An Investment-and-Marriage Model with Differential Fecundity

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Abstract

I build a stylized investment-and-marriage model to provide a surprising explanation of why in the United States and many other countries women attend college at higher rates but continue to earn lower average incomes than men. Differential fecundity and a general-equilibrium marriage-market effect form the basis of my explanation. The model can also account for the relationship between age at marriage and personal midlife income for men and women as well as the relationship between age at marriage and spousal midlife income for women. Empirical evidence and calibration results support my explanations for these facts.

Keywords: college gender gap, relationships between age at marriage and income

JEL: C78, D1

1 Introduction

In the United States and many other countries, more women than men go to college, but women continue to earn less than men on average. This paper proposes a stylized model that can explain the two opposing gender gaps with one gender asymmetry, *differential fecundity*: women stay fertile for a shorter period than men. In the model, women expect lower fertility if they choose to make income-improving career investments during their fertile years. Women's shorter fertility length directly deters them from career investments and contributes to the earnings gender gap. It is natural to think that women's shorter fertility length also deters them from college investments. One purpose of this paper is to demonstrate how women's shorter fertility length can surprisingly result in *more* women than men going to college through a general-equilibrium marriage-market

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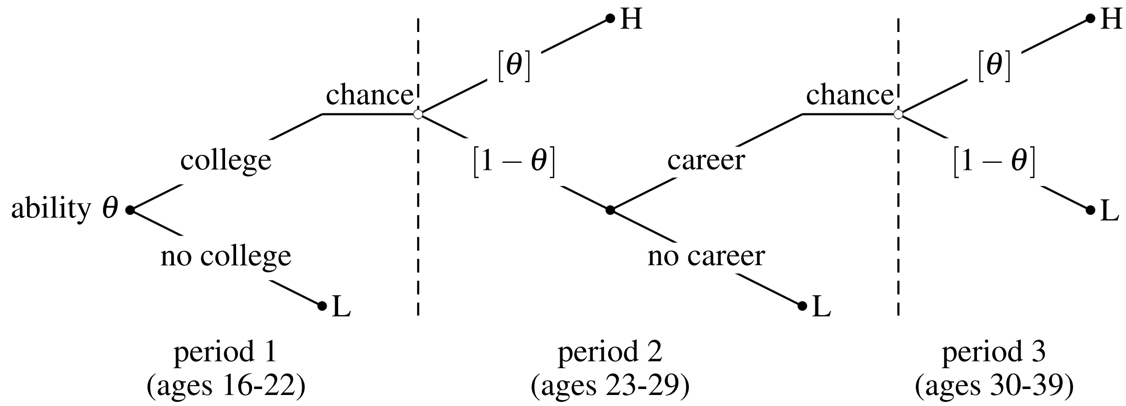
effect. In addition, the model also generates predictions consistent with the gender-specific relationships between age at marriage and midlife income for American men and women as well as the evolving relationship between age at marriage and spousal income for American women born throughout the twentieth century.

The model is as follows. There is an infinite number of discrete periods. A new generation of men and women enters the economy at the beginning of each period. Each agent is endowed with a heterogenous ability (i.e., probability of becoming a high-income earner after an investment), and makes investment and marriage decisions over the next three periods (i.e., ages 16-22, 23-29, and 30-39), summarized in figure 1.

Figure 1: An ability-θ agent's investment and marriage decisions.
Letter H denotes a high lifetime income and letter L denotes a low lifetime income.
An agent enters the marriage market immediately after lifetime income is determined.



In period 1, each agent decides between (i) entering the marriage market as a low-income earner and (ii) making a marriage-delaying college investment to potentially become a high-income earner. In period 2, anyone who went to college either gets a high-income offer and enters the marriage market as a high-income earner, or gets a low-income offer and decides between (i) entering the marriage market as a low-income earner and (ii) making a marriage-delaying career investment to potentially become a high-income earner. In period 3, anyone who made a career investment gets a final income offer and enters the marriage market either as a low-income earner or as a high-income earner. The total surplus of each marriage is determined by husband's and wife's income and fertility, while the division of the total surplus is determined by the supply and demand of these characteristics in the overlapping-generations marriage market. The key gender asymmetry is differential fecundity: men stay fertile throughout the three periods, but women become less fertile in the third period.

Because of differential fecundity, the optimal post-college career investments differ by gender. Facing a low-income offer after college, every man would make a career investment, but only a woman with a sufficiently high ability would do so, because the low expected income gain from a career investment for a woman with a low ability could not compensate for her fertility loss.

The model is flexible enough to allow men and women to differ other than the fertility length. However, even when it is the only gender asymmetry, in the unique equilibrium of our model, more women than men go to college and fewer women than men earn a high income. The reason that more women than men go to college is subtle, operating through a general-equilibrium marriage-market effect. As pointed out above, because of differential fecundity, college women are less likely than college men to make career investments. Consequently, high-income women are more scarce and hence more "valuable" than high-income men in the marriage market, providing an endogenously higher marriage-market incentive for women to go to college. Therefore, in summary, more women than men go to college because of an endogenously higher marriage-market incentive but fewer women than men end up with high incomes because of their shorter fertility length, sustaining the higher marriage-market incentive from college investments for women.

Besides providing an explanation for the college and earnings gender gaps, the model makes predictions consistent with the relationships between age at marriage and personal midlife income for American men and women born throughout the twentieth century. The model's prediction matches the observed hump-shaped relationship for men: in the model, period-2 grooms have a higher average income than period-1 grooms and period-3 grooms, because period-2 grooms are the high-income earners marrying immediately after college, period-1 grooms are low-income earners marrying without investing, and period-3 grooms consist of high-income and low-income earners marrying after career investments. The model's prediction can also match the positive relationship for women: in the model, period-1 brides earn significantly less than period-2 brides, who earn less than period-3 brides, because period-1 brides are low-income earners marrying without investing, period-2 brides are high-income earners and low-income low-ability earners marrying after college, and period-3 brides are high-ability women who earn a high income with high probabilities after career investments.

Finally, the model's prediction matches the hump-shaped relationship between age at marriage and spousal income for women (first documented by Low (2017)): in the model, period-2 brides (i) have a higher average spousal income than period-1 brides who are equally fertile but earn a lower average income, and (ii) have a higher average spousal income than period-3 brides who earn a similar average income but are less fertile. The model's prediction also matches the evolution that women who married between ages 16 and 22 had a higher/lower average spousal income than those who married in their thirties in pre-1950/post-1950 birth cohorts. I show that state mandates to cover or offer infertility treatments in insurances improved the marital outcome of late brides in the expense of early brides, supporting the explanation based on the tradeoff between income and "reproductive capital" emphasized by Low (2017): when fertility is a more/less important trait than income in the marriage market, fertile low-income women marry higher-/lower-income husbands than less fertile high-income women.

In summary, the paper makes three contributions. First, the paper provides the first explanation of the college and earnings gender gaps using only one gender difference, contributing to the line of research on the marriage-market impacts on college (Iyigun and Walsh, 2007; Chiappori et al., 2009; Ge, 2011; Lafortune, 2013; Bruze, 2015; Greenwood et al., 2016; Chiappori et al., 2017).[1] Second, the paper provides a detailed account and a unified explanation of the relationships between age at marriage and personal midlife income, based on labor-market shocks and differential fecundity, complementing the explanations based on search frictions in the marriage market (Becker, 1974) and information frictions in the marriage market (Bergstrom and Bagnoli, 1993).[2] Third, the paper provides a theory consistent with the previous fertility-based explanation of the relationship between age at marriage and spousal income for women (Low, 2017; Gershoni and Low, 2017), and shows that differential fecundity is able to explain even more gender differences in economic and marital outcomes than the previous literature suggests (Siow, 1998; Greenwood et al., 2003; Coles and Francesconi, 2011; Díaz-Giménez and Giolito, 2013).[3]

The rest of the paper is organized as follows. In section 2, I document the four stylized facts to be explained – (a) the college and earnings gender gaps, (b) the relationship between age at marriage and personal income for men, (c) the same relationship for women, and (d) the evolving relationship between age at marriage and spousal income for women. In section 3, I set up my stylized model, characterize its unique equilibrium, and present the model's implications on the four stylized facts, with the omitted proofs in appendix A. In section 4, I summarize the key empirical evidence and calibration results supporting my explanations of the stylized facts, with the omitted details in appendix B.

2 Documenting Four Stylized Facts

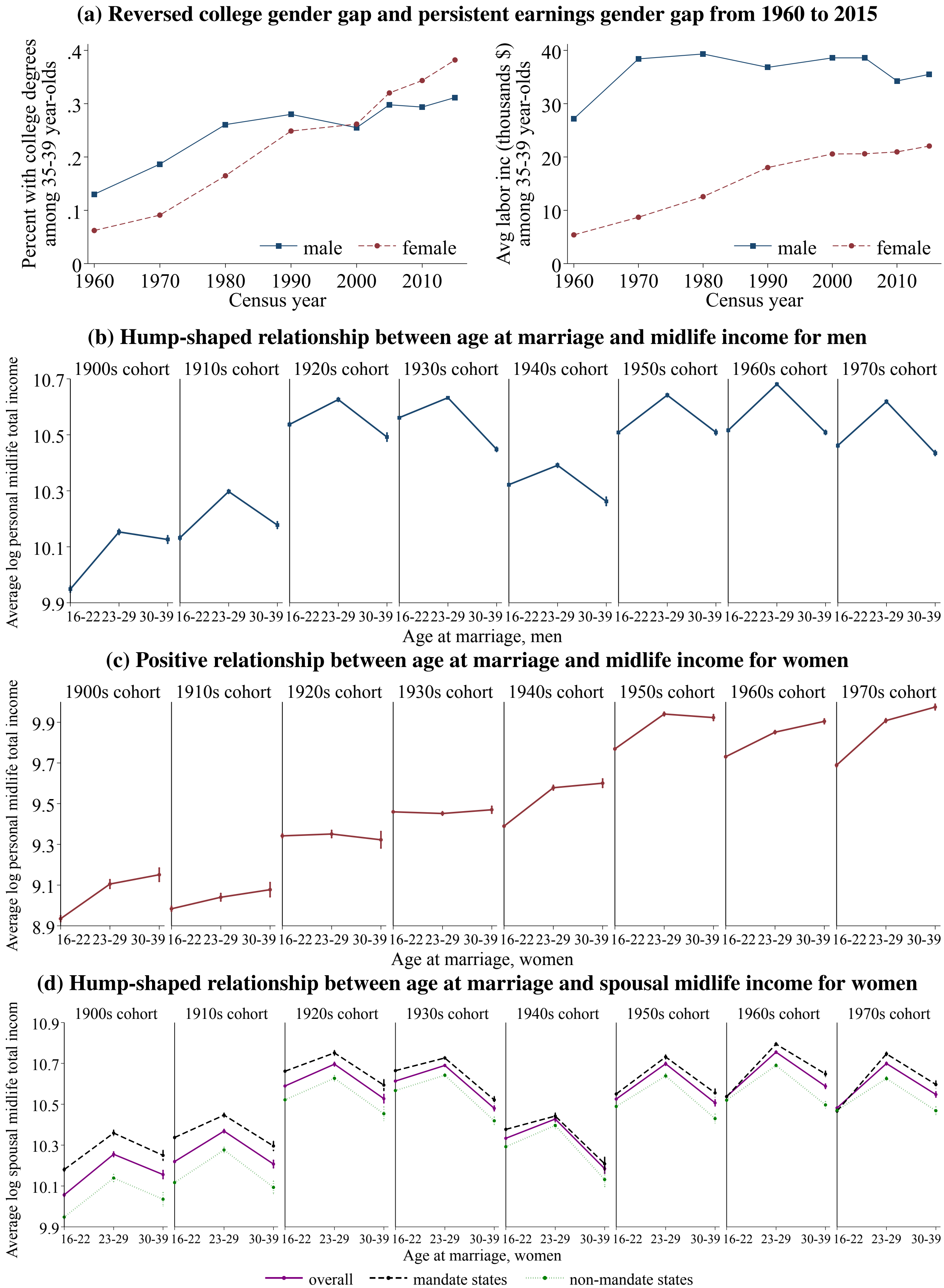
In this section, I document the education and income trends in the United States since 1960 as well as the relationships between age at marriage and incomes for Americans born between 1900 and 1979. Namely, I document the following four stylized facts to be explained by the model,

[1]Other explanations for the college gender gap include gender differences in distributions of noncognitive skills (Buchmann and DiPrete, 2006; Goldin et al., 2006; Becker et al., 2010a,b), in labor-market returns to college (Dougherty, 2005; Mulligan and Rubinstein, 2008; Hubbard, 2011), and in occupational choices (Charles and Luoh, 2003; Olivieri, 2014), and in opportunity costs of college (Chuan, 2018). Existing explanations for the earnings gender gap include gender differences in occupational choices (Bronson, 2013), in social roles (Goldin, 2014), and in career costs of children (Adda et al., 2017).

[2]The relationship between age at marriage and personal income has been documented for American men and women in the 1960 census (Keeley, 1974, 1977, 1979) and in the 1980 census (Bergstrom and Schoeni, 1996). It has also been documented for Taiwanese men in their 1989 census (Zhang, 1995) as well as for Canadians in their 1981 census and Brazilians in their 1991 census (Zhang, 2015). Relatedly, Oppenheimer (1988); Todd et al. (2005); Iyigun and Lafortune (2016) study age patterns at marriage.

[3]See also Siow and Zhu (2002); Schmidt (2005, 2007); Buckles (2007, 2008); Dessy and Djebbari (2010); Coles and Francesconi (2017, 2018); Bitler and Schmidt (2012); Garcia-Moran (2018).

Figure 2: Four stylized facts to be explained



summarized in figure 2: (a) a reversed college gender gap but a persistent earnings gender gap, (b) a hump-shaped relationship between age at marriage and midlife income for men, (c) a positive relationship between age at marriage and midlife income for women, as well as (d) (i) a hump-shaped relationship between age at marriage and spousal midlife income for women and (ii) a change in the relationship from one with a higher left shoulder to one with a higher right shoulder.

I use the decennial censuses of 1960, 1970, and 1980 as well as five-year American Community Surveys (ACS) 2010 and 2015 in the Integrated Public Use Microdata Series (IPUMS) USA (Ruggles et al., 2017). Age at first marriage is either reported directly (as variable AGEMARR) in these three decennial censuses or imputed from the year entering current marriage (variable YRMARR) in ACS since 2008 for those who have married once and stayed married. The measure of income is reported total pre-tax personal gains or losses from all sources in the previous calendar year, inflation-adjusted to 1999 USD (i.e., INCTOT×CPI99).[4] Midlife income is measured by income between ages 41 and 50 whenever possible.[5]

(a) A reversed college gender gap but a persistent earnings gender gap. The share of the college-educated among Americans aged 35-39 was higher for men before 2000 but was higher for women after 2000 (the left column of figure 2a), while the average labor income has been consistently higher for men than for women (the right column of figure 2a). The coexistence of these two opposite gender gaps is not uniquely American but a global phenomenon: in 2010, women went to college at higher rates than men in sixty-seven countries across all inhabited continents, but earned less than men on average in each of these countries (Becker et al., 2010a).[6]

(b) Hump-shaped relationship between age at marriage and midlife income for men. Men who married in their mid-twenties had a higher average midlife income than men who married earlier and later (figure 2b). To match the three periods in the model, I compare birth-year by birth-year the average incomes of *early grooms* (those who first married between ages 16 and 22), *middle grooms* (those who first married between ages 23 and 29), and *late grooms* (those who first married between ages 30 and 39). Middle grooms born in almost every year between 1900 and 1979 earned a statistically and economically significantly higher average midlife income than early and late grooms born in the same year; compared to middle grooms, on average, early grooms earned 13.1 percent less and late grooms earned 13.3 percent less (figure A1a).

