider the public good provider as a profit maximizer.[7] Accordingly, the provider's and user k's payoffs are respectively

$$\sum_{k=1}^{2} p^k - c(q) \quad \text{and} \quad \theta^k q - p^k,$$

where p^k is the payment from user k to the public good provider.

The collective value of the public good depends on the realized types of the two users. We are either in the *high-value* state (H) where both users have a high valuation for the public good, or in the *low-value* state (L) where both users have a low valuation, or in the *intermediate-value* state (M) where the two users have different valuations. For each of these collective-valuation states, we can find the first-best sizes of the public good that maximizes the sum of the provider's and the users' payoffs. The first-best public good sizes satisfy the *Samuelson condition*—the marginal cost of producing this first-best level is equal to the sum of the marginal values:

$$c'(q_H^*) = 2\theta_h, \quad c'(q_M^*) = \theta_h + \theta_l, \quad c'(q_L^*) = 2\theta_l.$$

If the public good provider could directly observe the users' valuations, she would choose to produce these first-best quantities to maximize the benefits of the public good net of its production costs. However, because these valuations are private information for the users, the provider has to give them the incentive to reveal their valuations truthfully. For this purpose, the provider offers a contract $\mathcal{C}$ that conditions the size of the public good and the payments from the users on their reports about valuations. In what follows, we denote by

[7] Our qualitative results remain unchanged if the provider is modeled as a welfare-maximizing government raising distortionary taxes to finance the good's production (as in Laffont and Tirole, 1993).

p_{ij} the payment charged to a user of type $i \in \{h, l\}$, paired with a user of type $j \in \{h, l\}$. Similarly, q_γ is the public good size when the users' reports indicate the collective-valuation state as $\gamma \in \{H, M, L\}$. Hence, a contract $\mathcal{C}$ is a collection of payments and public good sizes as below:

$$\mathcal{C} \equiv \{(p_{hh}, q_H),\ (p_{lh}, q_M),\ (p_{hl}, q_M),\ (p_{ll}, q_L)\}.$$

Finally, we assume that each user has an option to *opt out*, after learning the level of the public good and the required payment to the provider. If a user chooses to opt out, then the game ends without any public good provision and payments, so that all parties receive their reservation payoffs of zero.

The timing of the interaction is summarized as follows:

1. The public good provider offers contract $\mathcal{C}$ to the users.

2. Each user reports his valuation to the provider.

3. The provider reports the collective valuation to the users.

4. Payments are made and the public good is provided, if the users do not opt out.

In the next section, we analyze a non-manipulative public good provider, who would choose the public good and payment levels that would truthfully reflect the reported types of the users in stage 3. This benchmark case leads to the standard result that, under the second-best contract, the public good is underprovided and the high-valuation users get an information rent. In the subsequent section, we observe that such an underprovision invites the provider's manipulation in stage 3.[8] We then will demonstrate that the optimal manipulation-proof contract may exhibit overprovision of the public good and leave a positive rent even for a low-valuation user.

4 Non-Manipulative Public Good Provider

We discuss the benchmark—the public good sizes in the optimal contract when the provider cannot manipulate information reported from the users. Here, the provider's constraints in contracting for the public good provision are the users' participation and truthful reports on their valuation of the public good.

[8] We postulate that it is too costly for the users to directly communicate with each other.

The public good provider's expected payoff can be written as the expected payments that she will receive from the users net of the cost of producing the public good:

$$\begin{aligned} &\varphi^2 \left[2p_{hh} - c\left(q_H\right)\right] + \varphi\left(1-\varphi\right)\left[p_{hl} + p_{lh} - c\left(q_M\right)\right] \\ &+ \left(1-\varphi\right)\varphi\left[p_{lh} + p_{hl} - c\left(q_M\right)\right] + \left(1-\varphi\right)^2\left[2p_{ll} - c\left(q_L\right)\right] \end{aligned} \tag{$\mathcal{P}$}$$

$$= \underbrace{\left\{\varphi^2 2p_{hh} + 2\varphi\left(1-\varphi\right)p_{hl} + 2\varphi\left(1-\varphi\right)p_{lh} + \left(1-\varphi\right)^2 2p_{ll}\right\}}_{\text{expected payment from the users}} - \underbrace{\left\{\varphi^2 c\left(q_H\right) + 2\varphi\left(1-\varphi\right)c\left(q_M\right) + \left(1-\varphi\right)^2 c\left(q_L\right)\right\}}_{\text{expected cost of production}}$$

As mentioned above, the non-maipulative provider chooses the contract that maximizes her expected payoff subject to two sets of constraints for the users. The source of the first set of constraints is the voluntary participation of the users. The following pairs of *participation constraints* ensure that the high and low-valuation users would not opt out of the contract after learning the intended public good and the payment levels:

$$\theta_h q_H - p_{hh} \geq 0 \quad \text{and} \quad \theta_h q_M - p_{hl} \geq 0, \tag{PC_{hj}}$$

$$\theta_l q_M - p_{lh} \geq 0 \quad \text{and} \quad \theta_l q_L - p_{ll} \geq 0. \tag{PC_{lj}}$$

In addition, the provider's contract should give the users the incentive to reveal their true types, leading to the following *Bayesian incentive compatibility* conditions for the high and low-valuation users respectively:

$$\begin{aligned} &\varphi\left(\theta_h q_H - p_{hh}\right) + \left(1-\varphi\right)\left(\theta_h q_M - p_{hl}\right) \\ \geq\ &\varphi\left(\theta_h q_M - p_{lh}\right) + \left(1-\varphi\right)\left(\theta_h q_L - p_{ll}\right), \end{aligned} \tag{IC_h}$$

$$\begin{aligned} &\varphi\left(\theta_l q_M - p_{lh}\right) + \left(1-\varphi\right)\left(\theta_l q_L - p_{ll}\right) \\ \geq\ &\varphi \max\left\{\theta_l q_H - p_{hh}, 0\right\} + \left(1-\varphi\right)\max\left\{\theta_l q_M - p_{hl}, 0\right\}. \end{aligned} \tag{IC_l}$$

The 'max' operators on the right hand side (RHS) of IC_l reflect the possibility that a low-valuation user may misrepresent his type as type θ_h, and opt out after being informed of the other user's type (thus after learning the realized size of the public good and the payment level in the contract). As shown by Matthews and Postlewaite (1989) and Forges

(1999), quitting rights of the users require such strengthening of the incentive compatibility constraints. Notice that we do not need these 'max' operators on the RHS of IC_h, because PC_{lh} and PC_{ll} imply that opting out would be suboptimal for a high-valuation user after misrepresenting his type as θ_l.

When the public good provider cannot manipulate information from the users, she offers the second-best contract that maximizes her expected payoff ($\mathcal{P}$) subject to the participation and incentive compatibility constraints presented above. We characterize the optimal outcome in the following proposition.

Proposition 1 *The optimal contract $\mathcal{C}^n$ offered by the non-manipulative provider entails the public good levels identified by the following first-order conditions:*

$$\begin{aligned} c'(q_H^n) &= 2\theta_h, \\ c'(q_M^n) &= \max\left\{\theta_h + \theta_l - \frac{\varphi}{1-\varphi}\Delta\theta,\ 0\right\}, \\ c'(q_L^n) &= \max\left\{2\theta_l - 2\frac{\varphi}{1-\varphi}\Delta\theta,\ 0\right\}. \end{aligned}$$

A high-valuation user's expected rent is strictly positive unless $q_M^n = q_L^n = 0$, and a low-valuation user gets zero rent.

