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The Theoretical Case for Direct Democracy

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Abstract

Numerous formal studies have shown that information aggregation through voting is fragile in large elections when voters receive instrumental payoffs that depend only on the committee outcome. This raises the question of whether a body of "professional" voters, who are directly incentivized to vote informatively via instrumental payoffs that condition on their individual vote, can do better. Arguably, this latter case approximates representative democracy, where the probability that a legislator is re-elected depends on their individual voting record. Surprisingly, information is never consistently aggregated in representative democracy. That is, it is impossible to construct instrumental vote-contingent incentives that consistently aggregate information, since for any set of payoffs there are information structures (i.e. prior beliefs and precision of private signals) for which the committee decision is uninformative. This suggests that direct democracy, while fragile, may be the preferable institution for aggregating information.

Key words: Voting behavior; Information aggregation; Direct Democracy; Representative Democracy.

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1 Introduction

In 1785, Nicolas de Condorcet provided a strong theoretical case for democracy by showing that if all individuals hold private information that is more likely to be "right" than "wrong" and if all individuals vote according to their private information, then a sufficiently large committee that votes via a majority rule will choose the right option with arbitrary precision. That is, his analysis showed that collective decisions improve on individual decision-making by aggregating the private information of the voting population. More recently, Feddersen and Pesendorfer (1997)—following on the heels of Austin-Smith and Banks' (1996) seminal contribution—extended Condorcet's insight to a modern strategic setting and showed that in a common-values referendum, a majority vote aggregates information in large committees as long as committee members receive informative signals.

However, as subsequent research has shown, information aggregation through voting is fragile in the classic Condorcet setting, where voters only care about the correctness of the committee outcome—since the probability that any individual's vote influences the committee's decision becomes arbitrarily small in a large committee, voting behavior is very sensitive to information and payoff structures. Accordingly, information aggregation in large committees has been shown to fail due to, among others, the decision rule (Feddersen and Pesendorfer, 1998), vote-contingent payoffs (Dal Bo, 2007, Callander, 2007, Feddersen et al., 2009, and Midjord et al. 2017, Breitmoser and Valasek, 2017), and a failure of preference monotonicity (Bhattacharya, 2013). This suggests that while a majority vote has the desirable property of aggregating information for any information structure (i.e. prior beliefs and the precision of private signals), a property we refer to as *consistency*, information aggregation through voting in the standard setting fails to be *robust*, in the sense that it is not robust to small payoff deviations.

Given the frailty of information aggregation by a body of voters who are only motivated by the correctness of the collective decision (direct democracy), we raise the question of whether information is better aggregated by a voting body—committee henceforth—of professional voters (legislators) that are directly incentivized to vote informatively via payoffs that condition on their individual vote (representative democracy). We refer to this alternative model as representative democracy since elected representatives are also held responsible for their individual voting record. For example, many pundits identified Hillary Clinton's vote supporting military intervention in Iraq, which was predicated on the incorrect assumption that Iraq possessed sizable stores of weapons of mass destruction, as a key factor in her 2008 primary loss to Barak Obama, who voted against the war. This anecdote illustrates that the ex-post correctness of a representative's vote can have an instrumental effect by impacting the probability of re-election, or election to higher office.

Simple intuition would suggest that directly incentivizing committee members through vote-contingent payoffs can have a positive impact on information aggregation if these payoffs reinforce informative voting, thereby resulting in information aggregation that is both complete and robust. For example, assume each individual legislators' probability of re-election increases if they vote for the option that is shown to be correct ex-post. In this case, the legislator's payoff from voting informatively is not tied to the probability that their vote is pivotal, which provides a robust incentive to vote informatively. This intuition, however, is incomplete. As we show in this paper, there are no vote-contingent payoffs that result in information being consistently aggregated, including vote-contingent payoffs that reward committee members for voting correctly. That is, while for any information structure, there exists a set of vote-contingent payoffs that result in information aggregation, there are no vote-contingent payoffs that result in information aggregation *for any information structure.* Therefore, in contrast to direct democracy, the consistency of information aggregation fails generally for any set of vote-contingent payoffs.

For a simple illustration of our main finding, assume a large committee takes a decision between option a or option b by a majority vote. If the state is α, then option a is socially optimal, and if the state is β, then option b is socially optimal. The state of the world, however, is unknown and committee members have a prior belief that the probability that the state is equal to α is $2/3$. Additionally, each committee member receives an iid private signal of a or b that is informative of the state of the world. Specifically, the probability that they receive a signal of a (b) given a state of α (β)—we denote this probability as the precision of the signal—is greater than one-half. In this example, information will be aggregated if committee members have an incentive to vote according to their private signal (sincere voting).

Next, assume the committee consists of professionals that receive a payoff of 1 for matching their vote to the state. This simple vote-contingent payoff reinforces the incentive to vote informatively. However, whether the vote-contingent payoff results in information aggregation depends on the precision of the private signal: If the signal precision is high, say $4/5$, then each committee member maximizes their expected payoff by voting sincerely. In contrast, if the signal precision is low, say $3/5$, then committee members who receive a signal of b are better off voting for a. That is, since the informativeness of the private signal is not sufficient to outweight the prior—applying Bayes rule shows that $\Pr(\beta|s_i = b) = 3/7 < 1/2$ and the relative expected payoff from voting sincerely is negative given a signal of b—all agents maximize their expected vote-contingent payoffs by voting according to the prior.

This example shows that when the prior is informative and signal precision is low, then a simple vote-contingent payoff that rewards committee members if their vote is ex-post correct will not result in information aggregation. Instead, to incentivize sincere voting, the rewards for correctly voting for b must be higher than the rewards for correctly voting for a. That is, vote-contingent payoffs must provide a stronger incentive for voting for the option that is less likely ex ante. However, as we will show in detail below, this precludes the existence of vote-contingent payoffs that result in consistent information aggregation, since vote-contingent payoffs must be tailored to the specific information structure.

Our primary contribution is to the literature on information aggregation in committees. In contrast to other papers that have largely explored specific instances in which vote-contingent payoffs cause information aggregation to fail in large committees (cited above), we take the opposite approach and explore whether vote-contingent payoffs can lead to robust and consistent information aggregation. In this sense, our paper is closer to the work on electoral accountability and the political-agency theory of of political representation (see Besley, 2007). Our findings preclude the use of a simple system of rewards, such as reelection probabilities that only condition on the ex-post correctness of a legislator's vote, to incentivize information aggregation. Therefore, direct democracy, while non-robust, is the only system that consistently aggregates the information of the voting population.

The paper proceeds as follows. Section 2 introduces the model. In Section 3.1 we consider our benchmark of direct democracy. Section 3.2 presents our general method of characterizing equilibrium outcomes with vote-contingent payoffs (representative democracy) and presents our main result.

2 Model

Our model is based on the standard model of information aggregation introduced by Austen-Smith and Banks (1996). There are two states of the world $\omega \in \{\alpha, \beta\}$ where $\Pr(\alpha) \in (0, 1)$. A committee of $n > 2$ agents indexed by $i \in \{1, ..., n\}$ makes a decision between two choices, $x \in \{a, b\}$ by majority rule: If strictly more than half of the agents vote for a then $x = a$, and otherwise $x = b$. Each agent receives a private signal, $s_i \in \{a, b\}$, and then votes for either a ($v_i = a$) or b ($v_i = b$). The signals are i.i.d. conditional on ω and $\Pr(a|\alpha) = \Pr(b|\beta) = 1 - \varepsilon$, where $\varepsilon \in (0, \frac{1}{2})$.

Agents receive both "common-value" and "vote-contingent" payoffs. Common-value payoffs condition only on the committee decision and the state of the world: all agents receive a payoff of one if the committee decision matches the state of the world, and a payoff of zero otherwise. We label the model with common values only as "direct democracy," since it considers voters whose instrumental payoffs only depend on the committee decision.

Additionally, we consider the possibility of a body of professional voters, "representative democracy," with instrumental payoffs that are linked to each agent's individual vote. To be as general as possible, we allow these vote-contingent payoffs to condition not only on the agent's vote, but to also interact with the committee decision and the state of the world. That is, vote-contingent payoffs are represented by a function $k(v_i, x, \omega) : \{a, b\} \times \{a, b\} \times \{\alpha, \beta\} \to \mathbb{R}$.