[4]Similar relationships are obtained if inflation-adjusted wage income (i.e., INCWAGE×CPI99) is used instead.

[5]Since spousal income was not reported in the 1950 census, I use the income between ages 51 and 60 in the 1960 census for the 1900s birth cohort. Since age at marriage was not present in IPUMS USA between 1980 and 2008, income between ages 41 and 50 was not available for the 1940s and 1950s birth cohorts; I use the income between ages 61 and 70 for the 1940s birth cohort, and the income between ages 51 and 60 for the 1950s birth cohort.

[6]See Becker et al. (2010a,b) for worldwide college gender gap since 1960, Goldin et al. (2006) for American college gender gap in the twentieth century and Goldin (1990, 2006, 2014) for American earnings gender gap in the twentieth century.

(c) Positive relationship between age at marriage and midlife income for women. The later a woman married, the more she earned on average (figure 2c). Early brides had on average 11.8 percent less midlife income than middle brides born in the same year; middle brides had on average 2.1 percent less midlife income than late brides born in the same year, but the differences between middle and late brides were not statistically significant for the majority of the birth years (figure A1b). The observed gender difference in the relationships between age at marriage and personal income suggests that there is a fundamental gender asymmetry that results in gender-differential marriage timing and labor decisions.

(d) (i) Hump-shaped relationship between age at marriage and spousal midlife income for women. The husbands of the women who married in their mid-twenties earned a higher average midlife income than the husbands of those who married earlier and later (figure 2d). More precisely, the husbands of early brides and of late brides respectively earned 13.4 percent and 17.5 percent less midlife income on average than the husbands of middle brides (figure A1c).

(d) (ii) Change in the relationship from a higher left shoulder to a higher right shoulder. Early brides in the pre-1950 birth cohorts had higher-income husbands than late brides, but the pattern was reversed for post-1950 birth cohorts (figure 2d). This change was more pronounced in the thirteen states that passed mandates between 1985 and 1995 requiring infertility treatments to be covered or offered by insurance.[7] Because infertility treatment was (and still is) quite expensive, the laws reduced the costs for women and effectively extended the biological clock of the women in these states. We should expect that the marital outcome of early brides would have dropped and the marital outcome of late brides would have improved more in these thirteen states after the laws were passed. Indeed, we see that the right shoulder of the hump is raised above the left shoulder in the mandate states but not in the non-mandate states for the 1960s and 1970s birth cohorts (figure 2d). The average spousal income of late brides statistically significantly surpassed that of early brides born after 1960 in the mandate states but not in the non-mandate states (figure A1d). This observation suggests that gender-differential fertility length can contribute to explain the observed relationship between age at marriage and spousal income for women. I subsequently show that differential fecundity provides a unifying explanation of all the aforementioned observations.

3 Explaining the Stylized Facts

In this section, I first set up the model, then characterize the unique equilibrium of the model, and finally show how the predictions of the model are consistent with the four stylized facts documented above.

[7] The thirteen states are Maryland (1985), Arkansas, Hawaii, Massachusetts, Montana, Texas (1987), California, Connecticut, Rhode Island (1989), New York (1990), Illinois, Ohio (1991), and West Virginia (1995). See table 1 of Buckles (2007). See also Schmidt (2005, 2007); Bitler and Schmidt (2012).

3.1 Model

The model has an infinite number of discrete periods. At the beginning of each period, a unit mass of men and a unit mass of women enter the economy. Each agent is endowed with a heterogeneous ability $\theta \in [0,1]$. Let F_m and F_w denote the continuous and strictly increasing cumulative distribution of abilities for men and for women, respectively. Men and women make investment and marriage decisions over the next three periods of their lives to maximize their lifetime utilities. Think of the three periods as ages 16-22, 23-29, and 30-39. Each agent pays investment costs, receives a reservation payoff from working, and receives an additional endogenously determined marriage payoff if married. Each agent is risk-neutral and does not discount.

3.1.1 Investments

Figure 1 illustrates an agent's investment and marriage decisions. In period 1, each agent decides whether to go to college. Anyone who decides not go to college earns a low lifetime income and enters the marriage market immediately. Anyone who decides to go to college pays a cost, c_m for a male and c_w for a woman, and is assumed to delay marriage.[8]

In period 2, an ability-θ agent who went to college gets on the path to a high lifetime income with probability θ. Anyone who does not get on the path to a high lifetime income decides whether to make a career investment, which costs the same as a college investment. Examples of a career investment include obtaining additional training or education, working diligently on the current job, and switching to a new career path. Anyone who does not make a career investment earns a low lifetime income and enters the marriage market in the second period. Anyone who makes a career investment gets another chance to improve his/her lifetime income but delays marriage.

In period 3, an ability-θ agent who made a career investment enters the marriage market, either with a high lifetime income with probability θ or with a low lifetime income otherwise.

3.1.2 Differential Fecundity

A man is fertile for all three periods, but a woman is fertile for only the first two periods and is less fertile in the third period. In the marriage market, men are distinguished by income only, but women are distinguished by income and fertility. Let $T_m = \{H, L\}$ and $T_w = \{H, L, h, l\}$ denote the sets of marital characteristics for men and for women, respectively; letters h and H denote high-income types, letters l and L denote low-income types, uppercase letters denote fertile types, and lowercase letters denote less fertile types.

Income and fertility determine each agent's payoff as follows. Each income-y agent can generate a reservation utility of $z(y)$ without being married. An income-y_m man and an income-y_w fertility-ϕ_w woman would generate a total utility of $z(y_m, y_w, \phi_w)$ from marriage. Hence, the surplus

[8] The strategy of entering the marriage market while investing is assumed to be infeasible in the basic model. I present theoretical and empirical justifications for this assumption in section A.1, and relax the assumption in the calibration in section B.2.

due to marriage is $s(y_m, y_w, \phi_w) = z(y_m, y_w, \phi_w) - z(y_m) - z(y_w)$. Assume the marriage surplus is nonnegative, strictly increasing in income, and strictly increasing in fertility. Furthermore, assume the surplus is strictly supermodular in incomes and strictly supermodular in husband's income and wife's fitness. Formally, if we let $s_{\tau_m \tau_w} = s(\tau_m, \tau_w)$ denote the surplus of a type-τ_m man and a type-τ_w woman and define $\delta_{\tau_w} \equiv s_{H\tau_w} - s_{L\tau_w}$, then strict supermodularity in incomes means $\delta_H > \delta_L$ and $\delta_h > \delta_l$, and strict supermodularity in husband's income and wife's fitness means $\delta_H > \delta_h$ and $\delta_L > \delta_l$. The two assumptions together imply δ_H is the largest and δ_l is the smallest, and δ_h can be larger, smaller, or equal to δ_L. These supermodularity assumptions will help us pin down the stable matching patterns.[9]

3.1.3 The Marriage Market

Overlapping generations of men and women meet and bargain over the division of their marriage surplus until they reach a stable outcome in which no one can improve his or her payoff. Formally, a marriage market is described by distributions of marriage characteristics, $G_m = \{G_{m\tau_m}\}_{\tau_m \in T_m}$ and $G_w = \{G_{w\tau_w}\}_{\tau_w \in T_w}$, where $G_{m\tau_m}$ is the mass of type-τ_m men and $G_{w\tau_w}$ is the mass of type-τ_w women. A *stable outcome* of the marriage market (G_m, G_w) consists of *stable matching* $G = \{G_{\tau_m \tau_w}\}_{(\tau_m, \tau_w) \in T_m \times T_w}$ and *stable marriage payoffs* $v_m = \{v_{m\tau_m}\}_{\tau_m \in T_m}$ and $v_w = \{v_{w\tau_w}\}_{\tau_w \in T_w}$. Stable matching G satisfies *feasibility*: $\sum_{\tau_w} G_{\tau_m \tau_w} \leq G_{m\tau_m}$ for any $\tau_m \in T_m$ and $\sum_{\tau_m} G_{\tau_m \tau_w} \leq G_{w\tau_w}$ for any $\tau_w \in T_w$. Stable marriage payoffs v_m and v_w satisfy (i) *individual rationality*: $v_{m\tau_m} \geq 0$ for any $\tau_m \in T_m$ and $v_{w\tau_w} \geq 0$ for any $\tau_w \in T_w$ (every person receives at least as much as they would have if they had remained single); (ii) *pairwise efficiency*: $v_{m\tau_m} + v_{w\tau_w} = s_{\tau_m \tau_w}$ if $G_{\tau_m \tau_w} > 0$ (every married couple divides the entire marriage surplus); and (iii) *Pareto efficiency*: $v_{m\tau_m} + v_{w\tau_w} \geq s_{\tau_m \tau_w}$ for all $\tau_m \in T_m$ and $\tau_w \in T_w$ (no man-woman pair not married to each other can simultaneously improve their marriage payoffs by marrying each other). A stable outcome exists for any marriage market (theorem 2 of Gretsky et al. (1992)).

3.2 Unique Equilibrium

Define $\sigma_{m1}(\theta)$ and $\sigma_{m2}(\theta)$ as the probability of an ability-θ man investing in the first and second period, respectively, and define $\sigma_{w1}(\theta)$ and $\sigma_{w2}(\theta)$ for an ability-θ woman similarly. Strategies are summarized by functions $\sigma_m = (\sigma_{m1}, \sigma_{m2})$ and $\sigma_w = (\sigma_{w1}, \sigma_{w2})$. We say strategies σ_m and σ_w *induce* the marriage market (G_m, G_w) if the distributions of men's and women's marriage characteristics in each period are G_m and G_w, respectively, when men and women of every generation respectively choose strategies σ_m and σ_w.

Definition 1. A quadruple $(\sigma_m^*, \sigma_w^*, v_m^*, v_w^*)$ is an *equilibrium* if (i) $\sigma_m^*(\theta)$ and $\sigma_w^*(\theta)$ respectively maximize each ability-θ man's and each ability-θ woman's expected utility when the marriage

[9] Section A.2 provides a microfoundation of the marriage surplus function based on intra-household allocation of private and public goods.

payoffs are v_m^* and v_w^*; and (ii) v_m^* and v_w^* are stable marriage payoffs of the marriage market (G_m^*, G_w^*) induced by σ_m^* and σ_w^*. A quadruple $(\sigma_m^*, \sigma_w^*, v_m^*, v_w^*)$ is a *candidate equilibrium* if (i') condition (i) above holds, and (ii') v_m^* and v_w^* are stable marriage payoffs of *some* marriage market (G_m, G_w).

Any equilibrium is a candidate equilibrium, but a candidate equilibrium is not necessarily an equilibrium. I start by characterizing candidate equilibrium investments and marriage payoffs, and close off the model by finding the unique candidate equilibrium that is an equilibrium.

3.2.1 Equilibrium Investments

We solve for men's optimal investments by backward induction. Suppose an ability-θ man receives a low-income offer after college. If he decides to make a career investment, he incurs a cost c_m, and expects a lifetime income gain $\theta(z_{mH} - z_{mL})$ and a lifetime marriage gain $\theta(v_{mH} - v_{mL})$. An ability-θ man makes a career investment if and only if the expected gain outweighs the cost, that is, if and only if his ability is above

$$\theta_m := \frac{c_m}{z_{mH} - z_{mL} + v_{mH} - v_{mL}}. \tag{1}$$

A man goes through the same cost-benefit analysis to decide on optimal college investment. Therefore, in any equilibrium, any man with an ability above θ_m makes a college investment, and makes a career investment if he receives a low-income offer after college, while any man with an ability below θ_m makes no investment.[10]

We solve for women's optimal investments by backward induction, too. If an ability-θ woman who receives a low-income offer after college makes a career investment, her expected income gain is $\theta(z_{wH} - z_{wL})$ and her expected marriage gain is $\theta(v_{wh} - v_{wl}) - (v_{wL} - v_{wl})$, where the term $v_{wL} - v_{wl}$ represents her loss in marriage payoff due to fertility decline. She makes a career investment if and only if her ability θ is above

$$\theta_{w2} := \frac{c_w + v_{wL} - v_{wl}}{z_{wH} - z_{wL} + v_{wh} - v_{wl}}. \tag{2}$$

In contrast, a woman who makes a college investment does not expect an immediate fertility decline. An ability-θ woman makes a college investment if and only if her ability is above

$$\theta_{w1} := \frac{c_w}{z_{wH} - z_{wL} + v_{wH} - v_{wL}}. \tag{3}$$

Note that $\theta_{w1} < \theta_{w2}$: some women would not make a career investment. In summary, any woman whose ability is above θ_{w2} makes a college investment and, in case her college investment fails, makes a career investment; any woman whose ability is between θ_{w1} and θ_{w2} makes a college investment only; and any woman whose ability is below θ_{w1} makes no investment.

[10] Ability-θ_m men are indifferent between investing and not investing. Without loss of generality, we assume they invest whenever they are indifferent. It is without loss of generality because the distribution of abilities is atomless and there is measure 0 of ability-θ_m men, hence the stable outcome of the marriage market is not affected by the investment decisions of ability-θ_m men.

The induced distributions of marriage characteristics can be characterized straightforwardly from optimal investments. Type-H men consist of men with an ability above θ_m who receive a high-income offer either after college or after a career investment, so $G_{mH} = \int_{\theta_m}^{1} [\theta + (1-\theta)\theta] dF_m(\theta)$. Type-$L$ men consist of (i) all men with an ability below θ_m and (ii) men with an ability above θ_m who fail to receive a high income after college and career investments. Because there is a unit mass of men in each period's marriage market, the mass of low-income men is simply $G_{mL} = 1 - G_{mH}$. Type-H women are those with an ability above θ_{w1} who succeed right after college: $G_{wH} = \int_{\theta_{w1}}^{1} \theta dF_w(\theta)$. Type-$h$ women are those with an ability above θ_{w2} who succeed only after their career investment: $G_{wh} = \int_{\theta_{w2}}^{1} (1-\theta)\theta dF_w(\theta)$. Type-$L$ women consist of (i) all women with an ability below θ_{w1} and (ii) women with an ability between θ_{w1} and θ_{w2} who fail after college and do not make a career investment: $G_{wL} = F_w(\theta_{w1}) + \int_{\theta_{w1}}^{\theta_{w2}} (1-\theta) dF_w(\theta)$. Finally, type-$l$ women are those with an ability above θ_{w2} who fail to receive a high income after college and career investments: $G_{wl} = 1 - G_{wH} - G_{wL} - G_{wh}$.

3.2.2 Equilibrium Matching

Stable matching is described as follows. First, because the marriage surplus is assumed to be strictly supermodular in incomes, given two equally fertile women, a high-income woman almost surely marries a higher-income man than a low-income woman marries.[11] Second, because the surplus is assumed to be strictly supermodular in husband's income and wife's fertility, given two women with the same income, a fertile woman almost surely marries a higher-income man than a less fertile woman does. The two results together imply that (i) type-H women almost surely marry higher-income husbands and (ii) type-l women almost surely marry lower-income husbands than women of any other type. Whether a type-h woman or a type-L woman marries a higher-income husband depends on an additional condition. A type-h woman almost surely marries a man with a higher income than a type-L woman does if and only if $\delta_h > \delta_L$. In summary, stable matching is positive-assortative in men's income and women's type, provided that women's types are ranked according to (i) $H \succ h \succ L \succ l$ when $\delta_h > \delta_L$, (ii) $H \succ L \succ h \succ l$ when $\delta_L > \delta_h$, or (iii) $H \succ L \sim h \succ l$ when $\delta_L = \delta_h$.