Proof. See Appendix A. ■

If both users have high valuations, the optimal size of the public good coincides with the efficient one—conforming to the well-known "no distortion at the top" result of standard screening models. Incentive compatibility is the source of the information rent for the high-valuation user. As in the standard screening model, a user with high-valuation can command information rent by misrepresenting his type as the low-valuation. To prevent this user from misrepresenting his type, the provider must leave an information rent to him. The provider's optimal contract reduces the magnitude of this information rent by distorting the size of the public good downward whenever at least one of the users report a low type, i.e. whenever the collective valuation is low or intermediate. This underprovision of public good can take an extreme form of a *shut down* ($q_L^n = q_M^n = 0$), and a high-valuation user obtains no information rent. This indeed is the case when the likelihood that users are high-valuation type is sufficiently large. When that likelihood is not large enough, the public good levels are strictly positive, although they are distorted downwards.

The binding constraints in the non-manipulative provider's problem are the participation constraints of the low-valuation user, (PC_{lh}) and (PC_{ll}), and the incentive compatibility constraint of the high-valuation user, (IC_h)—see the proof of Proposition 1. The payments from the users are obtained from these binding constraints:

$$p_{lh} = \theta_l q_M, \quad p_{ll} = \theta_l q_L \quad \text{and}$$

$$\varphi p_{hh} + (1-\varphi)\, p_{hl} = \varphi[\theta_h q_H - \Delta\theta q_M] + (1-\varphi)[\theta_h q_M - \Delta\theta q_L].$$

Notice that, in the second-best contract $\mathcal{C}^n$, a high-valuation user's expected payment to the provider, $\varphi p_{hh} + (1-\varphi)\, p_{hl}$, leaves some degree of freedom in the determination of the individual levels of p_{hh} and p_{hl}. We point out this flexibility in allocation of the payment here, because it will be exploited in the next section, where manipulation by the public good provider is an issue.

Our discussion here on the public good size is summarized in the following corollary.

Corollary 1 *If the public good provider is non-manipulative, then the optimal contract entails only under-provision of the public good.*

5 Manipulative Public Good Provider

In the previous section, we derived the public good provider's optimal contract to the users under the assumption that she cannot manipulate the information revealed by the users. We now argue that this assumption is not innocuous—after learning that both users have low valuation, the provider may have an incentive to misrepresent this information in a way that is undetectable by the users.[9]

In order to see this, suppose that each user sends a message to the contract indicating that he has a low valuation for the public good. If the provider behaves truthfully and reports the collective valuation as low, the contract would commit her to producing public good level q_L in exchange for receiving payment p_{ll} from each of the users. The provider, however, would have another option if she is able to manipulate the information that she collects from one user when communicating with the other one. If she pretends to each user that the other user had reported to have a high valuation, she would instead commit to producing q_M and would receive p_{lh} from each of the users. For this manipulation not to be profitable, the provider's contract should satisfy the following *incentive compatibility constraint for the provider*:

$$2p_{ll} - c\,(q_L) \geq 2p_{lh} - c\,(q_M)\,. \qquad (PIC_L)$$

[9] Using the words in Akbarpour and Li (2019), the second-best contract is "not credible" in our model when the public good provider is manipulative.

As shown above, in the second-best contract $\mathcal{C}^n$ discussed in the previous section, the payments p_{ll} and p_{lh} are determined by the binding participation constraints (PC_{ll}) and (PC_{lh}), and they are $p_{ll}^n = \theta_l q_L^n$ and $p_{lh}^n = \theta_l q_M^n$. Accordingly, when both users' valuations are low ($\gamma = L$), the public good provider's payoff in $\mathcal{C}^n$ is $2\theta_l q^n - c(q^n)$, where $q^n \in \{q_L^n, q_M^n\}$ depending on whether or not she misrepresents the collective valuation of the public good. If the provider chooses to truthfully convey the information, then her payoff is:

$$2\theta_l q_L^n - c(q_L^n).$$

If, however, the provider falsifies the information received from one user when communicating with another user (she misrepresents the aggregate valuation as $\gamma = M$), then her payoff is:

$$2\theta_l q_M^n - c(q_M^n).$$

Notice that $2\theta_l q - c(q)$ is concave in q and it is maximized at the first-best level of the pubic good q_L^*. In the second best contract $\mathcal{C}^n$, q_L^n is set smaller than q_L^*. Notice, however, from Proposition 1 that, when the high and low valuations are equally likely ($\varphi = 1/2$), the second best level of q_M coincides with q_L^*, and thus:

$$2\theta_l q_M^n - c(q_M^n) = 2\theta_l q_L^* - c(q_L^*) > 2\theta_l q_L^n - c(q_L^n).$$

It follows from the continuity of the second-best contract that, as long as φ is sufficiently close to 1/2, the provider would prefer misrepresenting $\gamma = L$ as $\gamma = M$ under the second-best mechanism. We formalize this discussion with the following lemma.

Lemma 1 *The second-best contract $\mathcal{C}^n$ violates (PIC_L) if and only if $\varphi \in (\underline{\varphi}, \overline{\varphi})$, where $\overline{\varphi} \equiv (\theta_l + \theta_h)/(2\theta_h) > 1/2$ and $\underline{\varphi} \in (0, 1/2)$.*

Proof. See Appendix B. ■

Again, the second best contract $\mathcal{C}^n$ is prone to the public good provider's misrepresentation of the users' collective valuation—when both users report that their valuations are low to the provider, the provider has an incentive to claim to each user that the other user's valuation is high. According to Lemma 1, such an incentive of the provider is an issue for intermediate values of the likelihood that a user's valuation is high: $\varphi \in (\underline{\varphi}, \overline{\varphi})$. Within this interval, the public good level q_M is close enough to q_L^* such that the public good provider has an incentive to misrepresent $\gamma = L$ as $\gamma = M$ after she receives each user's information.

Figure 1 below illustrates the range of q_M within which the manipulation incentive of the public good provider arises.

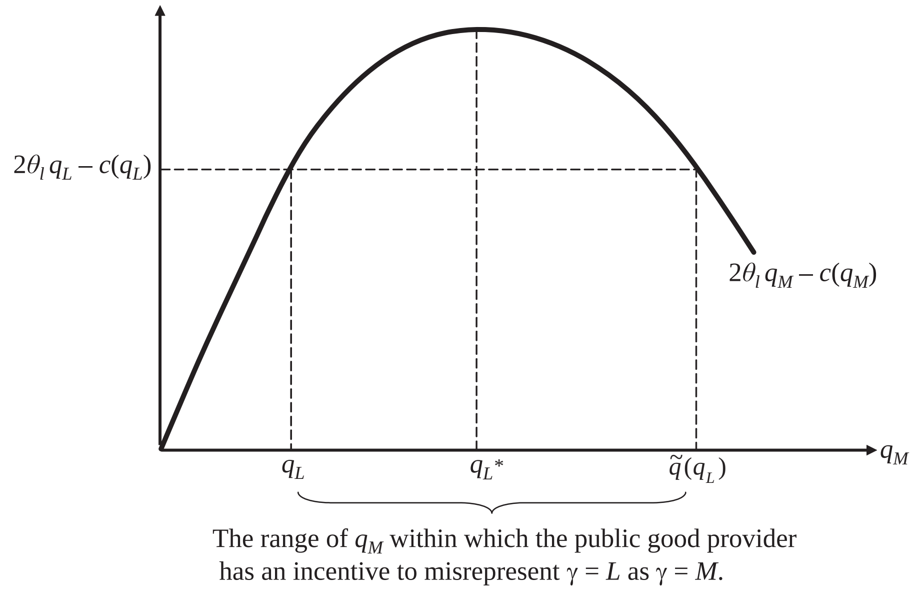

Fig 1. The public good sizes and the provider's incentive.