Asymptotically (as $n \to \infty$), however, we can normalize four out of the eight value of $k(v_i, x, \omega)$ to zero, and represent the vote-contingent payoffs as the *relative* payoff for voting "correctly:"

$$k_{\omega,x} \equiv k(v_i = \omega, x, \omega) - k(v_i \neq \omega, x, \omega). \tag{1}$$

For example, $k_{\alpha,b}$ is the relative vote-contingent payoff of voting for a given a majority decision for b, and a realized state of the world α.

Agents' payoffs are represented in the following expression (abusing notation we denote

$x = \omega$ when $(x, \omega) = (a, \alpha)$, or $(x, \omega) = (b, \beta)$):

$$U_i(v_i, x, \omega) = \mathbb{1}(x = \omega) + k_{\omega,x},$$

where the first term represents common-value payoffs.

A strategy for agent i is denoted by $\sigma_i = (\sigma_i(a), \sigma_i(b))$ such that $\sigma_i(a)$ is the probability that $v_i = a$ given $s_i = a$ and $\sigma_i(b)$ is the probability that $v_i = a$ given $s_i = b$. Given some n and strategy profile σ we let $Z^n_\alpha = Pr(x = a|\sigma, \alpha)$ indicate the probability that the committee chooses a when the state is α and $Z^n_\beta = Pr(x = a|\sigma, \beta)$ be the probability that the committee chooses a when the state is β. The pair (Z^n_α, Z^n_β) is denoted by Z^n.

Throughout the analysis we rely on the concept of symmetric Bayesian Nash Equilibrium:

Definition **1** (Symmetric Equilibrium)**.** *A pair σ^* is a symmetric equilibrium if, and only if, for all $i \in \{1, 2, ..., n\}$, $s_i \in \{a, b\}$, and σ_i: $E_\sigma[U(\sigma^*, x, \omega)|s_i] \geq E_\sigma[U(\sigma^*_{-i}, \sigma_i, x, \omega)|s_i]$.*

Additionally, we formally introduce our concept of consistent and robust information aggregation:

Definition **2** (Consistent Information Aggregation)**.** *We say that a given vector of payoffs $(k(v_i, x, \omega))$ results in consistent information aggregation if there exists a sequence of equilibria such that $Z^n \to (1, 0)$ as $n \to \infty$ for all $\{\varepsilon, \Pr(\alpha)\}$.*

Following Feddersen and Pesendorfer (1997), we are interested in identifying conditions such that information aggregation is achieved for all information structures ($\{\varepsilon, \Pr(\alpha)\}$).

Definition **3** (Robust Information Aggregation)**.** *Information aggregation is robust for a given vector of parameters $(k(v_i, x, \omega))$ if there exists a neighborhood of $(k(v_i, x, \omega))$ such that for all vote-contingent payoffs within the neighborhood, there exists a sequence of equilibria such that $Z^n \to (1, 0)$ as $n \to \infty$.*

That is, we define robustness in terms of robustness to small changes in voters' instrumental payoffs.

3 Analysis

First we introduce a main object of interest in our analysis, namely the relative expected utility that agent i receives from voting for a given i's signal and the strategy of all other agents. Formally, we denote this value by $\Phi^n_{s_i}$:

$$\Phi^n_{s_i}(\sigma_{-i}) \equiv E_{\sigma_{-i}}[U(v_i = a, x, \omega)|s_i] - E_{\sigma_{-i}}[U(v_i = b, x, \omega)|s_i].$$

In the following expression, we present a simplified equation for $\Phi^n_{s_i}$—letting piv_i indicate the event that agent i is pivotal for the final decision we get:

$$\Phi^n_{s_i}(\sigma_{-i}) = Pr(\alpha|s_i)\big[(1 + k(a, a, \alpha) - k(a, b, \alpha))Pr(piv_i|\alpha) + (k_{\alpha,a} - k_{\alpha,b})Pr(a, \neg piv_i|\alpha) + k_{\alpha,b}\big]$$
$$- Pr(\beta|s_i)\big[(1 + k(a, b, \beta) - k(a, a, \beta))Pr(piv_i|\beta) + (k_{\beta,a} - k_{\beta,b})Pr(a, \neg piv_i|\beta) + k_{\beta,b}\big]$$

The term $\Phi^n_{s_i}(\sigma_{-i})$ will feature heavily in our analysis below.

3.1 Benchmark: Direct Democracy

Here we consider a benchmark of the classic Condorcet model with no vote-contingent payoffs (direct democracy), and show that voting by majority results in consistent information aggregation. In this case, agents only consider the impact of their vote on the committee decision, and hence base their voting decision on the event that their vote is pivotal. Since $k(v_i, x, \omega) = 0$ the game is of common interest with diverse information and optimal equilibria yield asymptotically perfect decisions (McLennan, 1998). This leads us to the following result stemming from McLennan (1998) and Theorem 3 in Feddersen and Pesendorfer (1997).[1]

Proposition **1** (Consistency Direct Democracy)**.** *Given $k(v_i, x, \omega) = 0$ for all (v_i, x, ω) there exists a sequence of equilibria (σ^{n*}) such that $Z^n \to (1, 0)$ as $n \to \infty$ for all information structures $\{\varepsilon, \Pr(\alpha)\}$.*

The easiest way to explain the intuition behind Proposition 1 is when $Pr(\alpha) = \frac{1}{2}$ and n is uneven. Suppose all agents vote sincerely and thus when agent i is pivotal there are exactly $\frac{n-1}{2}$ signals for a and $\frac{n-1}{2}$ signals for b among all agents other than i. In this case, it is strictly optimal for agent i to vote sincerely for any ε as s_i determines which option is supported by the most signals. In all other cases (where i is not pivotal), the vote from agent i is inconsequential and the sincere strategy is then optimal. Given the sincere strategy profile and the law of large numbers the committee's mistake probability converges to zero as $n \to \infty$ for any information structure, which shows that direct democracy results in consistent information aggregation.

Next, we follow Feddersen (2009) and use the example of a simple "moral" payoff to illustrate the non-robustness of direct democracy. Assume a is a moral option and, correspondingly, $k_{\alpha,a} = k_{\beta,a} = \delta > 0$ for δ small. This gives us the following expression for $\Phi^n_{s_i}(\sigma_{-i})$:

$$\Phi^n_{s_i}(\sigma_{-i}) = (1 + \delta)Pr(piv_i|\alpha)Pr(\alpha|s_i) - Pr(piv_i|\beta)Pr(\beta|s_i) + \delta Pr(\neg piv_i)$$

By contradiction, assume there exists a sequence of equilibria $\sigma^{n*} \neq (1, 1)$ whereby $Pr(\neg piv_i)$ is bounded away from zero for all $n > n'$. For n large enough this gives us a contradiction as the pivotal probability converges uniformly to zero as $n \to \infty$ and $\delta Pr(\neg piv_i$ is bounded away from zero and thus optimal behavior prescribes $\sigma^n = (1, 1)$. It follows that under this incentive

[1] Proof can be requested from the authors as supplementary material to Midjord et al. (2017).

structure, it is not an equilibrium for large committees to vote sincerely. Intuitively, for a large enough committee, given a vanishing pivotal probability, all voters maximize their payoffs by voting according to the moral bias.

PROPOSITION **2** (Non-robustness Direct Democracy)**.** *For any $k_{\alpha,a} = \delta > 0$ (and all other vote-contingent payoffs being zero) any sequence of equilibria (σ^{n*}) have $Z^n \to (0,0)$ or $Z^n \to (1,1)$ as $n \to \infty$.*

This result demonstrates that information aggregation under direct democracy is not robust to small payoffs deviations.

3.2 Information Aggregation under Representative Democracy

We first present a novel approach that allows us to generically characterize equilibria of large committees with vote-contingent payoffs for any given signal structure. This approach allows us to characterize the set of equilibrium outcomes in a straightforward manner by identifying the probabilities $\Pr(X = a|\alpha)$ and $\Pr(X = a|\beta)$ that satisfy a simple set of conditions on the expression $\Phi_{s_i}(Z)$, defined as:

$$\Phi_{s_i}(Z) = Pr(\alpha|s_i)\big[k_{\alpha,a}Z_\alpha + k_{\alpha,b}(1 - Z_\alpha)\big] - Pr(\beta|s_i)\big[k_{\beta,a}Z_\beta + k_{\beta,b}(1 - Z_\beta)\big], \tag{2}$$

Loosely, since all probability terms involving piv_i converge uniformly to 0 as $n \to \infty$, $\Phi_{s_i}(Z)$ can be thought of as the limiting expression of the relative payoff of voting for option a as $n \to \infty$.