3.2.3 Equilibrium Marriage Payoffs

Stable marriage payoffs are characterized as follows. Because there is an equal mass of men and women in the marriage market, there is a positive mass of marriages between the bottom-ranked type-L men and type-l women. By pairwise efficiency, $v_{mL} + v_{wl} = s_{Ll}$. However, also because there is an equal mass of men and women, neither v_{mL} nor v_{wl} is determinate. Stable marriage payoffs can be determined only up to a constant. They can be characterized by differences

[11] The modifier "almost surely" is needed because the marriage market consists of a continuum of men and women, rather than a finite number of agents as in Shapley and Shubik (1972) and Becker (1973). It is a standard quantifier in the literature (Chiappori and Oreffice, 2008; Chiappori et al., 2012a,b).

between marriage payoffs of adjacently ranked types.

We first derive *men's stable marriage premium* $\pi_m \equiv v_{mH} - v_{mL}$. There are two cases. In the first case, the mass of high-income men is between the mass of women strictly higher-ranked than τ_w^* and the mass of women weakly higher-ranked than τ_w^*, for some female type τ_w^*. Type-τ_w^* women marry high-income men and low-income men with positive probabilities, so $v_{mH} + v_{w\tau_w^*} = s_{H\tau_w^*}$ and $v_{mL} + v_{w\tau_w^*} = s_{L\tau_w^*}$, which together imply $\pi_m = s_{H\tau_w^*} - s_{L\tau_w^*}$. In the second case, the mass of high-income men equals the mass of women weakly higher-ranked than τ_w^*. Women weakly higher-ranked than τ_w^* almost surely marry high-income men, and women strictly lower-ranked than τ_w^* almost surely marry low-income men. Since there is no type of women who marry both types of men with positive probabilities, π_m is indeterminate. It can take any value between $\delta_{\tau_w'}$ and $\delta_{\tau_w^*}$, where τ_w' is the female type ranked just below τ_w^*.[12] This indeterminacy in π_m will dissipate in equilibrium, however, when marriage payoffs and investments are jointly determined.

Now we derive the payoff differences between any two adjacently ranked female types τ_w and τ_w'. Either (i) type-τ_w and type-τ_w' women both marry type-τ_m men with positive probabilities so that $v_{w\tau_w} - v_{w\tau_w'} = s_{\tau_m\tau_w} - s_{\tau_m\tau_w'}$; or (ii) almost all type-τ_w women marry type-H men and almost all type-τ_w' women marry type-L men so that $v_{w\tau_w} - v_{w\tau_w'} = (s_{H\tau_w} - s_{L\tau_w'}) - \pi_m$. Hence, the difference between the marriage payoffs of any two types of women is either determinate or depends on π_m. Consequently, the difference between the marriage payoffs of any two types of women can be expressed as a function of π_m.

We see from the derivations above that the difference between the marriage payoffs of any two types can be represented by men's marriage premium π_m. Furthermore, the three optimal investment thresholds in equations (1)-(3) are uniquely determined by payoff differences, and thus, uniquely determined by π_m. Therefore, any candidate equilibrium can be simply represented by the one number of π_m.

3.2.4 Equilibrium Existence and Uniqueness

Theorem 1. An equilibrium exists. Equilibrium investments are uniquely determined, and equilibrium marriage payoffs are uniquely determined up to a constant.

The proof, presented in section A.3, follows three steps. First, I construct (i) a correspondence that represents the demand for high-income men in the marriage market and (ii) a function that represents the supply. Second, I argue that each intersection of the constructed demand and supply

[12] If $G_{mH} = G_{w,\succeq\tau_w^*}$, then the lowest type of women marrying high-income men with a positive probability is τ_w^*, and the highest type of women marrying low-income men is τ_w', the type ranked right below τ_w^*. By pairwise efficiency, a high-income man marrying a type-τ_w^* woman gets a payoff of $v_{mH} = s_{H\tau_w^*} - v_{w\tau_w^*}$. By Pareto efficiency, a low-income man is weakly better off staying in his current match than marrying a type-τ_w^* woman, so $v_{mL} \geq s_{L\tau_w^*} - v_{w\tau_w^*}$. The two conditions together imply the upper-bound $\delta_{\tau_w^*}$ for $v_{mH} - v_{mL}$. Because τ_w' is the highest-ranked type of women marrying low-income men with a positive probability, it follows that pairwise efficiency condition $v_{mL} = s_{L\tau_w'} - v_{w\tau_w'}$ and Pareto efficiency condition $v_{mH} \geq s_{H\tau_w'} - v_{w\tau_w'}$ imply the lower-bound $\delta_{\tau_w'}$ for $v_{mH} - v_{mL}$.

curves corresponds to an equilibrium. Third, I show that (i) the constructed demand and supply curves always intersect, proving equilibrium existence, and (ii) the demand curve is downward-sloping and the supply curve is upward-sloping, proving equilibrium uniqueness.

3.3 Explaining the Stylized Facts

(a) College and Earnings Gender Gaps

Proposition 1. Suppose investment costs c_m and c_w, labor-market opportunities F_m and F_w, income premiums $z_{mH} - z_{mL}$ and $z_{wH} - z_{wL}$, as well as the marriage surpluses s_{HL} and s_{LH} are all gender-symmetric, but women are less fertile in the third period. Strictly more women than men go to college in equilibrium. Strictly fewer women than men earn a high lifetime income in equilibrium if $G_{mH}(\delta_l) > G_{wH}(\delta_l) + G_{wh}(\delta_l)$.

Existing papers (cited in the introduction) have explained the college gender gap using gender differences in psychic and monetary costs of investments, in labor-market opportunities, in college income premiums, and in marital roles. Proposition 1 states that even in a model that does not include any of these gender differences, it could be the case that more women than men attend college. Adding any of these gender differences into the model would only reinforce the female-dominated college gender gap. Furthermore, the female-dominated college gender gap can be sustained even when gender differences that deter women's college investments are included.[13] Therefore, the model highlights a new fundamental force rooted in differential fecundity and propagated through the marriage market contributing to the global college gender gap. At the same time, the earnings gender gap is maintained, a result unattainable from previous models explaining the college gender gap without including additional gender differences.

While I present the formal proof of the proposition by contradiction in the appendix (section A.4), I provide an economic explanation here. Define the difference between the marriage payoffs of a fertile high-income earner and a fertile low-income earner, $\pi_i \equiv v_{iH} - v_{iL}$, $i = m, w$, as the marriage premium. The college ability cutoffs are simply determined by the investment cost divided by the income premium and the marriage premium, $\theta_i = c_i/(z_{iH} - z_{iL} + \pi_i)$, $i = m, w$.[14] When the setting is gender-symmetric, more women than men go to college if and only if the endogenous marriage premium π_i is higher for women than for men. If the marriage premiums π_m and π_w were exogenously fixed to be the same, the same number of men and women would go

[13] For example, it has been argued that women's college income premium is lower than men's. Dougherty (2005) shows that the college income premium for women (defined as the difference in log wages of non-college-educated female high school graduates and female college graduates) was higher than men's. Hubbard (2011) shows that the gender difference in the college income premiums was nonexistent after correcting for income top-coding bias which has previously underestimated men's college income premium. Estimates presented in figure 6 of DiPrete and Buchmann (2006) and figure 9 of Chiappori et al. (2017) provide additional evidence that the gender difference in college income premiums is not enough to explain the college gender gap.

[14] The shares of men and women going to college are $1 - F_i(c_i/(z_{iH} - z_{iL} + \pi_i^*))$, $i = m, w$.

to college, and fewer women than men would make a career investment because of differential fecundity. Consequently, fewer women than men earn a high income. However, the marriage premiums are endogenously determined in the model. Precisely because fewer women than men earn a high income, the women who earn a high income are more scarce and more "valuable" in the marriage market than the men who achieve the same feat. Women's endogenously higher marriage premium prompts more women than men to make a college investment. Hence, a key implication of the model is a higher marriage premium for women than for men. I will test and confirm this implication empirically in section B.1.1.

The two key drivers of our main result are differential fecundity and the endogenous division of the marriage surplus. In a model without differential fecundity, the setting is entirely gender-symmetric, so the same number of men and women would go to college, make career investments, and earn a high income. In a model that incorporates differential fecundity but omits the endogenous division of the marriage surplus (i.e., the marriage premiums are exogenously the same for the two genders), the same number of men and women would go to college, but fewer women than men would make a career investment and earn a high income; such a model would be able to explain the earnings gender gap but not the college gender gap.

Hence, the combination of differential fecundity and endogenous surplus division is needed to account for the opposite gender gaps. Differential fecundity directly reduces women's career investments but does not directly increase their college investments. College and career investments are not directly substitutes to improve income, but endogenous marriage surplus division makes these investments *strategic substitutes*.[15] Specifically, the decline in fertility directly discourages intermediate-ability college-investing women (namely, women with an ability close to θ^*_{w2}) from making career investments, and indirectly encourages lower-ability women (namely, women with an ability close to θ^*_{w1}) to go to college through endogenous marriage surplus division.

Furthermore, surplus supermodularity in incomes is necessary to explain the college gender gap. If the surplus is submodular in incomes, the same number of men and women would go to college. Surplus supermodularity in incomes is theoretically grounded in the intrahousehold allocation model presented in the appendix, and is empirically supported by our subsequent estimates of the marriage surplus function as well as findings on positive assortative matching in incomes and educations in the United States and other countries (Lam, 1988; Blossfeld and Timm, 2003; Schwartz and Mare, 2005; Stevenson and Wolfers, 2007; Greenwood et al., 2014; Siow, 2015; Greenwood et al., 2016; Chiappori et al., 2017).

(b) Relationship between Age at Marriage and Income for Men

Proposition 2. The relationship between age at marriage and income for men is hump-shaped in

[15] Thomas (2018) considers the possibility that college and career investments are direct substitutes.

equilibrium: the average income is the highest for middle grooms.

Early grooms in the model are the men with an ability below θ_m^*, and they earn a low lifetime income without making any investment. Middle grooms are the men with an ability above θ_m^* who get on the path to a high lifetime income after college. Late grooms are the remaining men with an ability above θ_m^* who fail to realize a high income after college and consequently make a career investment, and some of them receive a high income, and the rest of them receive a low income, so the average income is lower for late grooms than for middle grooms.

For the upward-sloping portion of the relationship, early grooms earn less than middle grooms on average because early grooms invest less than middle grooms. Bergstrom and Bagnoli (1993) also predict a positive relationship between age at marriage and income for men, but there is a difference between their model and the current model. In their model, high-income men wait to marry because they cannot credibly signal their earning abilities when they are young. In contrast, there is no private information in the current model. Even if a man can choose to marry during college, he weakly prefers to wait to marry until after he resolves his post-college income uncertainty (as shown in section A.1). The reason to delay marriage in this model is rooted in the inherent nature of the marriage market. A man who has uncertainty about his future lifetime income may not be able to marry the woman he could marry when he has a high lifetime income for sure, so he chooses to delay marriage.[16] Moreover, the downward-sloping portion of the relationship cannot be explained by Bergstrom and Bagnoli (1993) but can be explained by the current model.

For the downward-sloping portion of the relationship, middle grooms earn more than late grooms on average because middle grooms are the college-educated men who get on the path to a high lifetime income soon after college, and late grooms are the college-educated men who fail to do so and end up with a lower income on average. Becker (1974) and Keeley (1979) also predict a negative relationship, but their explanation is different. Whereas higher-ability men in their models marry earlier because they encounter less marriage-market friction, higher-ability men in the current model do so because they are less likely to encounter an adverse labor-market shock. Lower-income men involuntarily delay marriage due to marriage-market frictions in their models but voluntarily delay marriage due to labor-market shocks in the current model. Section B.1.2 will present patterns that can only be explained by the impacts of labor-market shocks on marriage timing. The calibration in section B.2 will quantify the respective impacts of marriage-market frictions and labor-market shocks on marriage timing.

(c) Relationship between Age at Marriage and Income for Women

Proposition 3. The relationship between age at marriage and income for women can be positive,

[16] The discussion in Oppenheimer (1988) that people would like to be sure of their career prospects before marrying is formalized by this model.

positive-then-flat, or hump-shaped in equilibrium.

In equilibrium, early brides earn a low income because they are the low-ability women (those with an ability below θ^*_{w1}) who do not go to college. Middle brides consist of all the intermediate-ability women (those with an ability between θ^*_{w1} and θ^*_{w2}) and the higher-ability women (those with an ability above θ^*_{w2}) who earn a high income right after college. Late brides are the higher-ability women who do not receive a high income after college. The model predicts that early brides earn less than middle brides and late brides, but the model does not make a definitive prediction about whether middle brides or late brides earn less.

In the model, early brides earn less than middle brides, because early brides are those who do not go to college but middle brides are those who go to college with many ending up with a high income. The impact of human capital investment on women's marriage timing is completely missing in Becker (1974) and and Keeley (1979) (which predict a positive relationship between age at marriage and income due to marriage-market frictions) and Bergstrom and Bagnoli (1993) (which predicts no relationship between age at marriage and income for women). The current model naturally incorporates this effect.

In the model, middle brides tend to earn less than late brides, because middle brides mostly consist of intermediate-ability women who fail to receive a high income after college but nonetheless choose to marry, but late brides are the high-ability women who do not receive a high income right out of college but receive a high income with a large probability after career investments. In short, labor-market shocks and the fertility-income tradeoff result in a positive selection in delayed marriage. Becker (1974) predicts a positive relationship between age at marriage and income driven by marriage-market frictions. Section B.1.3 will present evidence that can only be explained by labor-market shocks, and the calibration of section B.2 will quantify the respective impacts of marriage-market frictions and labor-market shocks on marriage timing.

(d) Relationship between Age at Marriage and Spousal Income for Women

Proposition 4. The relationship between age at marriage and spousal income for women is hump-shaped in equilibrium: the average spousal income is the highest for those who marry in the second period. The average spousal income is higher for early brides when $\delta_h > \delta_L$, and is higher for late brides when $\delta_L > \delta_h$.

In the model, early brides are fertile low-income earners, and middle brides consist of both fertile low-income earners and fertile high-income earners; since fertile high-income women's husbands almost always have a higher income than fertile low-income women's, middle brides are predicted to have a higher average spousal income than early brides. Late brides consist of both high-income and low-income earners, but they are less fertile than middle brides. Since (i) for any two women with the same income, the more fertile one marries a higher-income husband in equilibrium, and (ii) late brides do not earn significantly more than middle brides on average, the average spousal income is predicted to be lower for late brides than for middle brides.

The key to explaining whether early brides or late brides marry higher-income husbands is "non-assortative matching" in incomes (Low, 2017) in the marriage market. According to the model, early brides are fertile low-income earners, and late brides consist of less fertile women with a higher average income. If fertility is more important than income in the marriage market (i.e., $\delta_L > \delta_h$), type-L women marry higher-income husbands than type-h women, and consequently, early brides' average spousal income would be lower than late brides'. Otherwise (i.e., $\delta_h > \delta_L$), it is possible that less fertile high-income women's husbands have a higher income than fertile low-income women's, and the average spousal income is higher for less fertile women than for fertile low-income women.

The fact that late brides had higher-income husbands than early brides in the states with infertility mandates suggests that the evolution of the difference in average spousal income between early brides and late brides was at least partially driven by the change in the relative importance of income and fertility in the marriage market. These changes in the marriage market can be thought of as a decrease in the demand for and/or an increase in the supply of "reproductive capital" (Low, 2017) in the marriage market.

On the one hand, the demand for "reproductive capital" has decreased. First, the desired (and actual) family size has decreased; in the United States, the average desired number of children has declined from 3.6 to 2.6 from 1960 to 2010 (Livingston and Cohn, 2010). Many families have shifted from a demand for quantity of children to a demand for quality, as Becker and Lewis (1973) predicted. Women's fertility has become less of a concern in marriage decisions than women's income and education. Second, an increase in income gain from college and career investments also contributes to a decrease in the relative importance of fertility; the benefit of the career investment in the labor market outweighs the cost of delayed marriage in the marriage market.