It is noteworthy what would go wrong for the second-best contract $\mathcal{C}^n$ when (PIC_L) constraint is violated and the public good provider indeed manipulates the information that she receives from the low-valuation users. In that case, a user would end up with a higher level of the public good (q_M^n instead of q_L^n) and make a higher payment ($p_{lh} = \theta_l q_M^n$ instead of $p_{ll} = \theta_l q_L^n$) to the provider. Notice that a low-valuation user would be indifferent to this manipulation, because the binding participation constraints guarantee that he receives no rent whether the provider manipulates or not. Thus, the provider's misrepresentation is Pareto-improving *ex post*. The provider's manipulation incentive, however, is anticipated by the users, and as a result, $\mathcal{C}^n$ becomes no longer incentive compatible for a high-valuation user. To see this, consider the following binding (IC_h) in $\mathcal{C}^n$:

$$\varphi\left(\theta_h q_H^n - p_{hh}^n\right) + (1-\varphi)\left(\theta_h q_M^n - p_{hl}^n\right) = \varphi\left(\theta_h q_M^n - p_{lh}^n\right) + (1-\varphi)\left(\theta_h q_L^n - p_{ll}^n\right).$$

With the provider's misrepresentation, however, the RHS of the above equation becomes $\theta_h q_M^n - p_{lh}^n$, and after substituting for the payments we have:

$$\underbrace{\varphi\left(\theta_h q_H^n - p_{hh}^n\right) + (1-\varphi)\left(\theta_h q_M^n - p_{hl}^n\right)}_{=\varphi\Delta\theta q_M^n + (1-\varphi)\Delta\theta q_L^n} < \underbrace{\theta_h q_M^n - p_{lh}^n}_{=\Delta\theta q_M^n}.$$

The strict inequality above implies that (IC_h) will be violated—the real cost of the provider's manipulation is due to the violation of the incentive compatibility for a high-valuation user. Hence a high-valuation user, anticipating the provider's misrepresentation of the collective value, will require a larger information rent to reveal his private information truthfully.

In addition to the manipulation opportunity that we identified above for the low-value state, there is one more undetectable way for the provider to manipulate information. When both users report that they have high valuations, the provider can claim to each user that the other user reported his valuation as low. To ensure that the provider will not pursue this manipulation, the following incentive compatibility constraint should be satisfied in addition to (PIC_L):

$$2p_{hh} - c(q_H) \geq 2p_{hl} - c(q_M). \tag{PIC_H}$$

Notice that complying with constraints (PIC_L) and (PIC_H) presented above prevent the public good provider from all feasible manipulations, because any other form of manipulation is detectable by at least one user. For instance, when a user's valuation is low and the other user's is high, the provider cannot claim that the collective valuation is high, because such a false claim will be detected by the low-valuation user.

It is easier to curtail the provider's manipulation incentive in the high-value state in comparison to her manipulation incentives in the low-value state. In other words, (PIC_H) is a less demanding constraint than (PIC_L). As mentioned in the previous section, when the incentive compatibility constraint (IC_h) pins down the expected payment $\varphi p_{hh} + (1-\varphi) p_{hl}$ from a high-valuation user, it still leaves some degree of freedom in determining individual payment levels of p_{hh} and p_{hl}. In Lemma 2 below, we show that the manipulative provider's contract can make use of this freedom to satisfy (PIC_H) without violating the users' incentive compatibility or participation constraints, for the relevant levels of the public good.

Lemma 2 *Consider public good levels such that $q_L \leq q_M \leq q_H \leq q_H^*$, payments p_{ll} and p_{lh} satisfying (PC_{ll}) and (PC_{lh}), and $\varphi p_{hh} + (1-\varphi) p_{hl}$ given by binding (IC_h). There exist p_{hh} and p_{hl} that satisfy (PC_{hh}), (PC_{hl}), (IC_l) and (PIC_H).*

Proof. See Appendix C. ■

In light of the previous two lemmas, we can conclude that the optimal manipulation-proof contract is the second-best one, $\mathcal{C}^n$, if the value of φ is small or large ($\varphi \leq \underline{\varphi}$ or $\varphi \geq \overline{\varphi}$). The remaining task is identifying the optimal contract for an intermediate range of φ, where the public good provider's manipulation incentive is an issue. This contract should maximize the provider's expected payoff in $(\mathcal{P})$ subject to the providers' incentive compatibility constraints, as well as the users' participation constraints and the users' incentive compatibility constraints.

Since the second-best contract $\mathcal{C}^n$ violates (PIC_L) for $\varphi \in (\underline{\varphi}, \overline{\varphi})$, within this range of φ, perhaps the most natural contract that would eliminate the provider's incentive to manipulate is a *bunching* contract that does not distinguish between the case where both users report low valuation for the public good and the case where only one of them reports low valuation, i.e., $q_L = q_M$. In this way, a low-valuation user and the provider end up with the same public good and payment levels for both $\gamma = L$ and $\gamma = M$—with this bunching, there is no reason for the provider to misrepresent $\gamma = L$ as $\gamma = M$. With our next proposition, we show that this is indeed the optimal contract for the provider when a user is more likely to be the high-valuation type.

Proposition 2 *For $\varphi \in (1/2, \overline{\varphi})$, the optimal contract $\mathcal{C}^m$ offered by the manipulative provider entails a bunching outcome with under-provision of the public good:*

$$\begin{aligned} c'\left(q_H^m\right) &= 2\theta_h, \\ c'\left(q_M^m\right) &= c'\left(q_L^m\right) = \max\left\{2\theta_l - 2\frac{\varphi^2}{1-\varphi^2}\Delta\theta,\ 0\right\}. \end{aligned}$$

A high-valuation user's expected rent is strictly positive unless $q_M^m = q_L^m = 0$, and a low-valuation user gets zero rent.

Proof. See Appendix D. ■

When all users have low-valuation, the provider produces $q_L^m = q_M^m$ and receives $p_{ll}^m = p_{lh}^m$ $(= \theta_l q_L^m = \theta_l q_M^m)$ from each user regardless of she manipulates the reported information or not. It is straightforward to see that this arrangement sets the LHS and the RHS of PIC_L constraint equal to each other, thus eliminating the provider's incentive to manipulate information. Notice that the public good is under-provided in $\mathcal{C}^m$ ($q_M^m < q_M^*$ and $q_L^m < q_L^*$) for $\varphi \in (1/2, \overline{\varphi})$. When it is more likely that a user is the high-valuation type (the type receiving information rent), it is optimal for the provider to reduce the source of the users' information rents when removing her own manipulation incentive.