This structure allows us to define a *limit outcome* as a pair of conditional decision probabilities (Z_α, Z_β) that are consistent with the limiting values of a sequence of strategies σ^n that are best responses, given the expression for the limiting relative payoff of voting for option a, $\Phi_{s_i}(Z)$:

DEFINITION **4** (Limit Outcome)**.** *Given vote-contingent payoffs $k_{\omega,x}$, a pair $(Z_\alpha, Z_\beta) \in [0,1]^2$ is a limit outcome if, and only if, the following conditions are met:*

$$\begin{array}{ll} Z_\alpha = 1 & \text{if } \Phi_a(Z_\alpha, Z_\beta) > 0, \\ Z_\alpha \in [0,1] & \text{if } \Phi_a(Z_\alpha, Z_\beta) = 0, \\ Z_\alpha = 0 & \text{if } \Phi_a(Z_\alpha, Z_\beta) < 0. \end{array} \qquad \begin{array}{ll} Z_\beta = 1 & \text{if } \Phi_b(Z_\alpha, Z_\beta) > 0, \\ Z_\beta \in [0,1] & \text{if } \Phi_b(Z_\alpha, Z_\beta) = 0, \\ Z_\beta = 0 & \text{if } \Phi_b(Z_\alpha, Z_\beta) < 0. \end{array}$$

Consider the example of a limit outcome with $Z_\alpha = 1$ and $Z_\beta \in [0,1]$. Intuitively, in the limit agents with a signal of a must have a best response of voting for a, so that $Z_\alpha = 1$, and agents with a signal of b must be indifferent between voting for a and b, so that $Z_\beta \in [0,1]$. Therefore, for $Z_\alpha = 1$ and $Z_\beta \in [0,1]$ to be the limiting outcome of a sequence of equilibrium voting strategies, it must be the case that $\Phi_a(Z_\alpha, Z_\beta) > 0$ and $\Phi_b(Z_\alpha, Z_\beta) = 0$ for these values of (Z_α, Z_β).

This intuition suggests that the set of limiting values of (Z^n_α, Z^n_β) that result from limit equilibria is equivalent to the set of limit outcomes. However, this result must be proved formally, and is derived in the following result that establishes that (Z_α, Z_β) is a limit outcome if, and only if, it corresponds to the limit of (Z^n_α, Z^n_β) for a convergent sequence of finite equilibria:

Theorem **1** (Approximation of outcomes of large committees)**.** *Generically, in the space of all payoff vectors,*[2]

(1) Given any limit outcome (Z_α, Z_β), there exists a sequence of equilibria of the finite games (σ^{n*}), such that the associated sequences of decision probabilities Z^n_α and Z^n_β converge to Z_α and Z_β.

(2) The sequence of decision probabilities, (Z^n_α, Z^n_β), associated to any sequence of equilibria of the finite games, (σ^{n*}), must converge to the set of limit outcomes.

The proof of Theorem 1, and the proofs of all following formal results can be found in the Online Appendix. Intuitively, Theorem 1 results from the fact that, as $n \to \infty$, the terms in agent i's best response function that condition on i being pivotal converge uniformly to 0 for any set of strategies, resulting in an equivalence of the limiting "fixed points" of the best response functions represented by $\Phi^n_{s_i}$ and the fixed points of Φ_{s_i}.

To establish our main result, we utilize the result of Theorem 1 to first characterize the set of vote-contingent payoffs that aggregate information for a given signal structure, and then use this characterization to prove our impossibility result. To begin, we present a result that allows us to restrict attention to vote-contingent payoffs with $k_{\alpha,a} > 0$, $k_{\beta,b} > 0$; i.e. rewarding agents for matching their vote to the state.

Lemma **1** (Sincere Voting)**.** *Information aggregation is robust for a given vector of parameters $(k_{\omega,x})$ if and only if $Z = (1, 0)$ is a limit outcome under $(k_{\omega,x})$ with $\Phi_a > 0$ and $\Phi_b < 0$.*

Lemma 1 shows that the payoff vectors that result in robust information aggregation are equivalent to the payoff vectors that give agents a strict incentive to vote sincerely, defined as $v_i = s_i$, as $n \to \infty$ given $Z = (1, 0)$. Therefore, given Lemma 1 and the result of Theorem 1, we are able to characterize the set of vote-contingent payoffs that result in robust information aggregation by identifying the set of payoffs that satisfy $\Phi_a > 0$ and $\Phi_b < 0$:

Proposition **3** (Robustness Representative Democracy)**.** *Given an information structure $\{\Pr\alpha, \epsilon\}$, a necessary and sufficient condition for robust information aggregation with $k_{\alpha,a} > 0$, $k_{\beta,b} > 0$, is that:*

$$\frac{k_{\alpha,a}}{k_{\beta,b}} \in \left(\frac{Pr(\beta|s_i = a)}{Pr(\alpha|s_i = a)}, \frac{Pr(\beta|s_i = b)}{Pr(\alpha|s_i = b)} \right).$$

[2]That is, any payoff vector $(k_{\omega,x})$ to which the theorem does not apply is arbitrarily close to vectors to which it does apply. Moreover, while there are certain (non-generic) border cases to which Theorem 1 does not apply, Proposition 4 in the Appendix fully characterizes the set of these points and shows that it has measure zero. Importantly, Lemma 1 below also implies that these non-generic cases do not impact our main result of characterizing the set of vote-contingent payoffs that lead to robust information aggregation.

Proposition 3 states the conditions on vote contingent payoffs that are necessary and sufficient for information aggregation given $k_{\alpha,a} > 0$, $k_{\beta,b} > 0$. We should emphasize that it is only the relative size of the vote-contingent payoffs that matter—information aggregation can be achieved with vote-contingent payoffs that are very small (or very large), as long as they are balanced relative to the prior. In what follows we refer to any payoff vector which satisfies the conditions of Proposition 3 as *RIA* (Robust Information Aggregation) payoffs.

Our main result uses the characterization in Proposition 3 to show that there is no set of vote-contingent payoffs that result in consistent information aggregation.

THEOREM **2** (Non-Existence Consistent Information Aggregation)**.** *Given any set of vote-contingent payoffs* $\{k_{\omega,v_i}\}$ *with* $k_{\omega,v_i} > 0$*, information aggregation fails for some information structures* $\{\}$.

That is, Theorem 2 shows that, in contrast to direct democracy, under representative democracy there are no vote-contingent payoffs that consistently aggregate information. Intuitively, Theorem 2 follows from the fact that for any signal precision, there exists two priors ($\Pr(\alpha)$) such that the corresponding set of RIA payoffs are non-overlapping.

4 Conclusion

Our paper contrasts the ability of a body of voters to aggregate information through voting if the voters only receive instrumental payoffs based on the committee outcome (direct democracy) or if voters also receive instrumental payoffs based on their individual vote (representative democracy). We show that while representative democracy can result in robust information aggregation for any *given* information structure, there is no set of vote-contingent payoffs that results in consistent information aggregation (i.e. information aggregation for *every* information structure). This implies that direct democracy, while non-robust, may do a better job at consistently aggregating information.

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A Appendix: Proofs of Formal Results

Before presenting the proof for Theorem 1, we introduce some notation that will be helpful for the proof: we let $\mu_{\omega,n} = \sigma^n(a)(1-\varepsilon) + \sigma^n(b)\varepsilon$ be the probability that a randomly chosen agent votes for a given σ^n and ω. In the limit as $n \to \infty$ we have $\mu_\alpha = \sigma(a)(1-\varepsilon) + \sigma(b)\varepsilon$ and $\mu_\beta = \sigma(a)\varepsilon + \sigma(b)(1-\varepsilon)$.

Proof of Theorem 1: We prove part (1) of Theorem 1 by addressing three different kinds of *limit outcomes* separately. Lemma 2 addresses limit outcomes (Z_α, Z_β) at which both inequalities hold strictly, $\Phi_a \lesseqgtr 0$ and $\Phi_b \lesseqgtr 0$. Lemma 3 addresses limit outcomes at which exactly one of the inequalities holds with equality, at least one of Z_α and Z_β is interior (belongs to $(0,1)$) and its slope in the inequality that holds with equality is non-zero. Lemma 4 addresses the case in which both inequalities hold with equality and both Z_α and Z_β are interior. Proposition 4, presented immediately after this proof, shows that the set of points to which none of Lemmas 2, 3 and 4 apply has measure 0, thus establishing that the theorem holds generically, as pointed out by its statement.