On the other hand, an increase in the supply of "reproductive capital" has been achieved by advances in medical technology such as in-vitro fertilization, egg freezing, and more cost-effective maternal health services, all of which have resulted in a higher probability of staying fertile and conceiving. Older women can have children with less financial burden, more physical ease, and fewer adverse health effects than in the past. Gershoni and Low (2017) present causal evidence that policies that have made assistive reproductive technology less expensive and more accessible directly improved education, labor-market, and marital outcomes of Israeli women who married late. We show similar evidence for American women.

4 Supporting the Explanations, Summary

In this section, I summarize (i) the key distinguishing evidence supporting my explanations of the stylized facts and (ii) the results of calibration. I leave the details in the appendix.

4.1 Key Evidence

4.1.1 College and Earnings Gender Gaps

A key implication of the model is a higher for women than for men when more women than men go to college. I estimate marriage premiums from 1960 to 2015. Women's marriage premium was smaller than men's in 1960, 1970, and 1980, when fewer women than men, ages 35-39, graduated from college; and was greater than men's in 2010 and 2015, when more women than men, ages 35-39, graduated from college. I adopt the technique developed by Choo and Siow (2006) to exactly identify the marriage surplus function and compute the marriage premiums from the estimated marriage surplus function.

In the model, women's college investment rate and average income in equilibrium both increase if any or any combination of the following events happens: (i) women face a lower investment cost; (ii) women's labor-market opportunities improve; (iii) women's college income premium increases; and (iv) women's college marriage premium increases. These predictions are consistent with the empirical literature studying the rise of women's college enrollment and labor force participation.

4.1.2 Relationship between Age at Marriage and Income for Men

If labor-market shocks indeed delay marriages of the college-educated men as the model suggests, then we would expect that, compared to college-educated middle grooms, college-educated late grooms should have (i) a lower average income when they have just finished college and (ii) a higher income growth rate following college because they engage in more career investments. In contrast, if marriage-market friction is the sole determinant of marriage timing, then we would expect that college-educated late grooms should have a lower income growth rate following college than college-educated middle grooms. For the three cohorts tracked by National Longitudinal Study of Youth (NLSY), four-year-college-educated late grooms, the men who received exactly four years of postsecondary education and first married between ages 30 and 39, (i) earned a lower average income than four-year-college-educated middle grooms in their twenties, and (ii) caught up and earned almost as much as four-year-college-educated middle grooms in their thirties.

4.1.3 Relationship between Age at Marriage and Income for Women

If the labor-market shocks indeed delay marriages of the college-educated women as the model suggests, then we would expect that college-educated late brides should have a lower income in their twenties and a higher income in their thirties than college-educated early brides. The three cohorts of NLSY showed exactly those patterns: four-year-college-educated late brides earned a lower average income in their early twenties and a higher average income in their late twenties and early thirties than four-year-college-educated middle brides.

4.1.4 Relationship between Age at Marriage and Spousal Income for Women

If fertility and income are the two important factors that determine women's marital outcome, then a potential technological improvement in fertility should improve the relative marital outcome of late brides. The marital outcome of late brides, measured by spousal income and education ranks, indeed improved in the thirteen states that passed mandates to cover or offer infertility treatments in insurances between 1985 and 1995.

4.2 Calibration

First, I calibrate the benchmark model to examine the quantitative validity of the model. The calibration matches each targeted marriage-age distribution within 0.7 percent, and except for the average income of late grooms born in the 1930s and that of late brides in the 1960s, the calibration matches each of the targeted average incomes within 5 percent. Non-targeted average spousal incomes were also matched fairly well.

Second, I incorporate marriage-market frictions into the benchmark model to separate investment and marital timing decisions in order to match (i) age distributions at marriage by education level, (ii) average personal midlife income by age at marriage for men and for women, (iii) average spousal income by women's age at marriage, and (iv) men's and women's college enrollment rates, as well as to quantify the importance of labor-market shocks relative to marriage-market frictions in explaining marriage timing. With the calibration of the extended model, I quantify the relative impacts of marriage-market frictions and labor-market shocks on marriage timing decisions.

For the 1930s birth cohort, there was an estimated 17.1 percent chance that college-educated women who decided to enter the marriage market before age 30 involuntarily delayed their marriage until after age 30, slightly higher than the 21.2 percent chance that noncollege women involuntarily delayed their marriage until after age 30. Among the men and women who married between ages 30 and 39, among the college-educated, essentially all men delayed marriages due to labor-market shocks, and all women delayed marriages due to marriage-market frictions (consistent with the fact that a tiny portion of women in this cohort chose to make a career investment).

For the 1960s birth cohort, the chance for a college-educated man who decided to enter the marriage market between ages 23 and 29 not being able to marry before age 30 was 22.2 percent, and the chance that a college-educated woman who decided to enter the marriage market between ages 23 and 29 not being able to marry before age 30 was 23.8 percent. We find that 42.7 percent of college-educated men and 24.6 percent of college-educated women delayed their marriages due to labor-market shocks (and the rest delayed due to marriage-market frictions).

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A Omitted Discussions and Proofs

A.1 The Strategy of Entering the Marriage Market While Investing

I claim in footnote 8 that the strategy of entering the marriage market while investing is weakly dominated by the strategy of entering the marriage market after investing. I extend the basic model to allow this strategy, and show that the strategy is weakly dominated in the extended model.[17] Since lifetime income can only be high or low in the model, a man's marriage type can be simply represented by the probability of obtaining a high lifetime income when he enters the marriage market. The marriage surplus of a man who obtains a high income with probability θ_m and a fertile woman who obtains a high income with probability θ_w is

$$\widetilde{s}(\theta_m, \theta_w, +) = \theta_m \theta_w s_{HH} + \theta_m (1-\theta_w) s_{HL} + (1-\theta_m)\theta_w s_{LH} + (1-\theta_m)(1-\theta_w) s_{LL}.$$

The marriage surplus of a type-θ_m man and a less fertile type-θ_w woman is

$$\widetilde{s}(\theta_m, \theta_w, -) = \theta_m \theta_w s_{Hh} + \theta_m (1-\theta_w) s_{Hl} + (1-\theta_m)\theta_w s_{Lh} + (1-\theta_m)(1-\theta_w) s_{Ll}.$$

The marriage market in the extended model is organized in the same way as in the basic model. Namely, let $\widetilde{T}_m = [0,1]$ and $\widetilde{T}_w = [0,1] \times \Phi$ represent the expanded sets of marriage types, $\widetilde{G}_m$ and $\widetilde{G}_w$ distributions of marriage types, $\widetilde{G}$ a matching, and $\widetilde{v}_m$ and $\widetilde{v}_w$ marriage payoffs. The outcome $(\widetilde{G}, \widetilde{v}_m, \widetilde{v}_w)$ is stable in the marriage market $(\widetilde{G}_m, \widetilde{G}_w)$ if (1) (individual rationality) $\widetilde{v}_m \tau_m \geq 0$ for all $\tau_m \in \widetilde{T}_m$ and $\widetilde{v}_w \tau_w \geq 0$ for all $\tau_w \in \widetilde{T}_w$, (2) (pairwise efficiency) $\widetilde{v}_{m\tau_m} + \widetilde{v}_{w\tau_w} = \widetilde{s}_{\tau_m \tau_w}$ when $\widetilde{G}(\tau_m, \tau_w) > 0$, and (3) (Pareto efficiency) $\widetilde{v}_{m\tau_m} + \widetilde{v}_{w\tau_w} \geq \widetilde{s}_{\tau_m \tau_w}$ for any pair of $\tau_m \in \widetilde{T}_m$ and $\tau_w \in \widetilde{T}_w$.

I now show that the strategy of simultaneously investing and marrying is weakly dominated by investing and then marrying after income is realized for a college man. Let τ_w denote the type of woman an ability-θ man who has made the college investment and will not make the career investment marries in the stable matching. His stable marriage payoff is

$$\widetilde{v}_{m\theta} = \widetilde{s}_{\theta\tau_w} - \widetilde{v}_{w\tau_w} = p\widetilde{s}_{1\tau_w} + (1-\theta)\widetilde{s}_{0\tau_w} - \widetilde{v}_{w\tau_w}.$$

By Pareto efficiency, the marriage payoff of each man weakly exceeds what he would get if he marries a type-τ_w woman: for type-1 (i.e., high-income) and probability-0 (i.e., low-income) men, $\widetilde{v}_{m1} \geq \widetilde{s}_{1\tau_w} - \widetilde{v}_{w\tau_w}$ and $\widetilde{v}_{m0} \geq \widetilde{s}_{0\tau_w} - \widetilde{v}_{w\tau_w}$. If the same ability-$\theta$ man makes a college investment and marries after income is realized, his expected marriage payoff is $p\widetilde{v}_{m1} + (1-\theta)\widetilde{v}_{m0}$, which, by the two inequalities above, is greater than $p\widetilde{s}_{1\tau_w} + (1-\theta)\widetilde{s}_{0\tau_w} - \widetilde{v}_{w\tau_w}$, which is the expected payoff the man gets from simultaneously investing and marrying. The same argument applies to any man or any woman who chooses an investment strategy that results in a high income with probability θ, for any θ.

[17] Because income is uncertain when agents enter the marriage market while investing, their marriage characteristics are represented by distributions of incomes rather than realized incomes (Borch, 1962; Wilson, 1968; Chiappori and Reny, 2016; Zhang, 2017).

Empirically, most people chose not to marry when they were making human capital investments. First, 86 to 96 percents of college-educated men and 80 to 90 percents of college-educated women did not marry between ages 18 and 21, their college years (table A1), besides the two outliers, 1930s and 1940s birth cohorts, many of whom were rushed into marriage to avoid being drafted to the Vietnam War. Second, 78 to 92 percents of men with advanced degrees and 79 to 92 percents of women with advanced degrees did not marry between ages 22 and 23 (table A2).

Table A1: Proportion of college degrees marrying between ages 18 and 21

	1900s	1910s	1920s	1930s	1940s	1950s	1960s	1970s
Men	0.06	0.06	0.14	0.18	0.19	0.12	0.06	0.04
Women	0.10	0.12	0.23	0.29	0.29	0.20	0.12	0.10

Table A2: Proportion of advanced degrees marrying between ages 22 and 23

	1900s	1910s	1920s	1930s	1940s	1950s	1960s	1970s
Men	0.09	0.13	0.20	0.21	0.22	0.15	0.09	0.08
Women	0.08	0.13	0.20	0.21	0.20	0.15	0.12	0.11

A.2 A Microfoundation of the Marriage Surplus Function

The following intra-household public-good consumption problem justifies the monotonicity assumptions, the supermodularity assumptions, as well as transferable utilities in the marriage market. Low (2017) presents a similar microfoundation. Bergstrom and Cornes (1983); Chiappori and Gugl (2015); Chiappori et al. (2017) provide more general discussions of the microfoundation.

An unmarried income-y_m man who derives utility q_m from consuming q_m units of a composite private good derives a utility of $z(y_m) = y_m$, and an unmarried income-y_w woman who derives utility q_w from consuming q_w units of the same composite private good derives a utility of $z(y_w) = y_w$. A couple with children spends their income $y_m + y_w$ on q_m and q_w units of the private good as well as on Q units of a public good to derive a utility of $q_m(1+Q)$ for the husband and a utility of $q_w(1+Q)$ for the wife. To maximize joint utility $(q_m+q_w)(1+Q)$ subject to the budget constraint $q_m + q_w + Q \leq y_m + y_w$, the couple consumes $q_m + q_w = (y_m + y_w + 1)/2$ units of the private good and $Q = (y_m + y_w - 1)/2$ units of the public good for a joint utility $(y_m + y_w + 1)^2/4 = (y_m + y_w - 1)^2/4 + y_m + y_w$. A couple without children spends income on the composite private good only to derive a joint utility of $y_m + y_w$. With probability ϕ_w a woman with fitness level ϕ_w can have children. Therefore, the marriage surplus an income-y_m man and an income-y_w woman with fitness level ϕ_w generate is

$$\begin{aligned} s(y_m, y_w, \phi_w) &= z(y_m, y_w, \phi_w) - z(y_m) - z(y_w) \\ &= \phi_w[(y_m + y_w - 1)^2/4 + y_m + y_w] + (1 - \phi_w)(y_m + y_w) - y_m - y_w \\ &= \phi_w(y_m + y_w - 1)^2/4. \end{aligned}$$

The surplus is strictly increasing in y_m, y_w, and ϕ_w, when $y_m + y_w - 1 > 0$. The surplus is strictly supermodular in y_m and y_w as well as in y_m and ϕ_w. Moreover, any division of the marriage surplus between a couple can be achieved through the allocation of the private good. When there are children, q_m units of the private good are allocated to the husband and q_w units of the private good are allocated to the wife, where $q_m + q_w = (y_m + y_w + 1)/2$.

A.3 Proof of Theorem 1

Let $\theta_m(\pi_m)$, $\theta_{w1}(\pi_m)$, and $\theta_{w2}(\pi_m)$ denote the ability cutoffs characterizing optimal human capital investments when men's stable marriage premium is π_m (and women's stable marriage-payoff differences are pinned down by π_m). Let $G_m(\pi_m)$ and $G_w(\pi_m)$ denote the induced distributions of men's and women's marriage characteristics, respectively, when the investment strategies are the ones characterized by the ability cutoffs $\theta_m(\pi_m)$, $\theta_{w1}(\pi_m)$, and $\theta_{w2}(\pi_m)$. Let $\Pi_m(G_m, G_w)$ denote the set of men's stable marriage premiums (and associated stable marriage payoffs of women) in the marriage market (G_m, G_w). Construct the correspondence

$$D_{mH}(\pi_m) := \{G_{mH} \in [0,1] : \pi_m \in \Pi_m((G_{mH}, 1 - G_{mH}), G_w(\pi_m))\}.$$

For any $\pi_m \in [\delta_l, \delta_H]$, each element in the set $D_{mH}(\pi_m)$ is a mass G_{mH} of high-income men such that π_m is men's stable marriage premium in the marriage market $((G_{mH}, 1 - G_{mH}), G_w(\pi_m))$. Explicitly, (i) if $\pi_m = \delta_{\tau_w^*}$ for a certain type $\tau_w^* \in T_w$, then $D_{mH}(\pi_m) = [G_{w,\succ\tau_w^*}(\pi_m), G_{w,\succeq\tau_w^*}(\pi_m)]$; and (ii) if $\pi_m \in (\delta_{\tau_w'}, \delta_{\tau_w^*})$ for a certain pair of adjacently ranked types $\tau_w^* \in T_w$ and $\tau_w' \in T_w$, then $D_{mH}(\pi_m) = G_{w,\succeq\tau_w^*}(\pi_m)$.

We prove the claim that there exists an equilibrium in which men's stable marriage premium is π_m^* if and only if $G_{mH}(\pi_m^*) \in D_{mH}(\pi_m^*)$. First, the only if part. Suppose men's equilibrium marriage premium is π_m^*. The induced mass of high-income men is $G_{mH}(\pi_m^*)$, and the induced distribution of women's marriage characteristics is $G_w(\pi_m^*)$. Since $\pi_m^* \in \Pi_m((G_{mH}(\pi_m^*), 1 - G_{mH}(\pi_m^*)), G_w(\pi_m^*))$, by definition of $D_{mH}(\pi_m^*)$, we have $G_{mH}(\pi_m^*) \in D_{mH}(\pi_m^*)$. Reversely, if $G_{mH}(\pi_m^*) \in D_{mH}(\pi_m^*)$, then by definition of $D_{mH}(\pi_m^*)$, $\pi_m^* \in \Pi_m((G_{mH}(\pi_m^*), 1 - G_{mH}(\pi_m^*)), G_w(\pi_m^*))$, so π_m^* is men's equilibrium marriage premium.