In addition to the bunching presented in Proposition 2 above, there are other ways to keep the public good provider from manipulating information. To see this, consider the binding (PIC_L) presented below:

$$2p_{ll} - c\left(q_L\right) = 2p_{lh} - c\left(q_M\right).$$

From the equation above, instead of bunching the two outcomes for $\gamma = L$ and $\gamma = M$, the provider can reduce the payment p_{lh} in the RHS to discourage herself from information manipulation. Notice that, although lowering this payment relaxes the constraint, it comes at the cost of providing a low-valuation user with a strictly positive rent when he is paired with a high-valuation user. Another way to discourage the provider from manipulating information would be to inflate the level of public good q_M in the RHS, so that the value generated by it for low-valuation users would not justify the cost of producing it. As will be shown below, when a user is more likely to be the low-valuation type, the provider finds it optimal to use a combination of these two approaches to deal with her own incentive to manipulate information.

In order to present our results for $\varphi \in (\underline{\varphi}, 1/2)$, we first introduce the following condition.

Condition 1 $2\theta_l \hat{q}_L - c(\hat{q}_L) \leq 2\theta_l q_H^* - c(q_H^*)$, *where* $\hat{q}_L$ *is defined by:*

$$c'(\hat{q}_L) = \max\left\{2\theta_l - 2\frac{\varphi(1-\varphi)}{1-\varphi(1-\varphi)}\Delta\theta, 0\right\}.$$

Whether Condition 1 holds depends on the parameter values and the cost function's curvature. The next proposition presents the optimal outcome when the condition holds.

Proposition 3 *Suppose Condition 1 holds. For* $\varphi \in (\underline{\varphi}, 1/2)$, *the optimal contract* $\mathcal{C}^m$ *offered by the manipulative provider entails a bunching outcome with overprovision of the public good:*

$$q_H^m = q_H^* = q_M^m > q_M^* \text{ and } q_L^m = \hat{q}_L < q_L^*.$$

A high-valuation user receives a rent, and a low-valuation user receives a rent when paired with a high-valuation user.

Proof. See Appendix E. ■

As mentioned above, the manipulative public good provider can prevent herself from misrepresenting the users' collective valuation by distorting the size of public good and/or decreasing the payment from the low-valuation user when he is paired with a high-valuation user. Recall from Proposition 2 that, when it is more likely that a user is the high-valuation type ($\varphi > 1/2$), the optimal way to prevent the provider's manipulation is pooling the outcome for $\gamma = M$ with the outcome for $\gamma = L$. The bunching with public good under-provision effectively prevents the provider's manipulation, and at the same time, limiting her rent provision to a high-valuation user.

When it is more likely that a user is the low-valuation type ($\varphi < 1/2$), the optimal way to prevent the provider's manipulation entails increasing q_M, thus increasing the cost of production for $\gamma = M$, which in turn prevents the provider from misrepresenting $\gamma = L$ as $\gamma = M$ (a larger q_M implies a larger rent provision to a high-valuation user, but it is more likely that a user is the low-valuation type). Proposition 3 exhibits an extreme case. When it becomes significantly hard for the provider to incentivize herself for a truthful behavior, the provider must distort q_M^m upward all the way to q_H^m $(= q_H^*)$, and also give a strictly positive rent to a low-valuation user paired with a high-valuation user in the optimal contract. This implies an overproduction of the public good when the users have different valuations ($q_M^m > q_M^*$)—when a user is more likely to be the high-valuation type, as long as at least one of the users has a high valuation for the public good, the provider may prefer to set the production at the first-best level corresponding to all users having high valuation.

Comparison of $q_L^m (= \hat{q}_L)$ with the conditions defining the first-best and the second-best outcomes reveals that $\hat{q}_L$ is in between q_L^n and q_L^* for $\varphi \in (\underline{\varphi}, 1/2)$ and exactly equal to q_L^n for $\varphi = \underline{\varphi}$. This implies that Condition 1 is violated at $\varphi = \underline{\varphi}$.[10] When Condition 1 does not hold, the participation constraint (PC_{lh}) of the low-valuation user becomes binding in the optimal contract. As we have seen in Proposition 2, one way to satisfy constraints (PC_{lh}) and (PIC_L) simultaneously is setting the public good level q_M of the intermediate-value state equal to the public good level q_L of the low-value state. As illustrated in Figure 1, concavity of function $2\theta_l q - c(q)$ (together with the Inada condition that $\lim_{q\to\infty} c'(q) = \infty$) implies the existence of another level for q_M which achieves this objective but higher than the first-best public good level q_L^*. We define $\tilde{q}(q_L)$ as this higher level of q_M $(> q_L^*)$ that would satisfy the (PIC_L) constraint as an equality:

$$\tilde{q}(q_L) = \max\{q_M : 2\theta_l q_M - c(q_M) = 2\theta_l q_L - c(q_L)\}.$$

The following proposition presents the outcome in the optimal contract offered by the manipulative provider when Condition 1 is violated.

Proposition 4 *Suppose Condition 1 does not hold. For $\varphi \in (\underline{\varphi}, 1/2)$, the optimal contract $\mathcal{C}^m$ offered by the manipulative provider entails the following public good sizes:*

$$q_H^m = q_H^*,\ q_M^m = \tilde{q}(q_L^m) > q_L^*,\ \textit{where } q_M^m \gtreqless q_M^* \textit{ and } q_L^m < q_L^*.$$

A high-valuation user receives a rent, and a low-valuation users receives no rent.

[10] When $\varphi = \underline{\varphi}$, the second-best outcome satisfies the (PIC_L) constraint as an equality. Accordingly, $2\theta_l\hat{q}_L - c(\hat{q}_L) > 2\theta_l q_L^n - c(q_L^n) = 2\theta_l q_M^n - c(q_M^n) > 2\theta_l q_H^* - c(q_H^*)$.

Proof. See Appendix F. ■

Again, when a user is more likely to have low valuation ($\varphi < 1/2$), it may be optimal for the provider to prevent her own manipulation incentive by increasing q_M beyond the first-best level q_M^*. Proposition 4 exhibits cases where inducing the provider's truthful behavior is not as costly as in Proposition 3. Here, the provider leaves no rent to a low-valuation user by setting the payment from him as large as the value that this user gets from the public good. At the same time, to prevent the provider from manipulating collective information from the users, the optimal contract inflates the size of the public good large enough in the intermediate-value state. As a result, the optimal contract can still lead to an overprovision of the public good for $\gamma = M$.

To summarize, the propositions in this section characterized the optimal contract offered by the manipulative provider for the entire range of φ.[11] For the extreme values of φ, the provider's manipulability is not an issue and the optimal contract is the same as the second-best contract given in Proposition 1. If φ is larger than but close enough to 1/2, Proposition 2 yields the optimal contract, which bunches the low and intermediate collective valuations at the same public good level. If φ is smaller than but close enough to 1/2, the optimal contract is given either by Proposition 3 or by Proposition 4, depending on whether or not Condition 1 holds. For these latter values of φ, the public good can be overprovided and even the low-valuation users may receive a positive rent.[12]

The central message in this section is summarized in the following corollary.

Corollary 2 *If the public good provider is manipulative, then the optimal contract may entail over-provision of the public good.*

6 Numerical Example

We develop a numerical example here to demonstrate how our results can be used to identify the public good provider's optimal contract for different values of φ. The cost function for production of the public good is given as $c(q) = \frac{1}{5}q^5$, so that the marginal cost of production is $c'(q) = q^4$. Each user's valuation for (each unit of) the public good is either

[11] For completeness, we note that when $\varphi = 1/2$, there is a continuum of contracts maximizing the provider's expected payoff. The optimal public good and payment levels are given as in Proposition 3 for these contracts, except for the level of q_M which can take any value within the set $[\hat{q}_L, \min\{\tilde{q}(\hat{q}_L), q_H^*\}]$.