Part (2) of Theorem (1) is proved in Lemma 5.

Lemma 2. If (Z_α, Z_β) is a limit outcome at which the two inequalities (associated to Φ_a and Φ_b) hold strictly, then there exists a sequence of equilibria of the finite games (σ^{n*}) such that the associated sequences of decision probabilities (Z_α^n) and (Z_β^n) converge to Z_α and Z_β.

Proof of Lemma 2: If both inequalities hold strictly, then let $\sigma^{n*}(a) = 1$ if $\Phi_a(1, 0) > 0$, $\sigma^{n*}(a) = 0$ if $\Phi_a(1, 0) < 0$, $\sigma^{n*}(b) = 1$ if $\Phi_b(1, 0) > 0$, $\sigma^{n*}(b) = 0$ if $\Phi_b(1, 0) < 0$.

Due to convergence of $\Phi^n_{s_i}$ to Φ_{s_i} for all sufficiently large n, the inequalities associated to each of Φ^n_a and Φ^n_b will hold strictly when evaluated at $(\sigma^{n*}(a), \sigma^{n*}(b))$. This is the case, because given that $\varepsilon < 1/2$, (Z^n_α) and (Z^n_β) will converge to Z_α and Z_β as given in Definition 4 (of limit outcomes). Thus, for all sufficiently large n, σ^{n*} will be an equilibrium and (Z^n_α) and (Z^n_β) will converge to Z_α and Z_β as required. ∎

Lemma 3. If (Z_α, Z_β) is a limit outcome at which only one of two inequalities holds with equality (call it Φ_s), at least one of $Z_\alpha \in (0, 1)$ or $Z_\beta \in (0, 1)$ holds (call it Z_ω), and the slope of Z_ω in Φ_s is non-zero, then there exists a sequence of equilibria of the finite games (σ^n) such that the associated sequences of decision probabilities (Z^n_α) and (Z^n_β) converge to Z_α and Z_β.

Proof of Lemma 3: Assume that the inequality associated to Φ_s holds with equality and the one associated to $\Phi_{s'}$ holds strictly and let $Z_\omega \in (0, 1)$. We denote the state of the world different from ω by ω'. Then for all sufficiently small δ the inequality associated to $\Phi_{s'}$ evaluated at $(Z_\omega + \delta, Z_{\omega'})$, continues to hold strictly (and has the same direction as when evaluated at $(Z_\omega, Z_{\omega'})$), and $\Phi_s > 0$; and evaluated at $(Z_\omega - \delta, Z_{\omega'})$, the inequality associated to $\Phi_{s'}$ continues to hold strictly (and has the same direction as when evaluated at $(Z_\omega, Z_{\omega'})$), and $\Phi_s < 0$. This is because of the fact that Φ_a and Φ_b are linear functions of Z_α and Z_β and the slope of Z_ω in Φ_s is non-zero.[3]

Now, for all sufficiently large n it must be the case if we fix $Z^n = (Z_\omega - \delta, Z_{\omega'})$, or $Z^n = (Z_\omega + \delta, Z_{\omega'})$ then the inequalities above hold for Φ^n_a and Φ^n_b independently of the strategy σ_n that we use to evaluate the additional terms in Φ^n_a and Φ^n_b associated to the probabilities of the event piv_i. This is the case because of the uniform convergence to 0 of these probabilities. We split the rest of the proof into cases: (case I) $\Phi_a > 0$, $\Phi_b = 0$, (case II) $\Phi_a = 0$, $\Phi_b < 0$, (case III) $\Phi_a < 0$, $\Phi_b = 0$ and (case IV) $\Phi_a = 0$, $\Phi_b > 0$.

(case I) $(\Phi_a > 0, \Phi_b = 0)$
Then $\sigma(a) = 1$ and $Z_\alpha = 1$ and therefore $Z_\beta \in (0, 1)$. Notice that regardless of what $\sigma_n(b)$ is, we can have Z^n_α approximate 1 as well as we want by choosing n sufficiently large. Furthermore the quality of the approximation is increasing in $\sigma^n(b)$.[4] Evaluated at $(\sigma_n(a), \sigma_n(b)) = (1, 0)$, $\mu_\beta < 1/2$ and at $(\sigma_n(a), \sigma_n(b)) = (1, 1)$, $\mu_\beta > 1/2$ so for all sufficiently large n, Z^n_β is smaller than $Z_\beta + \delta$ when evaluated at $(1, 0)$ and larger than $Z_\beta + \delta$ when evaluated at $(1, 1)$. By continuity we can find $\bar{\sigma}_n(b)$ such that $Z^n_\beta = Z_\beta + \delta$. It follows that for all large enough n, $\Phi^n_a > 0$ and $\Phi^n_b > 0$ when evaluated at $(1, \bar{\sigma}_n(b))$. The key is that for any ϵ we can find an N such that for all $n > N$ we get $Z^n_\alpha > 1 - \epsilon$ and $Z^n_\beta = Z_\beta + \delta$ when evaluated at $(1, \bar{\sigma}_n(b))$.

Similarly, we can find $\underline{\sigma}_n(b)$ such that $Z^n_\beta = Z_\omega - \delta$ so that for all large enough n, $\Phi^n_a > 0$

[3]Note that the δ may need to be negative; when we say "for all small enough δ" we mean in absolute value.
[4]So we can establish the required threshold for n by considering $\sigma^n(b) = 0$.

and $\Phi_b^n < 0$ when evaluated in $(1, \underline{\sigma}_n(b))$. It follows that there exists $\sigma_n(b) \in (\underline{\sigma}_n(b), \bar{\sigma}_n(b))$ such that $\Phi_a^n > 0$ and $\Phi_b^n = 0$ when evaluated at $(1, \sigma_n(b))$. Pick n_1 large enough such that this is the case and notice that $(1, \sigma_n(b))$ is an equilibrium of the game with n_1 players. Furthermore, note that $Z_{n_1}^{\alpha} > 1 - \epsilon$ and $Z_{n_1}^{\beta} \in (Z_\beta - \delta, Z_\beta + \delta)$.[5] We now repeat all the process above but starting with $\delta/2$ instead of δ and $\epsilon/2$ instead of ϵ, and construct $n_2 > n_1$ and $\sigma_{n_2}(b))$. In step k we repeat the process above but starting with δ/k instead of δ and ϵ/k instead of ϵ, and construct $n_k > n_{k-1}$ and $\sigma_{n_k}(b))$. We thus obtain a subsequence of committee sizes and equilibria $(1, \sigma_{n_k}(b))$. We complete the sequence by using the method that we used for n_1 for all games with committee sizes between n_1 and n_2, the method for n_{k-1} for all games with committees of sizes between n_{k-1} and n_k. By construction $Z^n \to Z$.

(case II) ($\Phi_b < 0$, $\Phi_a = 0$)
Then $\sigma(b) = 0$ and $Z_\beta = 0$ and therefore $Z_\alpha \in (0, 1)$. Notice that regardless of what $\sigma_n(a)$ is, we can have Z_β^n approximate 0 as well as we want by choosing n sufficiently large. Furthermore the quality of the approximation is decreasing in $\sigma^n(a)$.[6] Evaluated at $(\sigma_n(a), \sigma_n(b)) = (0, 0)$, $\mu_\alpha < 1/2$ and at $(\sigma_n(a), \sigma_n(b)) = (1, 0)$, $\mu_\alpha > 1/2$ so for all sufficiently large n, Z_α^n is smaller than $Z_\alpha + \delta$ when evaluated at $(0, 0)$ and larger than $Z_\alpha + \delta$ when evaluated at $(1, 0)$. By continuity we can find $\bar{\sigma}_n(a)$ such that $Z_\alpha^n = Z_\alpha + \delta$. It follows that for all large enough n, $\Phi_a^n > 0$ and $\Phi_b^n < 0$ when evaluated at $(\bar{\sigma}_n(a), 0)$. The key is that for any ϵ we can find an N such that for all $n > N$ we get $Z_\beta^n < \epsilon$ and $Z_\alpha^n = Z_\alpha + \delta$ when evaluated in $(\bar{\sigma}_n(a), 0)$.