It follows from the claim above that an equilibrium exists if and only if the graph of function $G_{mH}(\cdot)$ and the graph of correspondence $D_{mH}(\cdot)$ intersect at least once. Equilibrium marriage-payoff differences and equilibrium investments are uniquely determined if and only if the graph of function $G_{mH}(\cdot)$ and the graph of correspondence $D_{mH}(\cdot)$ intersect once and only once. The existence of an equilibrium is guaranteed because $G_{mH}(\cdot)$ has a range $[0,1]$ and is continuous, and $D_{mH}(\cdot)$ has a range $[0,1]$ and is upperhemicontinuous.

It remains for us to prove equilibrium uniqueness. $G_{mH}(\pi_m) = \int_{\theta_m(\pi_m)}^1 \theta(2-\theta) dF_m(\theta)$ is strictly increasing in π_m because $\theta_m(\pi_m) = c_m/(z_{mH} - z_{mL} + \pi_m)$ is strictly decreasing in π_m. It suffices to show $D_{mH}(\pi_m)$ is weakly decreasing in the following sense: for any π_m and $\pi_m' > \pi_m$, $\max D_{mH}(\pi_m') \le \min D_{mH}(\pi_m)$. For the remainder of the proof, we mechanically show that $D_{mH}(\pi_m)$ is decreasing. Depending on $\delta_h > \delta_L$, $\delta_h < \delta_L$, or $\delta_h = \delta_L$, $D_{mH}(\pi_m)$ is characterized differently. We discuss the three cases separately.

Case 1. Suppose $\delta_L > \delta_h$. Explicitly,

$$D_{mH}(\pi_m) = \begin{cases} [G_{w,\succeq h}(\pi_m), 1] & \text{if } \pi_m = \delta_l \\ G_{w,\succeq h}(\pi_m) & \text{if } \pi_m \in (\delta_l, \delta_h) \\ [G_{w,\succeq L}(\pi_m), G_{w,\succeq h}(\pi_m)] & \text{if } \pi_m = \delta_h \\ G_{w,\succeq L}(\pi_m) & \text{if } \pi_m \in (\delta_h, \delta_L) \\ [G_{wH}(\pi_m), G_{w,\succeq L}(\pi_m)] & \text{if } \pi_m = \delta_L \\ G_{wH}(\pi_m) & \text{if } \pi_m \in (\delta_L, \delta_H) \\ [0, G_{wH}(\pi_m)] & \text{if } \pi_m = \delta_H \end{cases}.$$

It remains to show that (i) $G_{w,\succeq h}(\pi_m)$ is strictly decreasing when $\pi_m \in (\delta_l, \delta_h)$, (ii) $G_{w,\succeq L}(\pi_m)$ is strictly decreasing when $\pi_m \in (\delta_h, \delta_L)$, and (iii) $G_{wH}(\pi_m)$ is strictly decreasing when $\pi_m \in (\delta_L, \delta_H)$.

(i) To show $G_{w,\succeq h}(\pi_m) = 1 - \int_{\theta_{w2}(\pi_m)}^{1}(1-\theta)^2 dF_w(\theta)$ is strictly decreasing when $\pi_m \in (\delta_l, \delta_h)$, it suffices to show $\theta_{w2}(\pi_m)$ is strictly decreasing when $\pi_m \in (\delta_l, \delta_h)$. Men's stable marriage premium can be $\pi_m \in (\delta_l, \delta_h)$ only when $G_{mH} = G_{w,\succeq h}$. When $G_{mH} = G_{w,\succeq h}$, given men's stable marriage premium π_m, women's stable marriage-payoff differences are $v_{wL} - v_{wl} = s_{HL} - s_{Ll} - \pi_m$, $v_{wH} - v_{wL} = s_{HH} - s_{HL}$, and $v_{wh} - v_{wl} = s_{Hh} - s_{Ll} - \pi_m$, so

$$\begin{aligned} \theta_{w2}(\pi_m) &= \frac{c_w + (v_{wL} - v_{wl})}{z_{wH} - z_{wL} + (v_{wh} - v_{wl})} = \frac{c_w + (s_{HL} - s_{Ll} - \pi_m)}{z_{wH} - z_{wL} + (s_{Hh} - s_{Ll} - \pi_m)} \\ &= \frac{c_w + (s_{HL} - s_{Ll}) - \pi_m}{z_{wH} - z_{wL} + (s_{Hh} + s_{Ll}) - \pi_m}. \end{aligned}$$

Since $\theta_{w2}(\pi_m) < 1$, $\theta'_{w2}(\pi_m) < 0$ when $\pi_m \in (\delta_l, \delta_h)$.

(ii) To show $G_{w,\succeq L}(\pi_m) = 1 - \int_{\theta_{w2}(\pi_m)}^{1}(1-\theta) dF_w(\theta)$ is strictly decreasing when $\pi_m \in (\delta_h, \delta_L)$, it suffices to show $\theta_{w2}(\pi_m)$ is strictly decreasing when $\pi_m \in (\delta_h, \delta_L)$. Men's stable marriage premium can be $\pi_m \in (\delta_h, \delta_L)$ only when $G_{mH} = G_{w,\succeq L}$. When $G_{mH} = G_{w,\succeq L}$, given men's stable marriage premium π_m, women's stable marriage-payoff differences are $v_{wL} - v_{wl} = s_{HL} - s_{Ll} - \pi_m$, $v_{wH} - v_{wL} = s_{HH} - s_{HL}$, and $v_{wh} - v_{wl} = s_{Lh} - s_{Ll}$, so

$$\theta_{w2}(\pi_m) = \frac{c_w + (s_{HL} - s_{Ll} - \pi_m)}{z_{wH} - z_{wL} + (s_{Lh} - s_{Ll})}.$$

Therefore, $\theta_{w2}(\pi_m)$ is strictly decreasing when $\pi_m \in (\delta_h, \delta_L)$.

(iii) To show $G_{wH}(\pi_m) = \int_{\theta_{w1}(\pi_m)}^{1} \theta dF_w(\theta) + \int_{\theta_{w2}(\pi_m)}^{1}(1-\theta)\theta dF_w(\theta)$ is strictly decreasing when $\pi_m \in (\delta_L, \delta_H)$, it suffices to show $\theta_{w1}(\pi_m)$ and $\theta_{w2}(\pi_m)$ are strictly increasing when $\pi_m \in (\delta_L, \delta_H)$. Men's stable marriage premium is $\pi_m \in (\delta_L, \delta_H)$ only when $G_{mH} = G_{wH}(\pi_m)$. When $G_{mH} = G_{wH}$, given men's stable marriage premium π_m, women's stable marriage-payoff differences are $v_{wL} - v_{wl} = s_{LL} - s_{Ll}$, $v_{wH} - v_{wL} = s_{HL} - s_{Ll} - \pi_m$, and $v_{wh} - v_{wl} =$

$s_{Lh} - s_{Ll}$, so

$$\theta_{w1}(\pi_m) = \frac{c_w}{z_{wH} - z_{wL} + s_{HH} - s_{LL} - \pi_m},$$

and

$$\theta_{w2}(\pi_m) = \frac{c_w + (s_{LL} - s_{Ll})}{z_{wH} - z_{wL} + (s_{Lh} - s_{Ll})}.$$

Therefore, both $\theta_{w1}(\pi_m)$ and $\theta_{w2}(\pi_m)$ are increasing when $\pi_m \in (\delta_L, \delta_H)$.

Case 2. Suppose $\delta_h \geq \delta_L$. Explicitly,

$$D_{mH}(\pi_m) = \begin{cases} [G_{w,\succeq L}(\pi_m), 1] & \text{if } \pi_m = \delta_l \\ G_{w,\succeq L}(\pi_m) & \text{if } \pi_m \in (\delta_l, \delta_L) \\ [G_{w,\succeq h}(\pi_m), G_{w,\succeq L}(\pi_m)] & \text{if } \pi_m = \delta_L \\ G_{w,\succeq h}(\pi_m) & \text{if } \pi_m \in (\delta_L, \delta_h) \\ [G_{wH}(\pi_m), G_{w,\succeq h}(\pi_m)] & \text{if } \pi_m = \delta_h \\ G_{wH}(\pi_m) & \text{if } \pi_m \in (\delta_h, \delta_H) \\ [0, G_{wH}(\pi_m)] & \text{if } \pi_m = \delta_H \end{cases}.$$

It suffices to show that (i) $G_{w,\succeq L}(\pi_m)$ is strictly decreasing when $\pi_m \in (\delta_l, \delta_L)$, (ii) $G_{w,\succeq h}(\pi_m)$ is strictly decreasing when $\pi_m \in (\delta_L, \delta_h)$, and (iii) $G_{wH}(\pi_m)$ is strictly decreasing when $\pi_m \in (\delta_h, \delta_H)$.

(i) To show $G_{w,\succeq L}(\pi_m) = 1 - \int_{\theta_{w2}(\pi_m)}^{1} (1-\theta)^2 dF_w(\theta)$ is strictly decreasing when $\pi_m \in (\delta_l, \delta_L)$, it suffices to show $\theta_{w2}(\pi_m)$ is strictly decreasing when $\pi_m \in (\delta_l, \delta_L)$. Men's stable marriage premium can be $\pi_m \in (\delta_l, \delta_L)$ only when $G_{mH} = G_{w,\succeq L}$. When $G_{mH} = G_{w,\succeq L}$, given men's stable marriage premium π_m, women's stable marriage-payoff differences are $v_{wL} - v_{wl} = s_{HL} - s_{Ll} - \pi_m$, $v_{wH} - v_{wL} = s_{HH} - s_{HL}$, and $v_{wh} - v_{wl} = s_{Hh} - s_{Hl} - \pi_m$, so

$$\theta_{w2}(\pi_m) = \frac{c_w + (s_{HL} - s_{Ll} - \pi_m)}{z_{wH} - z_{wL} + (s_{Hh} - s_{Hl} - \pi_m)}.$$

Since $\theta_{w2}(\pi_m) < 1$, $\theta'_{w2}(\pi_m) < 0$ when $\pi_m \in (\delta_l, \delta_L)$.

(ii) To show $G_{w,\succeq h}(\pi_m) = \int_{\theta_{w1}(\pi_m)}^{1} \theta dF_w(\theta) + \int_{\theta_{w2}(\pi_m)}^{1} (1-\theta)\theta dF_w(\theta)$ is strictly decreasing when $\pi_m \in (\delta_L, \delta_h)$, it suffices to show both $\theta_{w1}(\pi_m)$ and $\theta_{w2}(\pi_m)$ are strictly increasing when $\pi_m \in (\delta_h, \delta_L)$. Men's stable marriage payoff can be $\pi_m \in (\delta_h, \delta_L)$ only when $G_{mH} = G_{w,\succeq h}$. When$G_{mH} = G_{w,\succeq h}$, given men's stable marriage premium π_m, women's stable marriage-payoff differences are $v_{wH} - v_{wL} = s_{HH} - s_{LL} - \pi_m$, $v_{wL} - v_{wl} = s_{LL} - s_{Ll}$, and $v_{wh} - v_{wl} = s_{Hh} - s_{Ll} - \pi_m$, so

$$\theta_{w1}(\pi_m) = \frac{c_w}{z_{wH} - z_{wL} + (s_{HH} - s_{LL} - \pi_m)}$$

and

$$\theta_{w2}(\pi_m) = \frac{c_w + (s_{LL} - s_{Ll})}{z_{wH} - z_{wL} + (s_{Hh} - s_{Ll} - \pi_m)}.$$

Therefore, both $\theta_{w1}(\pi_m)$ and $\theta_{w2}(\pi_m)$ are strictly increasing when $\pi_m \in (\delta_L, \delta_h)$.

(iii) To show $G_{wH}(\pi_m) = \int_{\theta_{w1}(\pi_m)}^{1} \theta dF_w(\theta)$ is strictly decreasing when $\pi_m \in (\delta_h, \delta_H)$, it suffices to show $\theta_{w1}(\pi_m)$ is strictly increasing when $\pi_m \in (\delta_h, \delta_L)$. Men's stable marriage premium can be $\pi_m \in (\delta_h, \delta_L)$ only when $G_{mH} = G_{wH}$. When $G_{mH} = G_{wH}$, given men's stable marriage premium π_m, women's stable marriage-payoff difference $v_{wH} - v_{wL} = s_{HH} - s_{LL} - \pi_m$, so

$$\theta_{w1}(\pi_m) = \frac{c_w}{z_{wH} - z_{wL} + s_{HH} - s_{LL} - \pi_m}.$$

Therefore, $\theta_{w1}(\pi_m)$ is strictly decreasing when $\pi_m \in (\delta_h, \delta_L)$.

Case 3. Suppose $\delta_h = \delta_L$. Types are ranked as $H \succ L \sim h \succ l$. Let $\tau_2 := L \sim h$. Explicitly,

$$D_{mH}(\pi_m) = \begin{cases} [G_{w,\succeq\tau_2}(\pi_m), 1] & \text{if } \pi_m = \delta_l \\ G_{w,\succeq\tau_2}(\pi_m) & \text{if } \pi_m \in (\delta_l, \delta_{\tau_2}) \\ [G_{wH}(\pi_m), G_{w,\succeq\tau_2}(\pi_m)] & \text{if } \pi_m = \delta_{\tau_2} \\ G_{wH}(\pi_m) & \text{if } \pi_m \in (\delta_{\tau_2}, \delta_H) \\ [0, G_{wH}(\pi_m)] & \text{if } \pi_m = \delta_H \end{cases}.$$

It remains to show that (i) $G_{w,\succeq\tau_2}(\pi_m)$ is strictly decreasing when $\pi_m \in (\delta_l, \delta_{\tau_2})$, and (ii) $G_{wH}(\pi_m)$ is strictly decreasing when $\pi_m \in (\delta_{\tau_2}, \delta_H)$.

(i) To show $G_{w,\succeq\tau_2}(\pi_m) = 1 - \int_{\theta_{w2}(\pi_m)}^{1} (1-\theta)^2 dF_w(\theta)$ is strictly decreasing when $\pi_m \in (\delta_l, \delta_{\tau_2})$, it suffices to show $\theta_{w2}(\pi_m)$ is strictly decreasing when $\pi_m \in (\delta_l, \delta_{\tau_2})$. Men's stable marriage premium can be $\pi_m \in (\delta_l, \delta_L)$ only when $G_{mH} = G_{w,\succeq\tau_2}$. When $G_{mH} = G_{w,\succeq\tau_2}$, given men's stable marriage premium π_m, women's stable marriage-payoff differences are $v_{wL} - v_{wl} = s_{HL} - s_{Ll} - \pi_m$, $v_{wH} - v_{wL} = s_{HH} - s_{HL}$, and $v_{wh} - v_{wl} = s_{Hh} - s_{Hl} - \pi_m$, so

$$\theta_{w2}(\pi_m) \quad = \quad \frac{c_w + s_{HL} - s_{Ll} - \pi_m}{z_{wH} - z_{wL} + s_{Hh} - s_{Hl} - \pi_m}.$$

Since $\theta_{w2}(\pi_m) < 1$, $\theta_{w2}(\pi_m)$ is strictly decreasing when $\pi_m \in (\delta_l, \delta_{\tau_2})$.