[12] In a single-agent setting, Beaudry (1994) shows that the privately informed principal may leave a rent to the agent without private information.

high $\theta_h = 2$, or low $\theta_l = 0.5$ with probabilities φ and $1 - \varphi$ respectively. The first-best levels of the public good can be found by equating the marginal cost with the sum of the users' marginal valuations (Samuelson condition) in the high-value, intermediate-value, and low-value states:

$$\begin{aligned} q_H^* &= (4)^{1/4} \approx 1.4142, \\ q_M^* &= (2.5)^{1/4} \approx 1.2574, \\ q_L^* &= (1)^{1/4} = 1. \end{aligned}$$

When the provider does not directly observe the users' valuations, but she is able to commit not to manipulate information, she can offer the second-best contract discussed in Section 3. The first-order conditions for the second-best levels of production are as below:

$$\begin{aligned} c'(q_H^n) &= 2\theta_h = 4, \\ c'(q_M^n) &= \max\left\{\theta_h + \theta_l - \frac{\varphi}{1-\varphi}\Delta\theta, 0\right\} = \max\left\{\frac{2.5 - 4\varphi}{1-\varphi}, 0\right\}, \\ c'(q_L^n) &= \max\left\{2\theta_l - 2\frac{\varphi}{1-\varphi}\Delta\theta, 0\right\} = \max\left\{\frac{1-4\varphi}{1-\varphi}, 0\right\}. \end{aligned}$$

They yield the following public good levels as a function of probability φ:

$$\begin{aligned} q_H^n &= q_H^* = (4)^{1/4}, \\ q_M^n &= \begin{cases} \left(\frac{2.5-4\varphi}{1-\varphi}\right)^{1/4} & \text{if } \varphi < 5/8 \\ 0 & \text{if } \varphi \geq 5/8 \end{cases}, \\ q_L^n &= \begin{cases} \left(\frac{1-4\varphi}{1-\varphi}\right)^{1/4} & \text{if } \varphi < 1/4 \\ 0 & \text{if } \varphi \geq 1/4 \end{cases}. \end{aligned}$$

The payments from low-valuation users are determined by the binding (PC_{ll}) and (PC_{lh}) constraints, and the expected payment from the high-valuation user is determined by the binding (IC_h) constraint.

To check if the second-best outcome would give the provider the incentive to manipulate, we consider the low-value state. The provider receives the ex-post payoff $2\theta_l q_L^n - c(q_L^n)$ by revealing the correct state, whereas misrepresenting the state as the intermediate one brings $2\theta_l q_M^n - c(q_M^n)$. When $\varphi \geq \overline{\varphi} = 5/8$, the provider is indifferent, because the second-best public good levels are equal to zero for both the low-value and the intermediate-value states. It follows from Lemma 1 that manipulation is not an issue for values of φ lower than a threshold level $\underline{\varphi}$ either. We can find the value of this threshold by equating the equilibrium payoff with the manipulation payoff:

$$\left(\frac{1-4\varphi}{1-\varphi}\right)^{1/4}-\frac{1}{5}\left(\frac{1-4\varphi}{1-\varphi}\right)^{5/4}=\left(\frac{2.5-4\varphi}{1-\varphi}\right)^{1/4}-\frac{1}{5}\left(\frac{2.5-4\varphi}{1-\varphi}\right)^{5/4}.$$

This yields the solution $\underline{\varphi}\approx 0.191478$.

The second-best outcome is manipulation proof if and only if $\varphi\geq\overline{\varphi}$ or $\varphi\leq\underline{\varphi}$. For intermediate levels of φ, we can follow Propositions 2-4 to identify the optimal manipulation-proof outcome. The optimal level of the public good in the high-value state will remain to be equal to its first-best level q_H^* regardless of the value of φ. The public good levels in the low-value and intermediate-value states will depend on φ as we explain below.

We start with $\varphi\in(1/2,\overline{\varphi}=5/8)$. We know from Proposition 2 that the optimal outcome will prescribe pooling the low-value and intermediate-value levels of the public good. The first-order condition in this proposition for this public good level is

$$c'(q_M)=c'(q_L)=\max\left\{2\theta_l-2\frac{\varphi^2}{1-\varphi^2}\Delta\theta,0\right\}=0 \text{ for } \varphi\geq 1/2,$$

implying that the provider prefers to shut down production ($q_M=q_L=0$) whenever at least one of the users reports to have a low value for the public good, in order to avoid manipulability of information. The payment levels are also equal to zero, unless both users report high types: $p_{ll}=p_{lh}=p_{hl}=0$.

To find the optimal manipulation-proof outcome for $\varphi\in(\underline{\varphi},1/2)$, we start with maximizing the provider's expected payoff subject to the participation constraints (for the users) and the incentive compatibility constraints (for the users as well as for the provider), except for the constraint (PC_{lh}) as in Proposition 3. The first-order conditions in this proposition reveal the public good levels as $q_M=q_H^*=(4)^{1/4}$ and $q_L=\hat{q}_L=\left(\frac{1-4\varphi(1-\varphi)}{1-\varphi(1-\varphi)}\right)^{1/4}$ in the intermediate-value and low-value states. The value of q_L is decreasing in φ for $\varphi<1/2$ and converges to zero as φ approaches to $1/2$. Transfers from the low-valuation users are given by the binding (PC_{lh}) and (PIC_L) constraints. Accordingly, the contract supporting this outcome would give the low-valuation user who is matched with a high-valuation one the possibility of having access to $q_M=(4)^{1/4}$ units of the public good in exchange for payment p_{lh}, where

$$p_{lh}=\theta_l q_L+\frac{c(q_M)-c(q_L)}{2}=\frac{1}{10}\left(\frac{4-\varphi(1-\varphi)}{1-\varphi(1-\varphi)}\right)\left(\frac{1-4\varphi(1-\varphi)}{1-\varphi(1-\varphi)}\right)^{1/4}+\frac{1}{10}(4)^{5/4}.$$

For the low-valuation user not to opt out of this contract (and for constraint (PC_{lh}) to be satisfied), p_{lh} must be smaller than $\theta_l q_M=\frac{1}{2}(4)^{1/4}$. The value of p_{lh} that we found above is decreasing in φ for $\varphi < 1/2$ and it converges to $\frac{1}{10}(4)^{5/4} < \theta_l q_M$ as φ approaches to 1/2. This discussion points to the existence of a cutoff value $\hat{\varphi}$ such that Proposition 3 yields the optimal manipulation-proof outcome for $\varphi \in (\hat{\varphi}, 1/2)$.[13] By solving $p_{lh} = \theta_l q_M$, we can find the approximate value of $\hat{\varphi}$ as 0.465 24.

For values of φ smaller than $\hat{\varphi}$ but larger than $\underline{\varphi}$, the solution to the relaxed problem in Proposition 3 violates the participation constraint PC_{lh}. It follows from Proposition 4 that the participation constraints are binding for the low-valuation users. Moreover, the values of $q_L < q_L^*$ and $q_M > q_L^*$ are determined to give the same ex-post payoff to the provider, regardless of she manipulates or not:

$$2\theta_l q_L - c(q_L) = 2\theta_l q_M - c(q_M).$$

In the context of our example, this means that q_L and q_M are two distinct roots of equation $q - \frac{1}{5}q^5 = \pi$, where π is the ex-post payoff of the provider in the low-value state.[14] As φ takes values between $\underline{\varphi}$ and $\hat{\varphi}$, q_L changes continuously between 0.73354 and 0.2832, and q_M changes continuously between 1.210 2 and $q_H^* = (4)^{1/4} \approx 1.414\,2$. Notice that a level of q_M larger than $q_M^* \approx 1.257\,4$ indicates overprovision of the public good.