Similarly, we can find $\underline{\sigma}_n(a)$ such that $Z_\alpha^n = Z_\alpha - \delta$ so that for all large enough n, $\Phi_a^n < 0$ and $\Phi_b^n < 0$ when evaluated in $(\underline{\sigma}_n(a), 0)$. It follows that there exists $\sigma_n(a) \in (\underline{\sigma}_n(a), \bar{\sigma}_n(a))$ such that $\Phi_a^n = 0$ and $\Phi_b^n < 0$ when evaluated at $(\sigma_n(a), 0)$. Pick n_1 large enough such that this is the case and notice $(\sigma_n(a), 0)$ is an equilibrium of the game with n_1 players. Furthermore, note that $Z_{n_1}^{\beta} < \epsilon$ and $Z_{n_1}^{\alpha} \in (Z_\alpha - \delta, Z_\alpha + \delta)$[7] We now repeat all the process above but starting with $\delta/2$ instead of δ and $\epsilon/2$ instead of ϵ, and construct $n_2 > n_1$ and $\sigma_{n_2}(a))$. In step k we repeat the process above but starting with δ/k instead of δ and ϵ/k instead of ϵ, and construct $n_k > n_{k-1}$ and $\sigma_{n_k}(a))$. We thus obtain a subsequence of committee sizes and equilibria $(\sigma_{n_k}(a), 0)$. We complete the sequence by using the method that we used for n_1 for all games with committee sizes between n_1 and n_2, the method for n_{k-1} for all games with committees of sizes between n_{k-1} and n_k. By construction $Z^n \to Z$.

(case III) ($\Phi_a < 0$, $\Phi_b = 0$)
Then $\sigma(a) = 0$ and $Z_\alpha = 0$ and therefore $Z_\beta \in (0, 1)$. Notice that regardless of what $\sigma_n(b)$ is, we can have Z_α^n approximate 0 as well as we want by choosing n sufficiently large. Furthermore the quality of the approximation is decreasing in $\sigma^n(b)$.[8] Evaluated at $(\sigma_n(a), \sigma_n(b)) = (0, 0)$, $\mu_\beta < 1/2$ and at $(\sigma_n(a), \sigma_n(b)) = (0, 1)$, $\mu_\beta > 1/2$ so for all sufficiently large n, Z_β^n is smaller than $Z_\beta + \delta$ when evaluated at $(0, 0)$ and larger than $Z_\beta + \delta$ when evaluated at $(0, 1)$. By

[5]Notice that δ was not necessarily positive. If $\delta < 0$ then the correct interval is just $(Z_\beta + \delta, Z_\beta - \delta)$.
[6]So we can establish the required threshold for n by considering $\sigma^n(a) = 1$.
[7]Notice that δ was not necessarily positive. If $\delta < 0$ then the correct interval is just $(Z_\beta + \delta, Z_\beta - \delta)$.
[8]So we can establish the required threshold for n by considering $\sigma^n(b) = 1$.

continuity we can find $\bar{\sigma}_n(b)$ such that $Z^n_\beta = Z_\beta + \delta$. It follows that for all large enough n, $\Phi^n_a < 0$ and $\Phi^n_b > 0$ when evaluated at $(0, \bar{\sigma}_n(b))$. The key is that for any ϵ we can find an N such that for all $n > N$ we get $Z^n_\alpha < \epsilon$ and $Z^n_\beta = Z_\beta + \delta$ when evaluated in $(0, \bar{\sigma}_n(b))$.

Similarly, we can find $\underline{\sigma}_n(b)$ such that $Z^n_\beta = Z_\omega - \delta$ so that for all large enough n, $\Phi^n_a < 0$ and $\Phi^n_b < 0$ when evaluated in $(0, \underline{\sigma}_n(b))$. It follows that there exists $\sigma_n(b) \in (\underline{\sigma}_n(b), \bar{\sigma}_n(b))$ such that $\Phi^n_a < 0$ and $\Phi^n_b = 0$ when evaluated at $(0, \sigma_n(b))$. Pick n_1 large enough such that this is the case and notice that $(0, \sigma_n(b))$ is an equilibrium of the game with n_1 players. Furthermore, note that $Z^\alpha_{n_1} <$ and $Z^\beta_{n_1} \in (Z_\beta - \delta, Z_\beta + \delta)$.[9] We now repeat all the process above but starting with $\delta/2$ instead of δ and $\epsilon/2$ instead of ϵ, and construct $n_2 > n_1$ and $\sigma_{n_2}(b))$. In step k we repeat the process above but starting with δ/k instead of δ and ϵ/k instead of ϵ, and construct $n_k > n_{k-1}$ and $\sigma_{n_k}(b))$. We thus obtain a subsequence of committee sizes and equilibria $(0, \sigma_{n_k}(b))$. We complete the sequence by using the method that we used for n_1 for all games with committee sizes between n_1 and n_2, the method for n_{k-1} for all games with committees of sizes between n_{k-1} and n_k. By construction $Z^n \to Z$.

(case IV) $(\Phi_b > 0,\ \Phi_a = 0)$
Then $\sigma(b) = 1$ and $Z_\beta = 1$ and therefore $Z_\alpha \in (0, 1)$. Notice that regardless of what $\sigma_n(a)$ is, we can have Z^n_β approximate 1 as well as we want by choosing n sufficiently large. Furthermore the quality of the approximation is increasing in $\sigma^n(a)$.[10] Evaluated at $(\sigma_n(a), \sigma_n(b)) = (0, 1)$, $\mu_\alpha < 1/2$ and at $(\sigma_n(a), \sigma_n(b)) = (1, 1)$, $\mu_\alpha > 1/2$ so for all sufficiently large n, Z^n_α is smaller than $Z_\alpha + \delta$ when evaluated at $(0, 1)$ and larger than $Z_\alpha + \delta$ when evaluated at $(1, 1)$. By continuity we can find $\bar{\sigma}_n(a)$ such that $Z^n_\alpha = Z_\alpha + \delta$. It follows that for all large enough n, $\Phi^n_a > 0$ and $\Phi^n_b > 0$ when evaluated at $(\bar{\sigma}_n(a), 1)$. The key is that for any ϵ we can find an N such that for all $n > N$ we get $Z^n_\beta > 1 - \epsilon$ and $Z^n_\alpha = Z_\alpha + \delta$ when evaluated in $(\bar{\sigma}_n(a), 1)$.

Similarly, we can find $\underline{\sigma}_n(a)$ such that $Z^n_\alpha = Z_\alpha - \delta$ so that for all large enough n, $\Phi^n_a < 0$ and $\Phi^n_b > 0$ when evaluated in $(\underline{\sigma}_n(a), 1)$. It follows that there exists $\sigma_n(a) \in (\underline{\sigma}_n(a), \bar{\sigma}_n(a))$ such that $\Phi^n_a = 0$ and $\Phi^n_b > 0$ when evaluated at $(\sigma_n(a), 1)$. Pick n_1 large enough such that this is the case and notice $(\sigma_n(a), 1)$ is an equilibrium of the game with n_1 players. Furthermore, note that $Z^\beta_{n_1} < \epsilon$ and $Z^\alpha_{n_1} \in (Z_\alpha - \delta, Z_\alpha + \delta)$.[11] We now repeat all the process above but starting with $\delta/2$ instead of δ and $\epsilon/2$ instead of ϵ, and construct $n_2 > n_1$ and $\sigma_{n_2}(a))$. In step k we repeat the process above but starting with δ/k instead of δ and ϵ/k instead of ϵ, and construct $n_k > n_{k-1}$ and $\sigma_{n_k}(a))$. We thus obtain a subsequence of committee sizes and equilibria $(\sigma_{n_k}(a), 1)$. We complete the sequence by using the method that we used for n_1 for all games with committee sizes between n_1 and n_2, the method for n_{k-1} for all games with committees of sizes between n_{k-1} and n_k. By construction $Z^n \to Z$. ∎

Lemma 4. Given any limit outcome (Z^*_α, Z^*_β), such that

[9] Notice that δ was not necessarily positive. If $\delta < 0$ then the correct interval is just $(Z_\beta + \delta, Z_\beta - \delta)$.

[10] So we can establish the required threshold for n by considering $\sigma^n(a) = 0$.

[11] Notice that δ was not necessarily positive. If $\delta < 0$ then the correct interval is just $(Z_\beta + \delta, Z_\beta - \delta)$.

(1) The two inequalities (associated to Φ_a and Φ_b) hold with equality.

(2) $Z_\alpha \in (0,1)$ and $Z_\beta \in (0,1)$,

then there exists a sequence of equilibria of the finite games (σ^{n*}) such that the associated sequences of decision probabilities (Z_α^n) and (Z_β^n) converge to Z_α^* and Z_β^*.