(ii) To show $G_{wH}(\pi_m) = \int_{\theta_{w1}(\pi_m)}^{1} \theta dF_w(\theta)$ is strictly decreasing when $\pi_m \in (\delta_{\tau_2}, \delta_H)$, it suffices to show $\theta_{w1}(\pi_m)$ is strictly increasing when $\pi_m \in (\delta_{\tau_2}, \delta_H)$. Men's stable marriage premium can be π_m only when $G_{mH} = G_{wH}$. When $G_{mH} = G_{wH}$, given men's stable marriage premium π_m, women's stable marriage-payoff difference $v_{wH} - v_{wL} = s_{HH} - s_{LL} - \pi_m$, so

$$\theta_{w1}(\pi_m) = \frac{c_w}{z_{wH} - z_{wL} + s_{HH} - s_{LL} - \pi_m}.$$

Therefore, $\theta_{w1}(\pi_m)$ is strictly increasing when $\pi_m \in (\delta_{\tau_2}, \delta_H)$. *QED*

A.4 Proof of Proposition 1

I first prove the college gender gap. Suppose by way of contradiction that weakly fewer women than men go to college in equilibrium: $1 - F_w(\theta_{w1}^*) \leq 1 - F_m(\theta_m^*)$. First, since $F_m = F_w$ by assumption, $F_w(\theta^*_{w1}) \geq F_m(\theta^*_m)$ implies $\theta^*_{w1} = c_w/(z_{wH} - z_{wL} + v^*_{wH} - v^*_{wL}) \geq \theta^*_m = c_m/(z_{mH} - z_{mL} + v^*_{mH} - v^*_{mL})$. Since $z_{wH} - z_{wL} = z_{mH} - z_{mL}$ by assumption, $v^*_{wH} - v^*_{wL} \leq v^*_{mH} - v^*_{mL}$.

Second, $\theta^*_{w2} > \theta^*_{w1}$, so strictly fewer women than men make a career investment in equilibrium. Since weakly fewer women go to college by our premise and strictly fewer women make a career investment, strictly fewer women than men earn a high income, i.e., $G^*_{wH} + G^*_{wh} < G^*_{mH}$. As a result, there is a positive mass of type-L women marrying high-income men. By pairwise efficiency, $v^*_{wL} = s_{HL} - v^*_{mH}$. Since there is always a positive mass of (H,H) couples, by pairwise efficiency, $v^*_{wH} = s_{HH} - v^*_{mH}$. The two pairwise efficiency conditions together imply $v^*_{wH} - v^*_{wL} = s_{HH} - s_{HL}$. By $s_{HL} = s_{LH}$, $v^*_{wH} - v^*_{wL} = s_{HH} - s_{HL} = s_{HH} - s_{LH} = \delta_H$. Because a positive mass of type-H men marries type-L women in equilibrium, $v^*_{mH} = s_{HL} - v^*_{wL}$. Furthermore, by Pareto efficiency, $v^*_{mL} \geq s_{LL} - v^*_{wL}$. The two conditions together imply $v^*_{mH} - v^*_{mL} \leq s_{HL} - s_{LL}$. Since the surplus is strictly super-modular in incomes, $v^*_{wH} - v^*_{wL} = \delta_H > \delta_L = v^*_{mH} - v_{mL}$.

The two conclusions, $v^*_{wH} - v^*_{wL} \leq v^*_{mH} - v^*_{mL}$ and $v^*_{wH} - v^*_{wL} > v^*_{mH} - v^*_{mL}$, contradict each other. Therefore, there must be strictly more women than men going to college.

I now prove the earnings gender gap. Consider the assumption $G_{mH}(\delta_l) > G_{wH}(\delta_l) + G_{wh}(\delta_l)$. It states that when men's stable marriage premium π_m is δ_l the lowest value possible, mass $G_{mH}(\delta_l)$ of high-income men is strictly greater than the mass $G_{wH}(\delta_l) + G_{wh}(\delta_l)$ of high-income women. That is, even when men have the smallest possible marriage premium $\pi_m = \delta_l = s_{Hl} - s_{Hl}$ and women have the largest possible marriage premium $\pi_w = s_{HH} - s_{HL}$, fewer women will end up with a high income than men. Therefore, the earnings gender gap always holds.

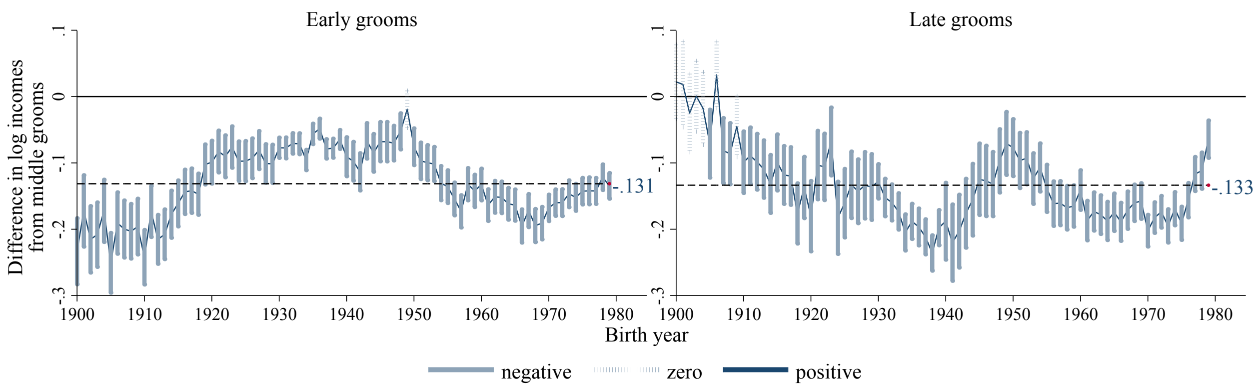
Without the assumption, I can show that there are weakly fewer fertile high-income women than high-income men in equilibrium. Suppose by way of contradiction that there are strictly fewer high-income men than fertile high-income women in equilibrium: $G^*_{mH} < G^*_{wH}$. As a result, low-income men marry type H women with a positive probability: $v^*_{mL} = s_{LH} - v^*_{wL}$. In addition, almost all high-income men marry type H women, so $v^*_{mH} = s_{HH} - v^*_{wL}$. The two conditions together imply $v^*_{mH} - v^*_{mL} = s_{HH} - s_{LH} = \delta_H$. Since low-income men marry both high-income men and low-income men with positive probabilities, $v^*_{wH} - v^*_{wL} = s_{LH} - s_{LL} = \delta_L$, where the second equality follows $s_{HL} = s_{LH}$. Since $v^*_{mH} - v^*_{mL} > v^*_{wH} - v^*_{wL}$, $\theta^*_m > \theta^*_{w1} > \theta^*_{w2}$. Since more men make college investments as well as career investments, there cannot be strictly fewer high-income men than high-income fertile women:

$$G^*_{mH} = \int_{\theta^*_m}^1 p dF_m(p) + \int_{\theta^*_m} p(1-p) dF_m(p) > \int_{\theta^*_{w1}}^1 p dF_w(p) + r \int_{\theta^*_{w2}}^1 p(1-p) dF_w(p) = G^*_{wH},$$

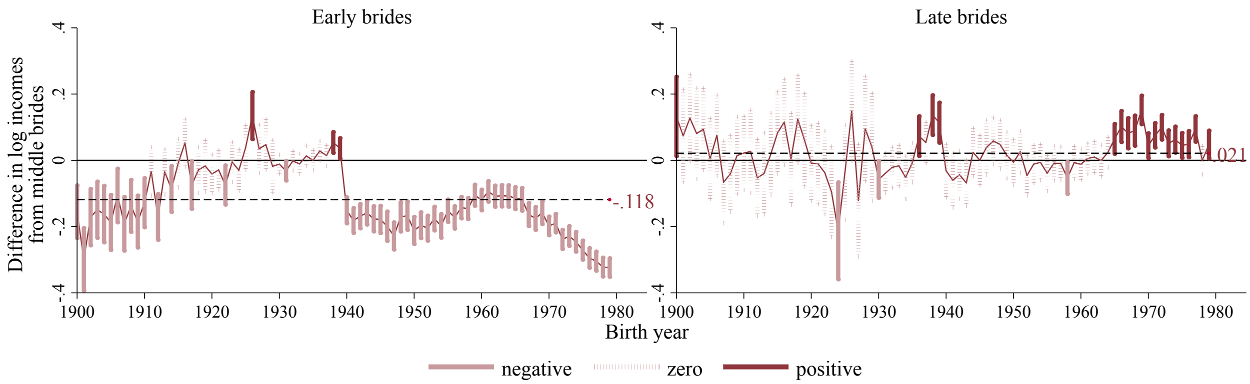
contradicting the premise. *QED*

Figure A1: Evolution of the average income differences between age-at-marriage groups

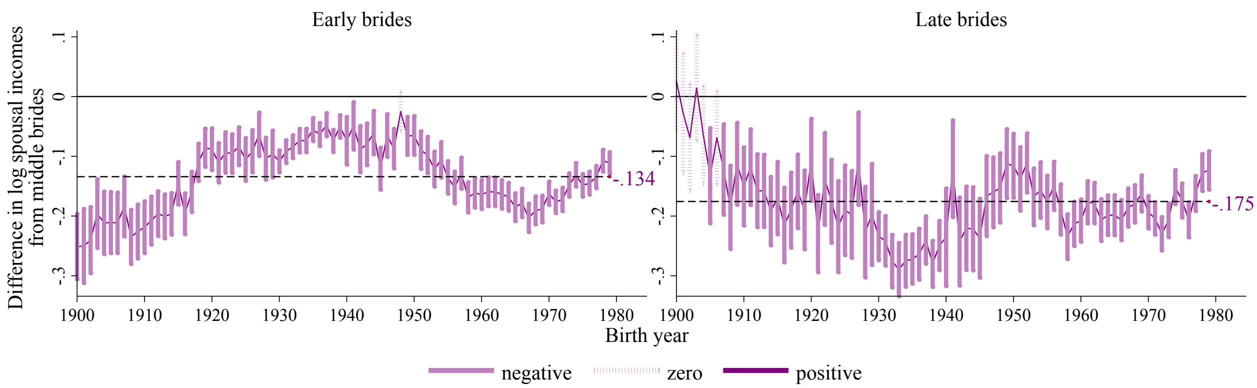
(a) Difference in average log personal midlife total income from middle grooms



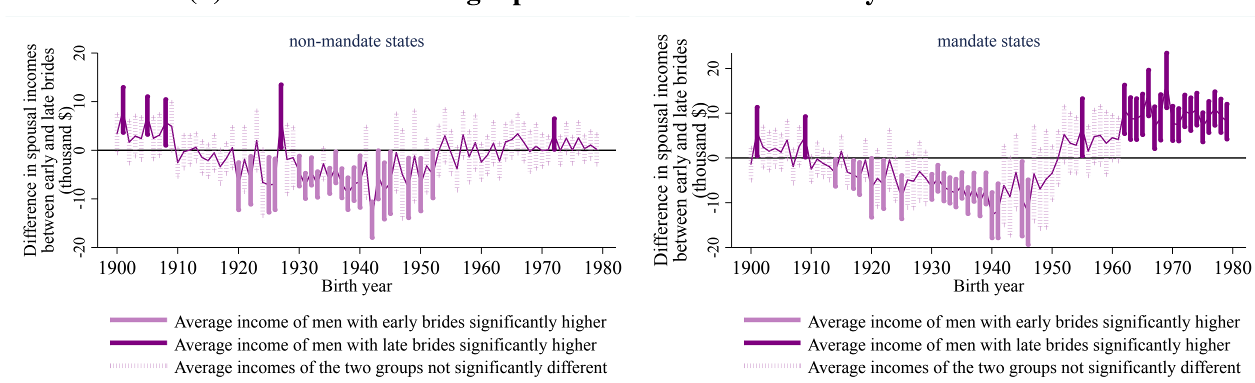
(b) Difference in average log personal midlife total income from middle brides



(c) Difference in average log spousal midlife total income from middle brides



(d) Difference in average spousal incomes between early and late brides



B Supporting the Explanations, Omitted Details

B.1 Empirical Evidence

B.1.1 College and Earnings Gender Gaps

The model predicts that women's marriage premium is higher than men's when more women than men go to college, but is lower than men's when fewer women than men go to college.

Claim 1. Suppose the marriage surpluses s_{HL} and s_{LH} are gender-symmetric and more men than women earn a high income in equilibrium. Women's marriage premium $\pi_w^* = s_{HH} - s_{HL}$ is larger than men's marriage premium $\pi_m^* = s_{HL} - s_{LL}$ in equilibrium.

Previous literature has presented evidence consistent with this prediction (Chiappori et al., 2009, 2017). However, in the previous papers, each individual's marriage type is education or age rather than income or fertility. Although income is positively correlated with education and fertility is negatively correlated with age, these papers do not provide direct evidence for our predictions. I directly test this key implication with data.

Figure B1 shows estimated marriage premiums from 1960 to 2015. The estimation is consistent with our prediction: women's marriage premium (i) was smaller than men's in 1960, 1970, and 1980, when fewer women than men, ages 35-39, graduated from college; and (ii) was greater than men's in 2010 and 2015, when more women than men, ages 35-39, graduated from college.[18] We adopt the technique developed by Choo and Siow (2006) to exactly identify the marriage surplus function, and compute the marriage premiums from the estimated marriage surplus function. We detail the estimation procedure below.

We only need to estimate s_{HH}, s_{HL}, and s_{LL} to compute the marriage premiums, because, according to claim 1, men's marriage premium is $\pi_m^* = s_{HL} - s_{LL}$, and women's marriage premium is $\pi_w^* = s_{HH} - s_{HL}$. We modify our matching model to adopt the technique of Choo and Siow (2006). The marriage payoff of a type-τ_m man i married to a type-τ_w woman is

$$v^i_{m\tau_m\tau_w} = z^m_{\tau_m\tau_w} - t_{\tau_m\tau_w} + \varepsilon^i_{\tau_m\tau_w},$$

where $z^m_{\tau_m\tau_w}$ is the systematic gross return to a type-τ_m man married to a type-τ_w woman, $t_{\tau_m\tau_w}$ is the transfer from a type-τ_m man to a type-τ_w woman, and $\varepsilon^i_{\tau_m\tau_w}$ is an independently and identically distributed random variable with a type I extreme-value distribution, i.e., $F(\varepsilon) = \exp[-\exp(-\varepsilon)]$. The marriage payoff of a type-τ_w woman j married to a type-τ_m man is

$$v^j_{w\tau_w\tau_m} = z^w_{\tau_m\tau_w} + t_{\tau_m\tau_w} + \varepsilon^j_{\tau_m\tau_w},$$

[18] We are not able to produce estimates for 1990 and 2000, because age at marriage, the information needed to construct the marriage market, was neither reported nor inferable in the censuses in these years. We are also not able to produce estimates for years prior to 1960, because spousal income – the information needed to construct marriage types and to compute the number of marriages between different marriage types – was not reported in the censuses.

where $z^w_{\tau_m\tau_w}$ is the systematic return to a type-τ_w woman married to a type-τ_m man, and $\varepsilon^j_{\tau_m\tau_w}$ is an i.i.d. random variable with a T1EV distribution. The payoff to man i who remains unmarried is

$$v^i_{\tau_m\emptyset} = z^m_{\tau_m\emptyset} + \varepsilon^i_{\tau_m\emptyset},$$

where $\varepsilon^i_{\tau_m\emptyset}$ is also an i.i.d. random variable with a T1EV distribution. The systematic marriage surplus for a type-τ_m man married to a type-τ_w woman is $s^m_{\tau_m\tau_w} = z^m_{\tau_m\tau_w} - z^m_{\tau_m\emptyset}$. Similarly, the systematic marriage surplus for a type-τ_w woman married to a type-τ_m man is $s^w_{\tau_m\tau_w} = z^w_{\tau_m\tau_w} - z^w_{\emptyset\tau_w}$.