Finally, for $\varphi = 1/2$, there is a continuum of optimal contracts. All these contracts involve shutting down production in the low-value state ($q_L = 0$). But the public good in state q_M can take any value between zero and $q_H^* = (4)^{1/4}$.

7 Welfare Effects

Having characterized the optimal manipulation-proof contract for all the parameter constellations, we now provide a discussion of the welfare effects of the manipulability of collective information. Our analysis indicates that the provider's opportunity to manipulate comes at a cost. When designing the contract, the provider has to persuade the users that she will not falsify the information that they will report to her. This consideration imposes a new incentive constraint for the public good provision contract, on top of the standard conditions securing the users' participation and their truthful reporting. It follows from Lemma 1 that, as long as there is a sufficient level of uncertainty about the users' valuations for

[13] For values of φ between $\hat{\varphi}$ and 1/2, following the proof of Lemma 2, we can set the transfers from the high-valuation agent p_{hh} and p_{hl} equal to each other: $p_{hh} = p_{hl} = \theta_l q_L + \varphi \frac{c(q_H) - c(q_L)}{2} + (1-\varphi)\theta_h (q_H - q_L)$.

[14] If x and y are two roots of this equation, then it must be that $\frac{1}{5}(x^5 - y^5) = x - y$. It follows from $x \neq y$ that $x^4 + x^3 y + x^2 y^2 + x y^3 + y^4 = 5$. Therefore, we can find a closed-form solution for q_M as a function of q_L by using the formula for the roots of a polynomial of degree 4.

the public good (as long as $\varphi \in (\underline{\varphi}, \overline{\varphi})$), this new constraint is violated by the second-best contract. In this case, the optimal manipulation-proof contract brings in a lower payoff for the provider relative to the second best.

The source of the users' payoffs in the second-best contract is their private information. A high-valuation user should be given an information rent, so that he would not choose to misreport his valuation. This information rent is increasing in q_L and q_M, the public good levels supplied for the low-valuation user. When high valuations are more likely than low valuations ($\varphi \in (1/2, \overline{\varphi})$), Proposition 2 tells us that the manipulation-proofness constraint would have different effects on these two public good levels: The optimal q_L is weakly higher and the optimal q_M is lower than their second-best values. Hence the effect of manipulability on the users' payoffs is ambiguous. By contrast, when low valuations are more likely ($\varphi \in (\underline{\varphi}, 1/2)$), we know from Propositions 3 and 4 that the optimal levels of both q_L and q_M are higher than in the second best. Therefore the high-valuation user is better off in this case, with the introduction of the provider's manipulation opportunities. Proposition 3 also points to the possibility that even the low-valuation user may receive a strictly positive payoff in the intermediate-value state. The provider tolerates leaving a rent to this user in order to strengthen her commitment not to misrepresent the low-value state as the intermediate-value one.

In sum, while the provider is hurt by having an opportunity to manipulate the information she receives from the users, the users themselves may strictly prefer to interact with a provider who is known to be capable of this manipulation. Examination of the change in the optimal levels of q_L and q_M would also give an idea on whether the increase in the users' payoffs compensate for the loss in the provider's. For example, if φ is in the interval $(\underline{\varphi}, 1/2)$ and there is no overprovision of the public good (if the optimal manipulation-proof level of q_M is lower than its first-best value q_M^*), then manipulability of information improves the sum of the expected payoffs of the provider and the users, because both q_L and q_M get closer to their first-best values under the optimal manipulation-proof contract.

Corollary 3 *Users, regardless of their valuation of the public good, may prefer to deal with a provider who is known to be capable of manipulation.*

8 Conclusion

In this paper, we have provided a rationale for an overprovision of public goods that is based on the presence of private information. In doing so, we analyzed contracting for a public good between a provider and users with private valuations for the public good. The users' private information causes a distortion in the size of the public good offered to them and such distortions may lead to the provider's incentive to manipulate. We have shown that, once the public good provider extracts the users' private information, she may have an incentive to misrepresent the collective information from the users.[15] Our results suggest that the provider's manipulation ability determines the direction of distortion in public good provision. If the provider is non-manipulative, her optimal contract underprovides the public good. If she is manipulative, however, public goods can be overprovided. In such cases, not only the high-valuation users of the public good, but also the low-valuation ones may obtain positive rents. Lastly, we have shown that all users, regardless of their types, can receive higher rents when the provider is manipulative, thus suggesting that, for strategic reasons, the users may want to contract with a provider who is capable of manipulating information.

We close the paper with a remark concerning our assumption that a user can opt out, if he anticipates a strictly negative payoff after receiving the provider's report about collective valuation of the public good. In the applications that we have in mind, we consider such limited liability as natural. Without such a limited liability of the users, the manipulative provider can still achieve the second-best outcome by trading off payments to low-valuation users. In particular, the binding participation constraint for a low-valuation user, with no limited liability, is:

$$\varphi\left(\theta_l q_M - p_{lh}\right) + \left(1-\varphi\right)\left(\theta_l q_L - p_{ll}\right) = 0.$$

As can be seen from the equation, the public good provider has an extra degree of freedom—she can make a low-valuation user's ex post payoff positive for $\gamma = M$ by decreasing p_{lh}, and negative for $\gamma = L$ by increasing p_{ll} without altering the public good sizes from the second-best level in each state. This allows the public good provider to achieve the second best even with aggregate information manipulation, but with the drawback that it violates the user's ex post participation constraint. Depending on the application, such a violation may however be infeasible.

[15] See Celik et al. (2019) for a study on the linkage between the principal's incentive to manipulate the information from the agents and the optimal structures of the organization. In that paper, unlike in the current paper, no over-production takes place at the optimum.

Appendix

A. Proof of Proposition 1

The non-manipulative provider's optimal contract $\mathcal{C}^n$ maximizes $(\mathcal{P})$ subject to (PC_{lh}), (PC_{ll}), (PC_{hh}), (PC_{hl}), (IC_h) and (IC_l). We follow the usual procedure of considering a relaxed problem with only the constraints (PC_{lh}), (PC_{ll}) and (IC_h), and ignoring the remaining three. Since the provider's payoff is increasing in p_{lh} and p_{ll} from low-valuation users and the expected payment $\varphi p_{hh}+(1-\varphi)\,p_{hl}$ from high-valuation users, (PC_{lh}), (PC_{ll}) and (IC_h) are binding. These binding constraints give the following expressions:

$$\begin{aligned} p_{ll} &= \theta_l q_L, \\ p_{lh} &= \theta_l q_M, \\ \varphi p_{hh} + (1-\varphi)\,p_{hl} &= \varphi[\theta_h q_H - \Delta\theta q_M] + (1-\varphi)[\theta_h q_M - \Delta\theta q_L]. \end{aligned}$$