Proof of Lemma 4: Note that Φ_a and Φ_b are both linearly increasing, or decreasing, in Z_α and Z_β where the slope (which is non-zero as $k_{\alpha,a} - k_{\alpha,b} = 0$ or $k_{\beta,b} - k_{\beta,a} = 0$ rules out (Z^*, σ^*)) of Φ_a (Φ_b) is steeper with respect to Z_α (Z_β). By this, there exists a constant δ such that $(Z_\alpha^* + \delta), (Z_\alpha^* - \delta) \in (0,1)$ and $Z'_\beta, Z''_\beta \in (0,1)$ such that $\Phi_a = (Z_\alpha^* + \delta)(k_{\alpha,a} - k_{\alpha,b})Pr(\alpha|s_i = a) + Z'_\beta(k_{\beta,b} - k_{\beta,a})Pr(\beta|s_i = a) + k_{\alpha,b}Pr(\alpha|s_i = a) - k_{\beta,b}Pr(\beta|s_i = a) = 0$ and $\Phi_a = (Z_\alpha^* - \delta)(k_{\alpha,a} - k_{\alpha,b})Pr(\alpha|s_i = a) + Z''_\beta(k_{\beta,b} - k_{\beta,a})Pr(\beta|s_i = a) + k_{\alpha,b}Pr(\alpha|s_i = a) - k_{\beta,b}Pr(\beta|s_i = a) = 0$ whereby $\Phi_b(Z_\alpha^* + \delta, Z'_\beta) > x$ and $\Phi_b(Z_\alpha^* - \delta, Z''_\beta) < -x$, where x is some positive constant. By the same token, there exists $\delta_m = \frac{\delta}{m}$, where $m = 1, 2, ...$, such that $(Z_\alpha^* + \delta_m), (Z_\alpha^* - \delta_m) \in (0,1)$ and $Z'_{\beta m}, Z''_{\beta m} \in (0,1)$ such that, for any m, we have $\Phi_a = (Z_\alpha^* + \delta_m)(k_{\alpha,a} - k_{\alpha,b})Pr(\alpha|s_i = a) + Z'_{\beta m}(k_{\beta,b} - k_{\beta,a})Pr(\beta|s_i = a) + k_{\alpha,b}Pr(\alpha|s_i = a) - k_{\beta,b}Pr(\beta|s_i = a) = 0$ and $\Phi_a = (Z_\alpha^* - \delta_m)(k_{\alpha,a} - k_{\alpha,b})Pr(\alpha|s_i = a) + Z''_{\beta m}(k_{\beta,b} - k_{\beta,a})Pr(\beta|s_i = a) + k_{\alpha,b}Pr(\alpha|s_i = a) - k_{\beta,b}Pr(\beta|s_i = a) = 0$ whereby $\Phi_b(Z_\alpha^* + \delta_m, Z'_{\beta m}) > x_m$ and $\Phi_b(Z_\alpha^* - \delta_m, Z''_{\beta m}) < -x_m$, where x_m is some positive constant. Moreover, $(Z_\alpha^* + \delta_m)$, $(Z_\alpha^* - \delta_m)$ converge to Z_α^* and $Z'_{\beta m}$, $Z''_{\beta m}$ converge to $Z_{\beta*}$ as $m \to \infty$.

Recall that

$$\begin{aligned}
\Phi_{s_i}^n(\sigma) =& (k(a,a,\alpha) - k(a,b,\alpha))Pr(piv_i|\alpha)Pr(\alpha|s_i) - (k(a,b,\beta) - k(a,a,\beta))Pr(piv_i|\beta)Pr(\beta|s_i) \\
&+ (k_{\alpha,a} - k_{\alpha,b})Pr(a, \neg piv_i|\alpha)Pr(\alpha|s_i) + (k_{\beta,b} - k_{\beta,a})Pr(a, \neg piv_i|\beta)Pr(\beta|s_i) \\
&+ k_{\alpha,b}Pr(\alpha|s_i) - k_{\beta,b}Pr(\beta|s_i)
\end{aligned}$$

where Φ_a^n and Φ_b^n are continuous in $\sigma(a)$ and $\sigma(b)$ and $Pr(a, \neg piv_i|\alpha)$ is a continuous, and strictly increasing, function of $\mu_\alpha = \sigma(a)(1-\varepsilon) + \sigma(b)\varepsilon$ and $Pr(a, \neg piv_i|\beta)$ is a continuous, and strictly increasing, function of $\mu_\beta = \sigma(a)\varepsilon + \sigma(b)(1-\varepsilon)$. For $\sigma = (1,1)$ we have $Pr(a, \neg piv_i|\alpha) = 1$ and $Pr(a, \neg piv_i|\beta) = 1$ and for $\sigma = (0,0)$ we have $Pr(a, \neg piv_i|\alpha) = 0$ and $Pr(a, \neg piv_i|\beta) = 0$. Consider some n and let $\mu_{\alpha,n}^*$ indicate the unique $\mu_{\alpha,n}$ such that $Pr(a, \neg piv_i|\alpha) = (Z_\alpha^* + \delta)$. Given $\mu_{\alpha,n}^*$, the highest possible $\sigma^n(a)$ is attained with $\sigma^n = (1, \frac{\mu_{\alpha,n}^* - (1-\varepsilon)}{\varepsilon})$ if $\mu_{\alpha,n}^* \geq (1-\varepsilon)$ and $\sigma^n = (\frac{\mu_{\alpha,n}^*}{(1-\varepsilon)}, 0)$ if $\mu_{\alpha,n}^* < (1-\varepsilon)$ and the lowest possible $\sigma^n(a)$ is attained with $\sigma^n = (0, \frac{\mu_{\alpha,n}^*}{\varepsilon})$ if $\mu_{\alpha,n}^* \leq \varepsilon$ and $\sigma^n = (\frac{\mu_{\alpha,n}^* - \varepsilon}{(1-\varepsilon)}, 1)$ if $\mu_{\alpha,n}^* > \varepsilon$. Choosing the highest possible $\sigma^n(a)$ gives the lowest feasible $Pr(a, \neg piv_i|\beta)$ and choosing the lowest possible $\sigma^n(a)$ gives the highest feasible $Pr(a, \neg piv_i|\beta)$. As $(Z_\alpha^* + \delta) \in (0,1)$ we must have that $\mu_{\alpha,n}^*$ converges to $\frac{1}{2}$ as $n \to \infty$ and thereby, for all sufficiently large n, the two relevant extremes are $\sigma^n = (\frac{\mu_{\alpha,n}^*}{(1-\varepsilon)}, 0)$ and $\sigma^n = (\frac{\mu_{\alpha,n}^* - \varepsilon}{(1-\varepsilon)}, 1)$. For these two extremes $Pr(a, \neg piv_i|\beta)$ converge to 0 and 1, respectively, as $n \to \infty$, and since $Z'_\beta \in (0,1)$ (because by assumption $k_{\beta,b} - k_{\beta,a} \neq 0$) then, for all sufficiently large n, $\Phi^n_a > 0$ for one of the extremes and $\Phi^n_a < 0$ for the other (note that $\Phi^n_a(\frac{\mu^*_{\alpha,n}}{(1-\varepsilon)}, 0) \to \Phi_a(Z^*_\alpha + \delta, 0)$ and $\Phi^n_a(\frac{\mu^*_{\alpha,n}-\varepsilon}{(1-\varepsilon)}, 1) \to \Phi_a(Z^*_\alpha + \delta, 1)$. By continuity of Φ^n_a in the σ-plane then, for all sufficiently large n, there exists an intermediate σ^n such that $\Phi^n_a(\sigma) = 0$. Moreover, given such σ^n whereby $\Phi^n_a(\sigma) = 0$ we have that

$$\begin{aligned}\Phi^n_a(\sigma) =&(k(a,a,\alpha) - k(a,b,\alpha))Pr(piv_i|\alpha)Pr(\alpha|s_i) - (k(a,b,\beta) - k(a,a,\beta))Pr(piv_i|\beta)Pr(\beta|s_i)\\ &+ (k_{\alpha,a} - k_{\alpha,b})(Z^*_\alpha + \delta)Pr(\alpha|s_i) + (k_{\beta,b} - k_{\beta,a})Pr(a,\neg piv_i|\beta)Pr(\beta|s_i)\\ &+ k_{\alpha,b}Pr(\alpha|s_i) - k_{\beta,b}Pr(\beta|s_i) = 0,\end{aligned}$$

and as $n \to \infty$ the pivotal terms uniformly converge to zero and we can conclude that, given our σ^n such that $\Phi^n_a(\sigma) = 0$, the term $Pr(a, \neg piv_i|\beta)$ must converge to Z'_β (where Z'_β is as defined above ensuring that $\Phi_a = 0$ given $Z_\alpha = (Z^*_\alpha + \delta)$).