Therefore, the total systematic marriage surplus of a type-τ_m man and a type-τ_w woman is $s_{\tau_m\tau_w} = s^m_{\tau_m\tau_w} + s^w_{\tau_m\tau_w}$. Following Choo and Siow (2006),

$$\widehat{s}_{\tau_m\tau_w} = 2\ln\left[\frac{\widehat{G}_{\tau_m\tau_w}}{\sqrt{\widehat{G}_{\tau_m\emptyset}\widehat{G}_{\emptyset\tau_w}}}\right],$$

where $\widehat{G}_{\tau_m\tau_w}$ is the estimated measure of marriages between type-τ_m men and type-τ_w women, $\widehat{G}_{\tau_m\emptyset}$ is the estimated measure of unmarried type-τ_m men, and $\widehat{G}_{\emptyset\tau_w}$ is the estimated measure of unmarried type-τ_w women. By claim 1, point estimates of the marriage premiums are $\widehat{\pi}^*_{mH} = \widehat{s}_{HL} - \widehat{s}_{LL}$ and $\widehat{\pi}^*_{wH} = \widehat{s}_{HH} - \widehat{s}_{HL}$. Standard errors of the marriage premiums are obtained from simulated measures of marriage characteristics.

What remains is to specify the marriage market and to assign each individual fertility and income marriage characteristics. We include in the marriage market all never-married individuals between ages 16-39 who were not in school, and all heterosexual couples who were both between ages 16-39 and who had both married for the first time within the two years under consideration. We categorize an agent as a high-income type if he or she earns more than the median personal labor income of the college graduates of the same age, and as a low-income agent otherwise. We treat men between ages 16-39 and women between ages 16-29 as fertile, and we treat women between ages 30-39 as less fertile.

The model also shows the factors that contribute to the rise of women's college enrollment and earnings over time.

Claim 2. Suppose more men than women earn a high income in equilibrium before and after the changes in the primitives of the model. Women's college investment rate and average income in equilibrium both increase if any or any combination of the following events happens: (i) women's investment cost c_w decreases; (ii) women's labor-market opportunities F_w (first-order stochastically) increase; (iii) women's income premium $z_{wH} - z_{wL}$ increases; and (iv) the surplus difference $s_{HH} - s_{HL}$ (women's equilibrium marriage premium π^*_w) increases.

Existing literature (cited in the introduction) has thoroughly studied how monetary and psychic college investment costs, labor-market opportunities, and income premium for women (as well as for men) have evolved over the past decades, and how these changes have contributed to the changes in the college and earnings gender gaps. Predictions (i)-(iii) are consistent with previous findings and do not add any new theoretical insights. The change in the marriage premium is relatively less studied. In equilibrium, women's marriage premium equals the difference between (i) the marriage surplus when a high-income man marries a high-income woman and (ii) the marriage surplus when he marries a low-income woman. Its increase is associated with technological and social changes that affect intrahousehold consumption and time-allocation decisions. Technological progress freed women from some household activities and made women with high earning abilities more valuable in the labor market as well as in the marriage market (Greenwood et al., 2014, 2016). In addition, an increasing focus on human capital of children also made highly educated and highly skilled women more valuable in the marriage market (Chiappori et al., 2009, 2017). We will empirically confirm that the marriage premium for women has indeed increased and has gradually surpassed the marriage premium for men over the last several decades.

B.1.2 Relationship between Age at Marriage and Income for Men

I present a piece of evidence that can only be explained by the labor-market shocks. If labor-market shocks indeed affect marriage timing as the model suggests, then we would expect that, compared to college-educated middle grooms, college-educated late grooms should have (i) a lower average income when they have just finished college and (ii) a higher income growth rate following college. Figure B2 shows exactly such patterns for the three cohorts tracked by National Longitudinal Surveys of Youth (NLSY). Four-year-college-educated late grooms, the men who received exactly four years of postsecondary education and first married between ages 30 and 39 (i) earned a lower average income than four-year-college-educated middle grooms in their twenties, and (ii) caught up and earned almost as much as four-year-college-educated middle grooms in their thirties.

B.1.3 Relationship between Age at Marriage and Income for Women

The model predicts that, because they receive an adverse labor-market shock and consequently make a career investment, compared to the college-educated women who marry before age thirty, the college-educated women who marry after age thirty have a lower post-college income initially and a steeper income gain afterwards, on average. Data matches this prediction: four-year-college-educated late brides had a lower average income right out of college but quickly caught up with and later on surpassed four-year-college-educated middle brides (figure B3).

B.1.4 Relationship between Age at Marriage and Spousal Income for Women

Figures B4a and B4b show that early brides' marital outcome deteriorated and late brides' marital outcome improved in mandate states, where the marital outcome is measured spousal income rank and spousal education rank. It is worth mentioning that the mandates did not significantly improve the average education and total income rank of late brides in those states (figures B4c and B4d), consistent with previous studies (Buckles, 2007). The result is also consistent with our

Figure B1: Marriage premiums from 1960 to 2015

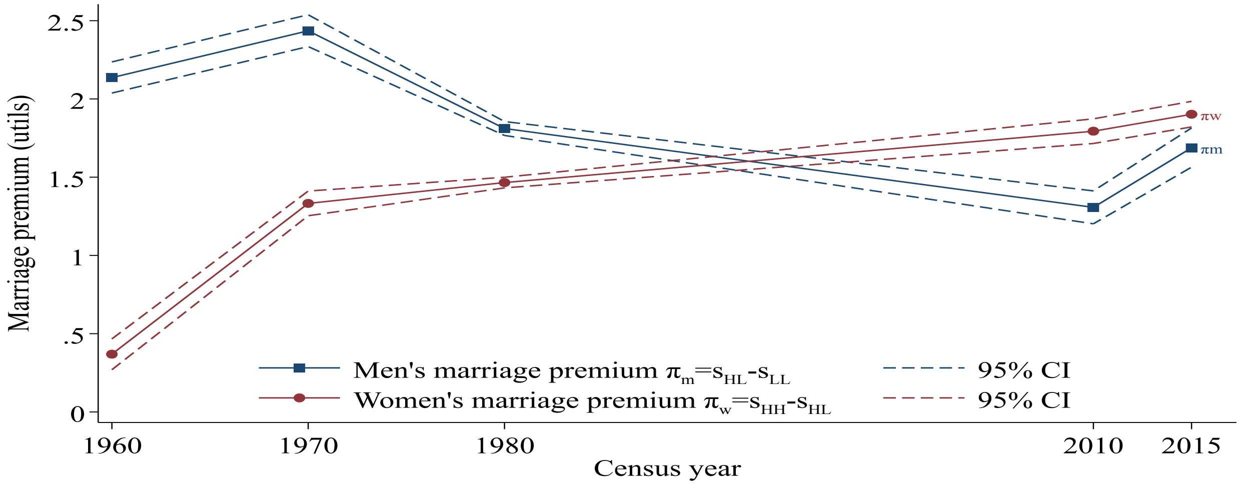


Figure B2: Income ratio of college late grooms to college middle grooms, NLSY

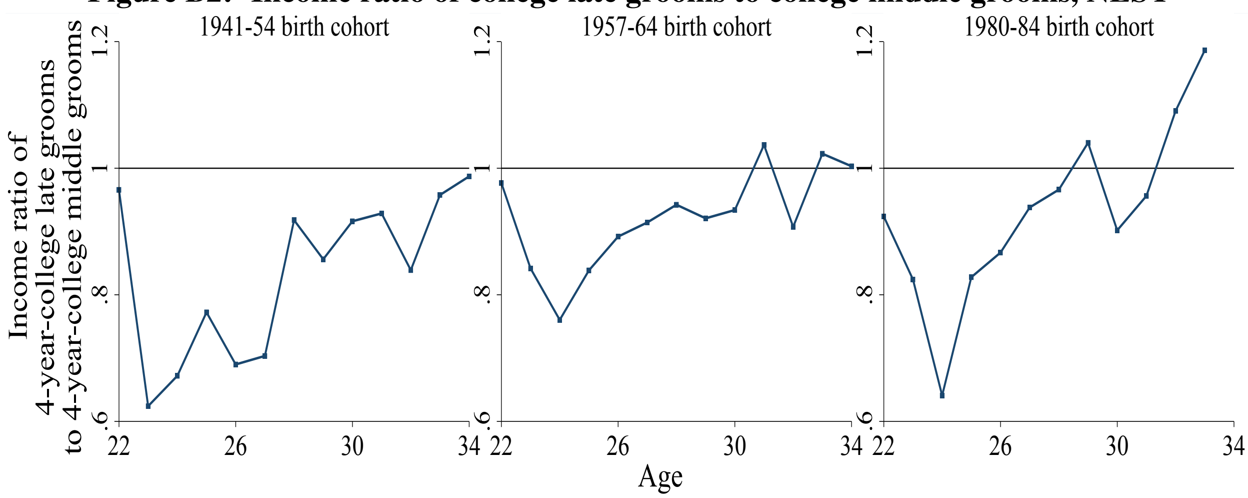
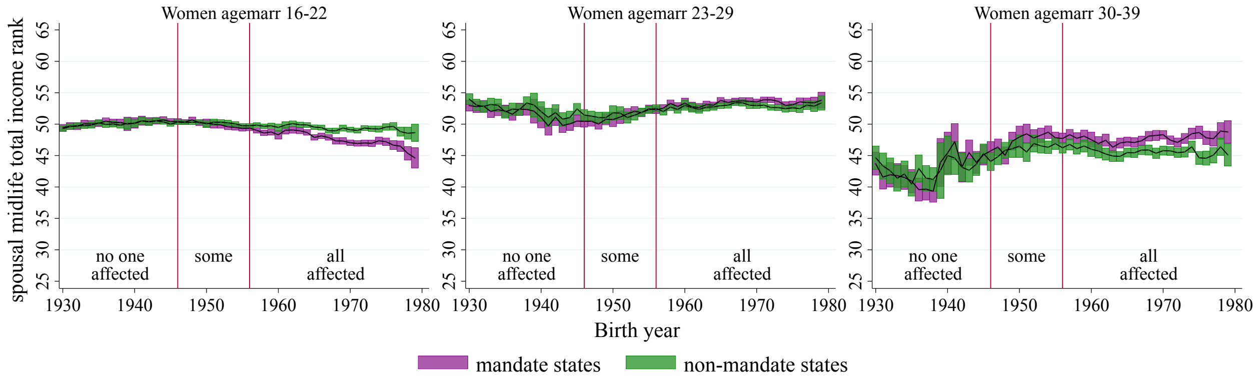


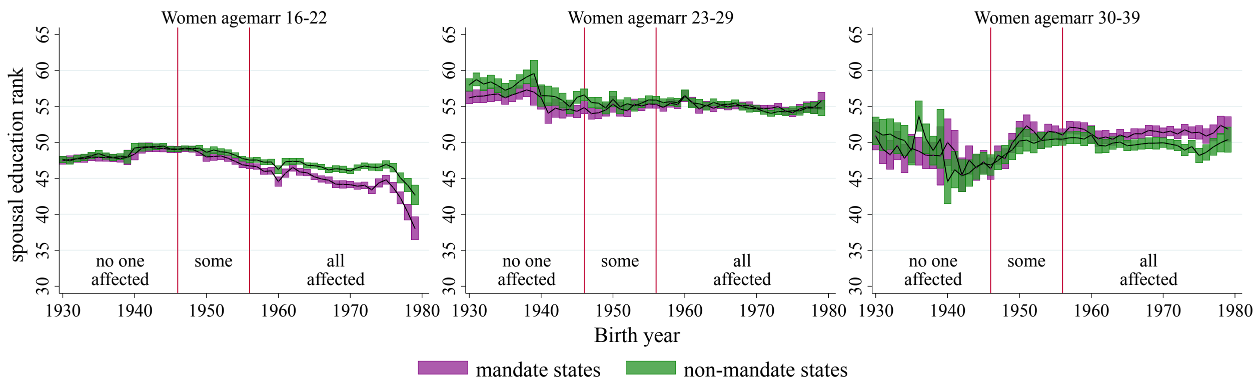
Figure B3: Income ratio of college late brides to college middle brides, NLSY

Figure B4: Income and education in mandate versus non-mandate states, 1930-1979

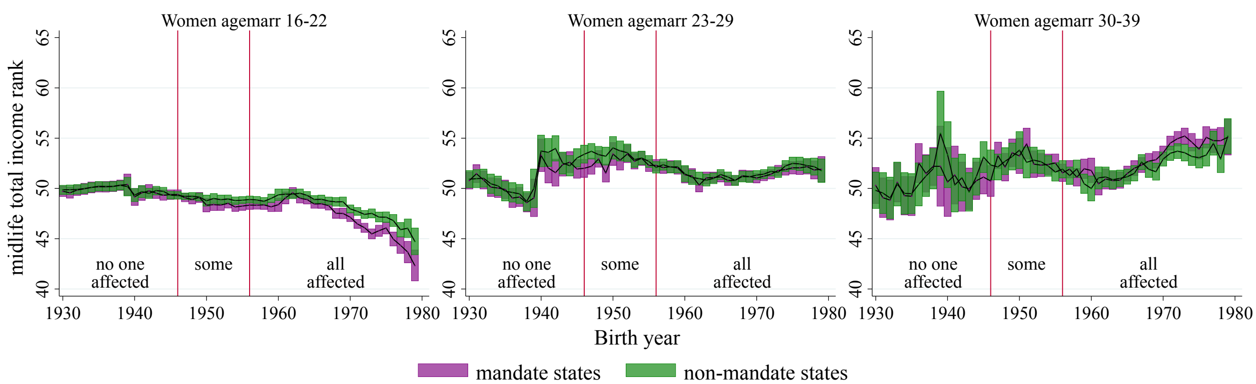
(a) Spousal income percentile rank by birth year



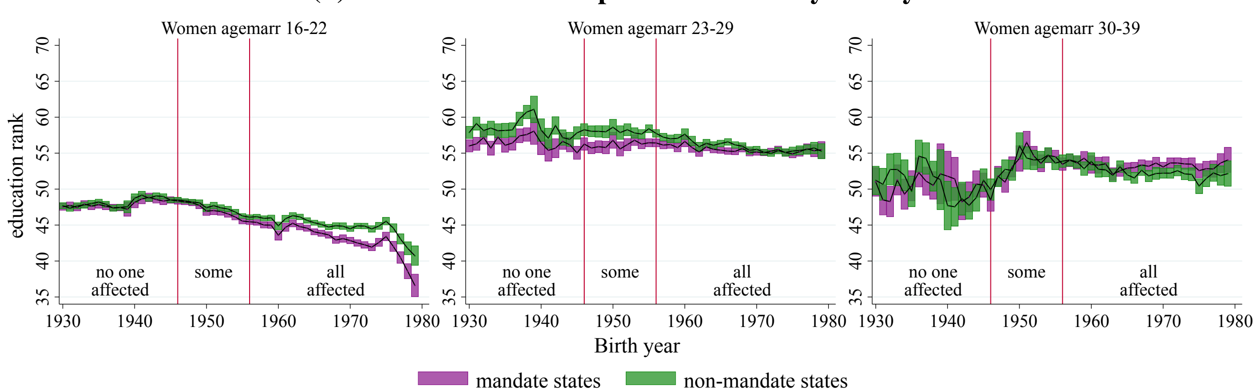
(b) Spousal education percentile rank by birth year



(c) Personal total income percentile rank by birth year



(d) Personal education percentile rank by birth year



theory: more women may make a career investment following the relaxation of the fertility constraint, but since these women have intermediate abilities and may have a relatively low chance of receiving a high income, late brides' average income may not increase.[19]

B.2 Calibration

B.2.1 Benchmark Model

Suppose the ability distributions for men and for women are beta distributions with parameters (α_m, β_w) and (α_w, β_w), respectively. Since the model predicts that those who marry in the first period are low-income earners, we use the average labor income of men and women who first married between ages 16 and 22 to estimate y_{mL} and y_{wL}, respectively. Since the model predicts that men who marry in the second period are high-income earners, we use the average labor income of men who married between ages 23 and 29 to estimate y_{mH}. We use the average labor income of the unmarried women to estimate y_{wH}. Total investment costs c_m and c_w are two years of low incomes. We use two years because the college-educated on average marry two years later than the non-college-educated.[20] Annual investment cost is total investment cost divided by 40. The marriage surplus in monetary terms is k times the marriage surplus in utils estimated in section B.1.1.