Maximizing the objective function in $\mathcal{P}$ after substituting out these payments yields q_L^n, q_M^n and q_H^n. From the expressions of the payments from the users, it follows that a high-valuation user's expected rent is strictly positive unless $q_M = q_L = 0$, and a low-valuation user's rent is always zero. What remains is showing that we can find individual levels of payments p_{hh} and p_{hl} that would satisfy the ignored constraints of (PC_{hh}), (PC_{hl}) and (IC_l). First, from the first order conditions for the optimal public good levels in the proposition, notice that $q_M^n = 0$ for $\varphi \geq (\theta_h+\theta_l)/2\theta_h$ and $q_L^n = 0$ for $\varphi \geq \theta_l/\theta_h$. Since $(\theta_h+\theta_l)/2\theta_h > \theta_l/\theta_h$, if $q_M^n = 0$ then $q_L^n = 0$. Also, for strictly positive public good levels, $q_H^n > q_M^n > q_L^n$. Thus, $q_H^n > q_M^n \geq q_L^n$ in any case. Consider now the levels of these payments which would satisfy (IC_h) in the ex-post sense:

$$\begin{aligned} p_{hh} &= p_{lh} + \theta_h\,(q_H - q_M)\,, \\ p_{hl} &= p_{ll} + \theta_h\,(q_M - q_L)\,. \end{aligned}$$

It follows from the monotonicity of the public good levels ($q_H^n > q_M^n \geq q_L^n$) that (PC_{hh}) and (PC_{hl}) are satisfied with these payments. Also, (IC_l) holds with zero on either side of the weak inequality. ■

B. Proof of Lemma 1

With the outcome in $\mathcal{C}^n$, we can re-write (PIC_L) by using the binding (PC_{lh}) and (PC_{ll}):

$$2\theta_l q_L^n - c\,(q_L^n) \geq 2\theta_l q_M^n - c\,(q_M^n)\,.$$

Function $2\theta_l q - c(q)$ is concave in q and maximized at q_L^*. It follows from the first-order conditions in Proposition 1 that $\bar{\varphi} \equiv (\theta_h + \theta_l)/2\theta_h$ is the lowest level of φ under which the provider chooses to shut down unless both users report high values. If $\varphi \geq \bar{\varphi}$, then $q_L^n = q_M^n = 0$ and (PIC_L) holds as an equality. For $\varphi \in [1/2,\ \bar{\varphi})$, the first order conditions of the optimal outcome in $\mathcal{C}^n$ implies $q_L^n < q_M^n \leq q_L^*$ and therefore (PIC_L) is violated. Similarly, when φ approaches to 0, q_L^n approaches to $q_L^* (< q_M^n)$ and (PIC_L) is satisfied. Existence of the threshold value $\underline{\varphi}$ follows from the fact that the left hand side of (PIC_L) decreases and its right hand side increases in φ on the interval $[0, 1/2]$. ■

C. Proof of Lemma 2

We first try setting payments p_{hl} and p_{hh} equal to the values that would satisfy a high-valuation user's incentive compatibility conditions in the ex-post sense: $p_{hh} = p_{lh} + \theta_h (q_H - q_M)$ and $p_{hl} = p_{ll} + \theta_h (q_M - q_L)$. The participation constraints for a low-valuation user, (PC_{lh}) and (PC_{ll}), imply that these payments also satisfy the participation constraints for a high-valuation user, (PC_{hh}) and (PC_{hl}). Constraint (IC_l) holds provided that (IC_h) is binding and the public good levels are monotonic $(q_L \leq q_M \leq q_H)$: Pretending to have high valuation would bring a lower payoff than the equilibrium payoff to a low-valuation user, regardless of the other user's type. So, if (PIC_H) is satisfied as well, then the proof is complete.

Suppose (PIC_H) is violated with the above values of p_{hl} and p_{hh}. In such a case, we increase p_{hh} and reduce p_{hl} such that both (IC_h) and (PIC_H) hold as equalities:

$$\begin{aligned} p_{hh} &= \varphi p_{lh} + \varphi \theta_h (q_H - q_M) + (1-\varphi) p_{ll} + (1-\varphi) \theta_h (q_M - q_L) + (1-\varphi) \frac{c(q_H) - c(q_M)}{2}, \\ p_{hl} &= \varphi p_{lh} + \varphi \theta_h (q_H - q_M) + (1-\varphi) p_{ll} + (1-\varphi) \theta_h (q_M - q_L) - \varphi \frac{c(q_H) - c(q_M)}{2}. \end{aligned}$$

Constraint (PC_{hl}) still holds, because we are reducing the payment p_{hl} that the user makes in this state of nature. Constraint (PC_{hh}) is satisfied as well, because $c(q_H) - c(q_M) \leq 2\theta_h (q_H - q_M)$ under convexity of $c(\cdot)$, and therefore:

$$\begin{aligned} p_{hh} &\leq \varphi \theta_l q_M + \theta_h (q_H - q_M) + (1-\varphi) \theta_l q_L + (1-\varphi) \theta_h (q_M - q_L) \\ &= \theta_h q_H - \varphi (\theta_h - \theta_l) q_M - (1-\varphi)(\theta_h - \theta_l) q_L \leq \theta_h q_H. \end{aligned}$$

Showing (IC_l) holds is more involved for this case because of the 'max' operators representing the user's opportunity to opt out of the contract. First notice that the expected equilibrium payoff of the low-valuation user is higher than the expected payoff of pretending to be high-valuation and opting in the contract regardless of the other user's type. This is due the fact that (IC_h) is binding and the public good levels are monotonic $(q_L \leq q_M \leq q_H)$. What remains to show is the suboptimality of imitating a high-valuation user and then opting out depending on the type of the other user. This imitation is not profitable when the other user has high valuation, because p_{hh} is now higher than $p_{lh} + \theta_h (q_H - q_M)$. On the other hand, in the case that the other user has low valuation, the imitation payoff is:

$$\begin{aligned} & \theta_l q_M - p_{hl} \\ = \;& \theta_l q_M - \varphi p_{lh} - \varphi \theta_h (q_H - q_M) - (1-\varphi) p_{ll} - (1-\varphi) \theta_h (q_M - q_L) + \varphi \frac{c(q_H) - c(q_M)}{2} \\ \leq \;& \theta_l q_M - \varphi p_{lh} - (1-\varphi) p_{ll} - (1-\varphi) \theta_h (q_M - q_L) \\ = \;& \varphi (\theta_l q_M - p_{lh}) + (1-\varphi)(\theta_l q_L - p_{ll}) - (1-\varphi)(\theta_h - \theta_l)(q_M - q_L), \end{aligned}$$

where the inequality follows from the convexity of $c(\cdot)$ again. Because this payoff is smaller than the expected equilibrium payoff of $\varphi (\theta_l q_M - p_{lh}) + (1-\varphi)(\theta_l q_L - p_{ll})$ for a low-valuation user, constraint (IC_l) is satisfied. ■

D. Proof of Proposition 2

For proof of the proposition, we first consider a relaxed problem in Lemma 3 below where we look for the outcome that maximizes the provider's objective function in $(\mathcal{P})$ subject to (IC_h), (PC_{lh}), (PC_{ll}) and (PIC_L) constraints, ignoring (IC_l), (PC_{hh}), (PC_{hl}) and (PIC_H) constraints—we will also refer to this lemma for proofs of all remaining propositions.