By the parallel arguments we can fix $Pr(a, \neg piv_i|\alpha) = (Z^*_\alpha - \delta)$ and, for all sufficiently large n, there exists a σ^n such that $\Phi^n_a(\sigma) = 0$ and $Pr(a, \neg piv_i|\beta)$ converges to Z''^β for $n \to \infty$. Similarly if we consider $Pr(a, \neg piv_i|\alpha) = (Z^*_\alpha + \delta')$ and $Pr(a, \neg piv_i|\alpha) = (Z^*_\alpha - \delta')$ for $\delta' \in [0, \delta]$. For sufficiently large n this constitutes a span of strategies.

Now fix $Pr(a, \neg piv_i|\alpha) = (Z^*_\alpha + \delta)$ and $Pr(a, \neg piv_i|\alpha) = (Z^*_\alpha - \delta)$ then, for sufficiently large n, call it n_1, there exists σ^{n_1} such that $\Phi^{n_1}_a(\sigma) = 0$ and $\Phi^{n_1}_b(\sigma) < 0$ and another σ^{n_1} such that $\Phi^{n_1}_a(\sigma) = 0$ and $\Phi^{n_1}_b(\sigma) > 0$ and by continuity there exists σ^{n_1} such that $\Phi^{n_1}_a(\sigma) = 0$ and $\Phi^{n_1}_b(\sigma) = 0$. We now repeat the process starting with δ_2 and $-\delta_2$ and we have $n_2 > n_1$ and σ^{n_2} such that $\Phi^{n_2}_a(\sigma) = 0$ and $\Phi^{n_2}_b(\sigma) = 0$. We do this for any δ_m and $-\delta_m$ and construct $n_m > n_{m-1}$ and we have a subsequence of committee sizes and equilibria with associated Z^{n*}_α and Z^{n*}_β converging to Z^*_α and Z^*_β. We complete the sequence by using the method that we used for n_1 for all games with committee sizes between n_1 and n_2, the method for n_{m-1} for all games with committees of sizes between n_{m-1} and n_m. ∎

Lemma 5. *The sequence of decision probabilities, (Z^n_α, Z^n_β), associated to any sequence of equilibria of the finite games, (σ^n), must converge to the set of limit outcomes.*

Proof of Lemma 5: Let (σ^n) be a sequence of equilibria of the finite games and suppose $(Z^n_\alpha, Z^n_\beta) = (\Pr^n(a|\alpha), (\Pr^n(a|\beta))$ is the associated sequence of decision probabilities. Suppose that there exists $\epsilon > 0$ such that there is an infinite subsequence of terms $(\Pr^n(a|\alpha), (\Pr^n(a|\beta))$ which are at least ε away from any pair (Z_α, Z_β) that is a limit outcome. Because all the terms in this subsequence are bounded above and below (by $(0, 0)$ and $(1, 1)$), it must have a convergent subsequence. Call it's limit point (Y_α, Y_β). by construction we thus have that for any limit outcome (Z_α, Z_β), $||(Z_\alpha, Z_\beta) - (Y_\alpha, Y_\beta)|| \geq \varepsilon$. So (Y_α, Y_β) must violate at least one of the four conditions that define a limit outcome. Suppose that it violates (1a). That is, suppose that $\Phi_a(Y_\alpha, Y_\beta) > 0$ yet $Y_\alpha < 1$. Let $h = 1 - Y_\alpha$ This means that for all sufficiently large n we must have $\Phi_a(Z^n_\alpha, Z^n_\beta) > 0$ and $Z^n_\alpha < 1 - h/2$, but this is a contradiction, since $\Phi_a(Z^n_\alpha, Z^n_\beta) > 0$

implies that $\sigma_n(a) = 1$ and thus $Z_\alpha^n \to 1$. The other 3 cases are anlogous. ▮

Proposition 4 *(Theorem 1 applies generically). The set of parameter vectors with the property that there exists some limit outcome (Z_α, Z_β) at which Theorem 1 does not apply has measure 0. That is, Theorem 1 holds generically in $\mathbb{R}^8$.*

Proof of Proposition 4: The only cases to which the argument presented in Lemmas 2,3 and 4 do not apply involve one of the following two conditions:

1. A vector of parameters $(k(v_i, x, \omega))$ and a limit outcome such that only one of $\Phi_{s_i}(Z_\alpha, Z_\beta)$ ($s_i = a$ or $s_i = b$) is 0, $Z_\alpha \notin (0, 1)$ and $Z_\beta \notin (0, 1)$.
2. A vector of parameters $(k(v_i, x, \omega))$ and a limit outcome such that only one of $\Phi_{s_i}(Z_\alpha, Z_\beta)$ ($s_i = a$ or $s_i = b$) is 0, (call it Φ_s), only one of Z_α or Z_β is interior (call it Z_ω) and the multiplier of Z_ω in Φ_s is 0.
3. Both $\Phi_{s_i}(Z_\alpha, Z_\beta) = 0$ (for $s_i = a$ and $s_i = b$) and either (or both) $Z_\alpha \notin (0, 1)$ or $Z_\beta \notin (0, 1)$.

We proceed by showing that the set of vectors $(k(v_i, x, \omega))$ for which each of the three conditions above can hold is a subspace of $\mathbb{R}^8$ of dimension strictly less than 8, and therefore of measure 0 in $\mathbb{R}^8$. In fact, we will show the stronger property that the sets of vectors $(k_{\omega,x})$ for which each of the conditions above can hold is a subspace of $\mathbb{R}^4$ of dimension strictly less than 4, and therefore of measure 0 in $\mathbb{R}^4$. Since the union of finitely many sets of measure 0 is of measure 0 the result follows.

(Condition 1) Let s be such that $\Phi_s(Z_\alpha, Z_\beta) = 0$. Assume that $Z_\alpha = 0$ and $Z_\beta = 0$. Then it follows that $Pr(\alpha|s)k_{\alpha,b} - Pr(\beta|s)k_{\beta,b} = 0$ which given that $Pr(\alpha|s) > 0$ and $Pr(\beta|s) > 0$ defines a subspace of dimension 3 in $\mathbb{R}^4$. Similarly in case $Z_\alpha = 1$ and $Z_\beta = 0$ then the analogous condition is $Pr(\alpha|s)k_{\alpha,a} - Pr(\beta|s)k_{\beta,b} = 0$. In case $Z_\alpha = 0$ and $Z_\beta = 1$ then it is $Pr(\alpha|s)k_{\alpha,b} - Pr(\beta|s)k_{\beta,a} = 0$. In case $Z_\alpha = 1$ and $Z_\beta = 1$ then it is $Pr(\alpha|s)k_{\alpha,a} - Pr(\beta|s)k_{\beta,a} = 0$.

(Condition 2) Let s be such that $\Phi_s(Z_\alpha, Z_\beta) = 0$. Assume that Z_α is interior and the multiplier of Z_α in Φ_s is 0. Then $k_{\alpha,a} - k_{\alpha,b} = 0$ which defines a subspace of dimension 3 in $\mathbb{R}^4$. Similarly, if Z_β is interior and the multiplier of Z_β in Φ_s is 0 then $k_{\beta,a} - k_{\beta,b} = 0$.

(Condition 3) Suppose that $\Phi_a(Z_\alpha, Z_\beta) = 0$ and $\Phi_b(Z_\alpha, Z_\beta) = 0$ If $k_{\alpha,a} - k_{\alpha,b} = 0$ or $k_{\beta,a} - k_{\beta,b} = 0$ then as above we have subspaces of dimension 3 in $\mathbb{R}^4$. Otherwise, suppose $Z_\alpha \notin (0, 1) = 0$. It follows by solving for Z_β in each of the equations $\Phi_a(Z_\alpha, Z_\beta) = 0$ and $\Phi_b(Z_\alpha, Z_\beta) = 0$, that,

$$Z_\beta = \frac{Z_\alpha(k_{\alpha,a} - k_{\alpha,b})Pr(\alpha|a) - k_{\beta,b}Pr(\beta|a) + k_{\alpha,b}Pr(\alpha|a)}{(k_{\beta,a} - k_{\beta,b})}$$
$$= \frac{Z_\alpha(k_{\alpha,a} - k_{\alpha,b})Pr(\alpha|b) - k_{\beta,b}Pr(\beta|b) + k_{\alpha,b}Pr(\alpha|b)}{(k_{\beta,a} - k_{\beta,b})}$$

If $Z_\alpha = 0$, then $k_{\alpha,b}(Pr(\alpha|a) - Pr(\alpha|b)) + k_{\beta,b}(Pr(\beta|b) - Pr(\beta|a)) = 0$. Given that $Pr(\alpha|a) > Pr(\alpha|b)$ and $Pr(\beta|b) > Pr(\beta|a)$, this equation defines a 3 dimensional subspace in $\mathbb{R}^4$. Similarly, if $Z_\alpha = 1$, then $k_{\alpha,a}(Pr(\alpha|a) - Pr(\alpha|b)) + k_{\beta,b}(Pr(\beta|b) - Pr(\beta|a)) = 0$.