Twelve moments are targeted: observed percentage and average midlife labor income of early, middle, and late grooms (denoted by $\widehat{G}_{ma}$ and $\widehat{y}_{ma}$, $a \in \{1,2,3\}$), and of early, middle, and late brides ($\widehat{G}_{wa}$ and $\widehat{y}_{wa}$, $a \in \{1,2,3\}$). Define the penalty function with five arguments, α_m, α_w, β_m, β_w, and k and find the parameters to minimize it:

$$D_1(\alpha_m, \alpha_w, \beta_m, \beta_w, k) = \sqrt{\sum_{i \in \{m,w\}} \left[\sum_{a=1}^{3} \left| (G_{ia} - \widehat{G}_{ia})/\widehat{G}_{ia} \right|^2 + \sum_{a=1}^{3} \left| (y_{ia} - \widehat{y}_{ia})/\widehat{y}_{ia} \right|^2 \right]}.$$

We find the parameters for the 1930s and the 1960s birth cohorts, respectively. Table B1 shows the fit of the model. The calibration matches each targeted marriage-age distribution within 0.7 percent, and except for the average income of late grooms born in the 1930s and that of late brides in the 1960s, the calibration matches each of the targeted average incomes within 5 percent. The non-targeted average spousal incomes are also matched fairly well. Table B2 shows the estimated parameters of the model. Labor-market opportunities are estimated to be much greater for women born in the 1960s and slightly lower for men born in the 1960s (figure B5), consistent with the results in Coles and Francesconi (2018).

[19] In contrast, the improvements in Israeli women's education and earnings were more pronounced (Gershoni and Low, 2017), partially because Israelis go to college later due to mandatory military services.

[20] Coles and Francesconi (2017) use four years of minimum wages.

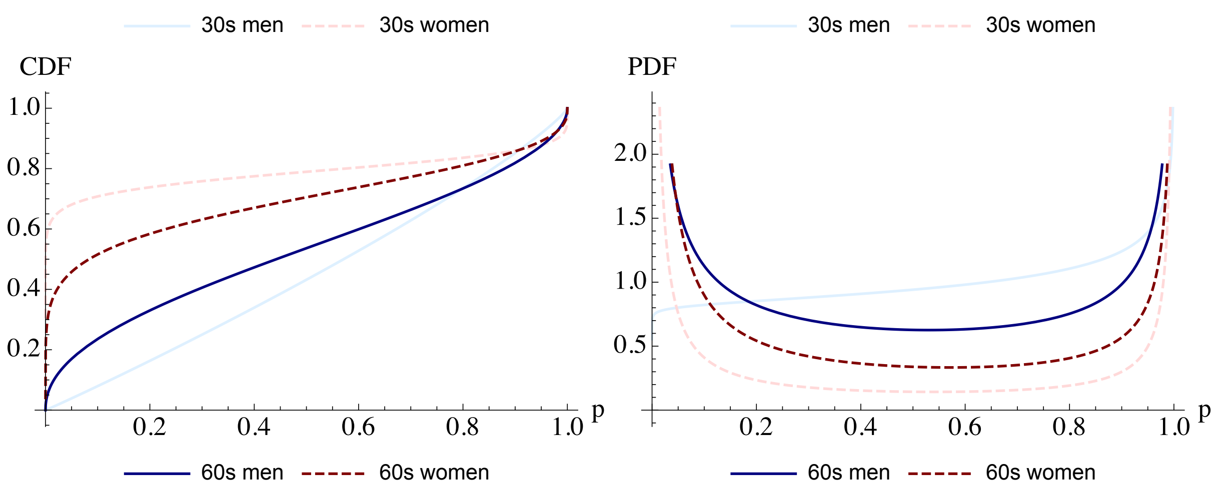
Table B1: Fit of the benchmark model

moments	30s target	30s model	difference	60s target	60s model	difference
G_{m1}	0.48476	0.484927	0.0346%	0.30756	0.307632	0.0233%
G_{m2}	0.411344	0.411096	-0.0605%	0.451633	0.451472	-0.0356%
G_{m3}	0.103896	0.103977	0.0782%	0.240807	0.240896	0.037%
G_{w1}	0.740591	0.740641	0.00673%	0.4494	0.450621	0.272%
G_{w2}	0.206928	0.206863	-0.0314%	0.381204	0.378867	-0.613%
G_{w3}	0.0524809	0.0524961	0.0289%	0.169396	0.170512	0.659%
y_{m1}	40209.7	40209.7	0.%	44571.6	44571.6	0.%
y_{m2}	43820.8	43820.8	0.%	56434.2	56434.2	0.%
y_{m3}	37442.	42783.4	14.3%	48376.5	49895.	3.14%
y_{w1}	12049.	12049.	0.%	20091.	20091.	0.%
y_{w2}	12457.2	12066.3	-3.14%	24627.8	24216.	-1.67%
y_{w3}	12886.1	13445.7	4.34%	26080.1	28028.3	7.47%
average			1.83%			1.16%
x_{w1}	41269.2	41445.5	0.427%	47016.2	48873.3	3.95%
x_{w2}	45269.5	43806.4	-3.23%	61434.7	54923.9	-10.6%
x_{w3}	35537.5	41977.3	18.1%	53644.3	49131.2	-8.41%

Table B2: Estimated parameters of the benchmark model

	1930s	1960s		1930s	1960s
F_m	Beta(1.02,0.837)	Beta(0.462,0.523)	s_{HH}	\$43.8148	\$5782.23
F_w	Beta(0.0535,0.196)	Beta(0.165,0.373)	s_{HL}	\$39.2241	\$4296.6
c_m	\$2010.48	\$2228.58	s_{Hh}	\$14.6042	\$3370.09
c_w	\$602.452	\$1004.55	s_{Hl}	\$21.0646	\$2055.66
y_{mH}	\$43820.8	\$56434.2	s_{LH}	\$28.4427	\$3604.2
y_{mL}	\$40209.7	\$44571.6	s_{LL}	\$32.6076	\$3302.76
y_{wH}	\$14902.3	\$40741.3	s_{Lh}	\$0.	\$0.
y_{wL}	\$12049.	\$20091.	s_{Ll}	\$14.6112	\$1750.35

Figure B5: Estimated CDF and PDF of abilities in the benchmark model



B.2.2 Extended Model

Not everyone who decides to marry can get married right away, and not everyone who goes to college waits to marry after college. To separate college attendance and marriage timing, I extend the model to allow college men and women to marry in the first period and noncollege men and women to marry in the second and third periods.

The ability distributions are beta distributions as in the benchmark model. We use the average incomes of noncollege men and women as low incomes y_{mL} and y_{wL}, respectively, and the average incomes of college men and women as high incomes y_{mH} and y_{wH}, respectively. The total investment cost is the opportunity cost in the form of two years of low incomes. The annual investment cost is the total investment cost divided by 40. The surplus in monetary terms is again k times the surplus in utils estimated in section B.1.1. Now, frictions. First, not all noncollege men and women marry between ages 16 and 22. The actual probabilities that they married after age 22 are taken from the data. Let h_{iNa}, $i \in \{m,w\}$, $a \in \{1,2\}$, denote the hazard rate of a noncollege man or a noncollege woman marrying in period a. Let h_{iC2} denote the hazard rate of a college man or a college woman marrying between ages 23 and 29. Second, not all college men and college women delay marriage until after college: let h_{iC1}, $i \in \{m,w\}$, denote the probability that a college man or a college woman marries between ages 16 and 22.

We target seventeen moments: the college enrollment rates of men and women (denoted by G_{mC} and G_{wC}, respectively), the average incomes of men who married early, middle, and late brides (denoted by x_{w1}, x_{w2}, and x_{w3}, respectively), as well as the twelve moments targeted in the benchmark model.

To estimate the seven parameters $(\alpha_m, \alpha_w, \beta_m, \beta_w, k, \mu_m, \mu_w)$, we define the penalty function

$$D_2 = \sqrt{\sum_{i\in\{m,w\}} \left[\left| \frac{G_{iC} - \widehat{G}_{iC}}{\widehat{G}_{i,\text{col}}} \right|^2 + \sum_{a=1}^{3} \left| \frac{G_{ia} - \widehat{G}_{ia}}{\widehat{G}_{ia}} \right|^2 + \sum_{a=1}^{3} \left| \frac{y_{ia} - \widehat{y}_{ia}}{\widehat{y}_{ia}} \right|^2 \right] + \sum_{a=1}^{3} \left| \frac{x_{wa} - \widehat{x}_{wa}}{\widehat{x}_{wa}} \right|^2}.$$

We find the parameters to minimize the penalty.

We test the performance of the extended model for the 1930s and 1960s birth cohorts, too. Table B3 shows how well the model matches the data. The average error between targeted and calibrated moments is 1.71 percent for the 1930s birth cohort, and is 1.51 percent for the 1960s birth cohort. Table B4 shows the model's calibrated parameters.

We can examine the relative importance of labor-market shocks to marriage-market frictions in influencing marriage timing.

For the 1930s birth cohort, there was an estimated 17.1 percent chance that college-educated women who decided to enter the marriage market before age 30 involuntarily delayed their marriage until after age 30, slightly higher than the 21.2 percent chance that noncollege women involuntarily delayed their marriage until after age 30. Among the men and women who married

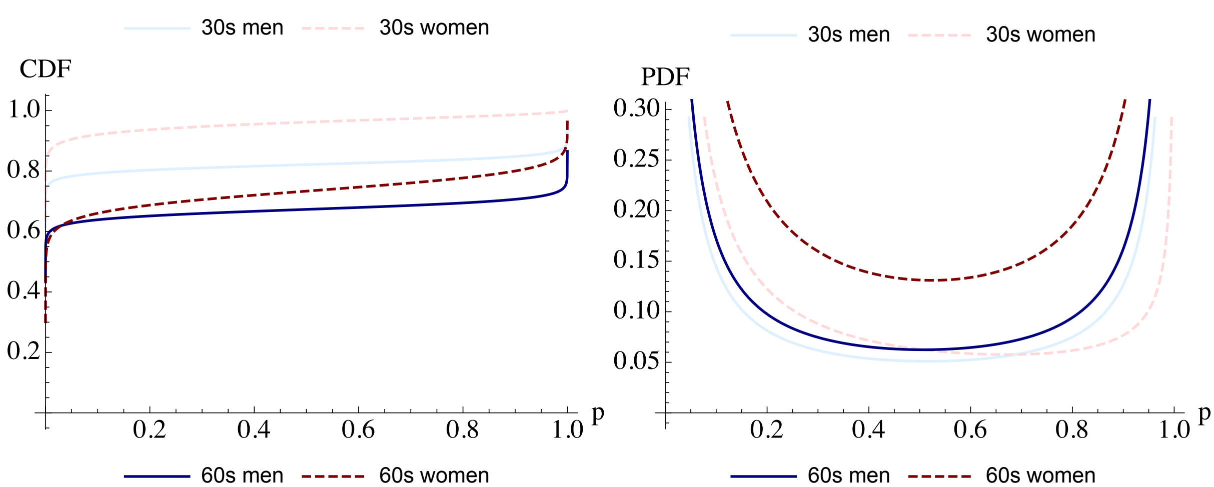
Table B3: Fit of the extended model

moments	30s target	30s model	difference	60s target	60s model	difference
G_{m1}	0.48476	0.484451	-0.0637%	0.30756	0.307372	-0.0613%
G_{m2}	0.411344	0.412559	0.295%	0.451633	0.452309	0.15%
G_{m3}	0.103896	0.102989	-0.872%	0.240807	0.24032	-0.202%
G_{w1}	0.740591	0.740591	0.000051%	0.4494	0.449534	0.0299%
G_{w2}	0.206928	0.206847	-0.0393%	0.381204	0.380081	-0.295%
G_{w3}	0.0524809	0.0525618	0.154%	0.169396	0.170385	0.584%
$G_{m,\mathrm{col}}$	0.218733	0.220363	0.745%	0.379722	0.380819	0.289%
$G_{w,\mathrm{col}}$	0.119257	0.119255	-0.00131%	0.390058	0.389479	-0.148%
y_{m1}	40209.7	39603.7	-1.51%	44571.6	44730.5	0.357%
y_{m2}	43820.8	43915.8	0.217%	56434.2	56524.6	0.16%
y_{m3}	37442.	38350.9	2.43%	48376.5	48589.3	0.44%
y_{w1}	12049.	11696.3	-2.93%	20091.	20510.	2.09%
y_{w2}	12457.2	12739.2	2.26%	24627.8	25169.9	2.2%
y_{w3}	12886.1	12421.	-3.61%	26080.1	24207.1	-7.18%
x_{w1}	41269.2	41155.8	-0.275%	46138.3	47051.6	1.98%
x_{w2}	45269.5	42290.6	-6.58%	58701.2	55594.8	-5.29%
x_{w3}	35537.5	38066.9	7.12%	48666.8	50699.8	4.18%
average	->	->	1.71%	->	->	1.51%

Table B4: Estimated parameters of the extended model

	1930s	1960s		1930s	1960s
F_m	Beta(0.0165,0.0757)	Beta(0.0244,0.0501)	y_{mH}	\$62730.8	\$84142.6
F_w	Beta(0.0236,0.519)	Beta(0.0507,0.138)	y_{mL}	\$36320.4	\$34021.1
c_m	\$1816.02	\$1701.05	y_{wH}	\$31524.1	\$40741.3
c_w	\$553.415	\$798.058	y_{wL}	\$11068.3	\$15961.2
s_{HH}	\$135315.	\$40212.5	h_{mC1}	0.336696	0.200993
s_{HL}	\$121137.	\$29880.7	h_{mC2}	0.999999	0.777833
s_{Hh}	\$45102.6	\$23437.3	h_{mN1}	0.526214	0.372798
s_{Hl}	\$65054.6	\$14296.1	h_{mN2}	0.795684	0.641768
s_{LH}	\$87840.5	\$25065.4	h_{wC1}	0.527418	0.307963
s_{LL}	\$100703.	\$22969.	h_{wC2}	0.829379	0.761695
s_{Lh}	\$0.	\$0.	h_{wN1}	0.769455	0.539849
s_{Ll}	\$45124.3	\$12172.8	h_{wN2}	0.788497	0.675267

Figure B6: Estimated CDF and PDF of abilities in the extended model



between ages 30 and 39, among the college-educated, essentially all men delayed marriages due to labor-market shocks, and all women delayed marriages due to marriage-market frictions (consistent with the fact that a tiny portion of women in this cohort chose to make a career investment).

For the 1960s birth cohort, the chance for a college-educated man who decided to enter the marriage market between ages 23 and 29 not being able to marry before age 30 was 22.2 percent, and the chance that a college-educated woman who decided to enter the marriage market between ages 23 and 29 not being able to marry before age 30 was 23.8 percent. We find that 42.7 percent of college-educated men and 24.6 percent of college-educated women delayed their marriages due to labor-market shocks (and the rest delayed due to marriage-market frictions).

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