Lemma 3 *Suppose $\varphi \in (\underline{\varphi}, \overline{\varphi})$. At the solution to the relaxed problem, payment levels are given by the binding (IC_h), (PC_{ll}) and (PIC_L) constraints. The public good levels q_H, q_M and q_L are chosen to maximize:*

$$\begin{aligned} \varphi^2 [2\theta_h q_H - c(q_H)] + \varphi (1-2\varphi) [2\theta_l q_M - c(q_M)] + (1 - \varphi(1-\varphi)) [2\theta_l q_L - c(q_L)] \\ +2\varphi(1-2\varphi)(\theta_h - \theta_l) q_M - 2\varphi(1-\varphi)(\theta_h - \theta_l) q_L, \end{aligned} \tag{$\widetilde{\mathcal{P}}$}$$

subject to

$$2\theta_l q_M - c(q_M) \geq 2\theta_l q_L - c(q_L). \tag{$\widetilde{PC}_{lh}$}$$

Proof. Because the objective function is decreasing in p_{hh}, p_{hl}, p_{ll} and constraint (PIC_L) is relaxed with a lower value of p_{ll}, constraints (IC_h) and (PC_{ll}) are binding for the outcome solving the relaxed problem. It follows from Lemma 1 that (PIC_L) is binding for $\varphi \in (\underline{\varphi}, \overline{\varphi})$. We can rewrite the expected payment to the provider by substituting in these constraints:

$$\begin{aligned}&\varphi^2 2p_{hh} + 2\varphi(1-\varphi)p_{hl} + 2\varphi(1-\varphi)p_{lh} + (1-\varphi)^2 2p_{ll}\\ = \;&2\theta_l q_L + \varphi^2 2\theta_h q_H + \varphi(1-2\varphi)2\theta_h q_M - \varphi(1-\varphi)2\theta_h q_L + \varphi c(q_M) - \varphi c(q_L).\end{aligned}$$

Once the expected cost of public good provision is taken into account, the provider's objective function reduces to the objective in $(\widetilde{\mathcal{P}})$. Similarly, constraint (PC_{lh}) can be rewritten as $(\widetilde{PC}_{lh})$ after substituting in the binding constraints of (PIC_L) and (PC_{ll}). ■

We now move on to the proof of Proposition 2. We will start with ignoring (IC_l), (PC_{hh}), (PC_{hl}) and (PIC_H) constraints and maximizing the provider's objective function subject to (IC_h), (PC_{lh}), (PC_{ll}) and (PIC_L) constraints only, as in Lemma 3. The solution to the relaxed problem will be the one identified by the proposition. Since the solution in the proposition satisfies the hypothesis of Lemma 2, there exists an outcome that solves the relaxed problem and that satisfies the ignored constraints.

For $\varphi > 1/2$, the objective function $(\widetilde{\mathcal{P}})$ is convex in q_M. Therefore $(\widetilde{PC}_{lh})$ constraint is satisfied as an equality at the solution to this maximization. This equality holds when $q_M = q_L$. Given concavity of function $2\theta_l q - c(q)$, the equality may also be satisfied when one variable is strictly higher than the other. This will not be the case for the outcome solving the maximization problem: Holding $2\theta_l q_M - c(q_M)$ and $2\theta_l q_L - c(q_L)$ constant, the objective function is decreasing in both q_L and q_M (when $\varphi > 1/2$). This proves that the optimal outcome is a bunching outcome $(q_M = q_L)$. The first order condition yields:

$$c'(q_M) = c'(q_L) \geq 2\theta_l - 2\frac{\varphi^2}{1-\varphi^2}\Delta\theta,$$

where the weak inequality holds as equality if $q_M = q_L > 0$.

Finally, Lemma 2 implies that we can find individual levels of p_{hl} and p_{hh} (for instance, $p_{hl} = \theta_l q_L$ and $p_{hh} = \theta_l q_L + \theta_h(q_H - q_L)$) that satisfy the ignored (IC_l), (PC_{hh}), (PC_{hl}) and (PIC_H) constraints. ■

E. Proof of Proposition 3

Ignoring (IC_l), (PC_{hh}), (PC_{hl}) and (PIC_H) constraints, it follows from Lemma 3 that the provider's problem turns into maximization of $(\widetilde{\mathcal{P}})$ by choosing q_H, q_M and q_L subject to $(\widetilde{PC}_{lh})$. When we ignore $(\widetilde{PC}_{lh})$ constraint as well, the problem is an unconstrained optimization problem and the first order conditions yield the values of outputs q_H, q_M, and

q_L as stated in the proposition. The payments p_{ll}, p_{lh}, and $\varphi p_{hh} + (1-\varphi)\, p_{hl}$ are given by the binding (PC_{ll}), (PIC_L) and (IC_h) constraints:

$$\begin{aligned} p_{ll}^m &= \theta_l q_L^m, \\ p_{lh}^m &= \theta_l q_L^m + \frac{c(q_H^m) - c(q_L^m)}{2}, \\ \varphi p_{hh}^m + (1-\varphi)\, p_{hl}^m &= \theta_l q_L^m + \varphi \frac{c(q_H^m) - c(q_L^m)}{2} + (1-\varphi)\, \theta_h (q_H^m - q_L^m). \end{aligned}$$

The solution satisfies the ignored $(\widetilde{PC}_{lh})$ constraint because:

$$2\theta_l q_M - c(q_M) = 2\theta_l q_H^* - c(q_H^*) \geq 2\theta_l \hat{q}_L - c(\hat{q}_L).$$

The existence of the individual values of p_{hh} and p_{hl} satisfying the ignored (IC_l), (PC_{hh}), (PC_{hl}) and (PIC_H) constraints follow from Lemma 2. For instance, setting these payments equal to each other would work:

$$p_{hh} = p_{hl} = \theta_l q_L + \varphi \frac{c(q_H) - c(q_L)}{2} + (1-\varphi)\, \theta_h (q_H - q_L). \blacksquare$$

F. Proof of Proposition 4

Following the proof of the previous propositions, we maximize the provider's objective function, ignoring (IC_l), (PC_{hh}), (PC_{hl}) and (PIC_H) constraints. The payments p_{ll}, p_{lh}, and $\varphi p_{hh} + (1-\varphi)\, p_{hl}$ are given by the binding (PC_{ll}), (PC_{lh}), (PIC_L) and (IC_h) constraints:

$$\begin{aligned} p_{ll}^m &= \theta_l q_L, \\ p_{lh}^m &= \theta_l q_M^m, \\ \varphi p_{hh}^m + (1-\varphi)\, p_{hl}^m &= \theta_l q_L^m + \varphi \theta_l (q_M^m - q_L^m) + (1-\varphi)\, \theta_h (q_M^m - q_L^m) + \varphi \theta_h (q_H^m - q_M^m) \end{aligned}$$

From Lemma 3, this problem turns into maximization of $(\widetilde{\mathcal{P}})$ by choosing q_H, q_M and q_L subject to $(\widetilde{PC}_{lh})$. The constraint is binding—otherwise, the first order conditions yield that $q_M = q_H^*$ and $q_L = \hat{q}_L$, violating $(\widetilde{PC}_{lh})$ since:

$$2\theta_l q_M - c(q_M) = 2\theta_l q_H^* - c(q_H^*) < 2\theta_l \hat{q}_L - c(\hat{q}_L).$$

Holding $2\theta_l q_L - c(q_L)$ and $2\theta_l q_M - c(q_M)$ constant, the objective function is decreasing in q_L but increasing in q_M (for $\varphi < 1/2$). This proves that $q_L < q_L^*$ and $q_M = \tilde{q}(q_L)$. The possibility of $q_M > q_M^*$ is demonstrated with a numerical example in Appendix G.

Lemma 2 implies that we can find individual levels of p_{hl} and p_{hh} that satisfy the ignored (IC_l), (PC_{hh}), (PC_{hl}) and (PIC_H) constraints. $\blacksquare$

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