If on the other hand $Z_\beta \notin (0,1) = 0$ we solve for Z_α in each of the equations $\Phi_a(Z_\alpha, Z_\beta) = 0$ and $\Phi_b(Z_\alpha, Z_\beta) = 0$, and proceed just as above. ∎

The following Proposition, which relies on Proposition 4, is the basis for Lemma 1 in the main text.

Proposition 5. *Given any vector of parameters $(k(v_i, x, \omega))$ at which Theorem 1 does not apply for limit outcome $(Z_\alpha, Z_\beta) = (1, 0)$, and for any ε, there exists a vector of parameters $(k'(v_i, x, \omega))$ at which Theorem 1 does apply for $(Z_\alpha, Z_\beta) = (1, 0)$ and such that (1) $\Phi_a(Z = (1, 0)) < 0$ or $\Phi_b(Z = (1, 0)) > 0$ and (2) $||(k(v_i, x, \omega)) - (k'(v_i, x, \omega))|| < \varepsilon$.*

Proof of Proposition 5: Note that,

$$\Phi_{s_i}(Z_\alpha = 1, Z_\beta = 0) = k_{\alpha,a} Pr(\alpha|s_i) - k_{\beta,b} Pr(\beta|s_i)]$$

Suppose that $\Phi_a(Z_\alpha = 1, Z_\beta = 0) \neq 0$ and $\Phi_b(Z_\alpha = 1, Z_\beta = 0) = 0$. Then by perturbing the parameters just slightly so that $k'_{\beta,b} = k_{\beta,b} - \delta$ (with $\delta > 0$) we can guarantee that at the new parameters, $\Phi_a(Z_\alpha = 1, Z_\beta = 0) \neq 0$ and $\Phi_b(Z_\alpha = 1, Z_\beta = 0) > 0$. It follows that since both inequalities hold strictly at these new parameters, Theorem 1 holds for $(Z_\alpha, Z_\beta) = (1, 0)$ and $\Phi_b(Z_\alpha = 1, Z_\beta = 0) > 0$.

Analogously, suppose that $\Phi_a(Z_\alpha = 1, Z_\beta = 0) = 0$ and $\Phi_b(Z_\alpha 1, Z_\beta = 0) \neq 0$.Then by perturbing the parameters just slightly so that $k'_{\alpha,a} = k_{\alpha,a} - \delta$ (with $\delta > 0$) we can guarantee that at the new parameters, $\Phi_a(Z_\alpha = 1, Z_\beta = 0) < 0$ and $\Phi_b(Z_\alpha = 1, Z_\beta = 0) \neq 0$. It follows that since both inequalities hold strictly at these new parameters, Theorem 1 holds for $(Z_\alpha, Z_\beta) = (1, 0)$ and $\Phi_a(Z_\alpha = 1, Z_\beta = 0) < 0$.

Suppose that $\Phi_a(Z_\alpha = 1, Z_\beta = 0) = 0$ and $\Phi_b(Z_\alpha 1, Z_\beta = 0) = 0$. Then by perturbing the parameters just slightly so that $k'_{\beta,b} = k_{\beta,b} - \delta$ (with $\delta > 0$) we can guarantee that at the new parameters, $\Phi_a(Z_\alpha = 1, Z_\beta = 0) > 0$ and $\Phi_b(Z_\alpha = 1, Z_\beta = 0) > 0$. It follows that since both inequalities hold strictly at these new parameters, Theorem 1 holds for $(Z_\alpha, Z_\beta) = (1, 0)$ and $\Phi_b(Z_\alpha = 1, Z_\beta = 0) < 0$.

By Proposition 4, with the above we have covered all possible parameter vectors at which Theorem 1 does not apply for limit outcome $(Z_\alpha, Z_\beta) = (1, 0)$. ∎

Remark: As stated in Lemma 1 of the main text it follows from Proposition 5 that information aggregation cannot be robust at payoff vectors for which Theorem 1 does not apply.

Proof of Lemma 1. It is clear that if Theorem 1 applies to a given payoff vector at which $(Z_\alpha, Z_\beta) = (1,0)$ as a limit outcome at which $\Phi_a(Z = (1,0)) > 0$ and $\Phi_b(Z = (1,0)) < 0$, then this vector supports robust information aggregation.

By Theorem 1 any vector of parameters $(k_{\omega,x})$ to which it applies and which does not have $(Z_\alpha, Z_\beta) = (1,0)$ as a limit outcome does not support information aggregation to begin with, and therefore cannot support robust information aggregation either. Furthermore for robust information aggregation to be supported by payoff vectors which do have $(Z_\alpha, Z_\beta) = (1,0)$ as a limit outcome it must be the case that $\Phi_a(Z = (1,0)) > 0$ and $\Phi_b(Z = (1,0)) < 0$. If this is not the case then in any open ball around $(k_{\omega,x})$ there will be a payoff vector $(k'_{\omega,x})$ under which either $\Phi_a(Z = (1,0)) < 0$ or $\Phi_b(Z = (1,0)) > 0$ and to which Theorem 1 applies (because it applies generically in the payoffs' space as shown by Proposition 4 above). It follows that $(Z_\alpha, Z_\beta) = (1,0)$ is not a limit outcome for $(k'_{\omega,x})$. By the argument above $(k'_{\omega,x})$ does not support information aggregation, and therefore $(k_{\omega,x})$ does not support robust information aggregation.

The only possible concern is thus with payoff vectors $(k_{\omega,x})$ at which Theorem 1 does not apply. As shown by Proposition 5 above, in any open ball around $(k_{\omega,x})$ there must exist payoff vectors to which Theorem 1 does apply and at which at least one of $\Phi_a(Z = (1,0)) < 0$ or $\Phi_b(Z = (1,0)) > 0$ holds. It follows that robust information aggregation is not supported by $(k_{\omega,x})$. ▮

Proof of Proposition 3. Proposition 3 follows from Lemma 1 and Theorem 1, which show that to robustly aggregate information, it must be the case that $\Phi_a(Z = (1,0)) > 0$ and $\Phi_b(Z = (1,0)) < 0$. Using the expressions for $\Phi_a(Z = (1,0)) > 0$ and $\Phi_b(Z = (1,0)) < 0$ and simplifying gives the condition for robust information aggregation. ▮

Proof of Theorem 2. Consider any set of vote-contingent payoffs $\{k_{\omega,v_i}\}$ with $k_{\alpha,a}, k_{\beta,b} > 0$. Next, consider a prior $Pr(\alpha)$ such that $\frac{(1-Pr(\alpha))}{Pr(\alpha)} \neq \frac{k_{\alpha,a}}{k_{\beta,b}}$. Note that

$$\left(\frac{Pr(\beta|s_i = a)}{Pr(\alpha|s_i = a)}, \frac{Pr(\beta|s_i = b)}{Pr(\alpha|s_i = b)}\right) = \left(\frac{(1-Pr(\alpha))\varepsilon}{Pr(\alpha)(1-\varepsilon)}, \frac{(1-Pr(\alpha))(1-\varepsilon)}{Pr(\alpha)\varepsilon}\right),$$

and thus for ε' sufficiently high we have that

$$\frac{k_{\alpha,a}}{k_{\beta,b}} \notin \left(\frac{(1-Pr(\alpha))\varepsilon'}{Pr(\alpha)(1-\varepsilon')}, \frac{(1-Pr(\alpha))(1-\varepsilon')}{Pr(\alpha)\varepsilon'}\right)$$

By Proposition 3, this implies that information aggregation fails for $\{\varepsilon', Pr(\alpha)\}$. ▮